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Development of quasi-isodynamic stellarators

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Abstract

Theoretical stellarator research from MHD-stable stellarators via quasi-helically symmetric ones to Wendelstein 7-X, quasi-axisymmetric tokamaks and quasi-isodynamic stellarators is sketched. Research strategy, computational aspects and various favorable properties are emphasized. The results found, but only together with the completion of according experimental devices and their scientific exploitation, may form a basis for selecting the confinement geometry most viable for fusion.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The subject of toroidal magnetic confinement is only a small part of this special issue of Plasma Physics and Controlled Fusion so that a brief presentation of the confinement system discussed in this paper, a stellarator, see figure 1 (right), may be appropriate. Magnet coils enclose a toroidal domain in which they create a magnetic field. Fieldlines encircle this domain toroidally and, simultaneously, poloidally and so form magnetic surfaces which, in turn, fill the domain inside an outer surface with a set of nested surfaces, not visible in figure 1. Distinct from the tokamak situation, the magnetic field is mainly provided by the external coils. In view of the goal, fusion by toroidal magnetic confinement, the trajectories of charged particles in this magnetic field should be confined.

2. Exploration of stellarator configurational space

The path of exploration of stellarator configurational space, starting in 1979 and leading to quasi-isodynamic stellarators, is sketched. The general strategy was long-term risky basic research, proceeding independently of prevalent conjectures. The decisions on the theoretical research to be undertaken were, on the one hand, goal-oriented, stellarator configurations viable for application to fusion, and, on the other hand, constrained by computational theory and computing power available at a given time. By way of example, the risk that configurations could possibly be discovered for which feasible coils would not exist was accepted until 1986 [1].
After several years of analytic work, starting from Mercier’s expansion about the magnetic axis of a stellarator, a promising MHD stability problem had emerged which needed a computational optimization in a four-dimensional configurational space comprising three geometrical variables of the configuration and the plasma pressure gradient. It resulted in a stability limit estimate of an average \( \beta \) of 10% for a configuration with 10 periods, aspect ratio 20 and a geometry of flux surfaces as shown in figure 1 (left) along one period, vertical elliptical cross-section at the point of strongest curvature with triangle-pointing-outward shaped boundary, horizontal elliptical cross-section at the point of weakest curvature with similar-triangle-shaped boundary.

This result was sufficiently promising to trigger a greater computational effort which, of course, had its own specific needs and rules. With the background of the results of 1979 and the pioneering equilibrium tools of Garabedian et al [3] and Hirshman and Lee [4], less questionable results in the form of nonlinear equilibria with smooth boundaries, see figure 2 (left, top), could be established [5]: stability criteria could be satisfied for \( \langle \beta \rangle \approx 0.05 \). The essential computational point was that relatively large steps could be taken in a ten-dimensional configurational space parametrizing the geometry of the boundary. Beyond its own relevance it triggered another computational discovery: the approach to look for a high stable \( \beta \) value led to a significant reduction of the toroidal Fourier component of the strength of B in magnetic coordinates [6], i.e. Boozer coordinates. The reason is that it is this Fourier component that drives the parallel current density known as Pfirsch–Schlüter current density, which, in turn, drives MHD instability. Also, the spectrum of the field strength showed the dominance of the helical curvature as compared with all higher Fourier components. So, it trigged the search for equilibria with only helical components of the same helicity which would result in helical symmetry in magnetic coordinates of the strength of the magnetic field.

The result [7], see figure 2 (left, bottom), shows the dominance of helical curvature by more than an order of magnitude, see figure 2 (right). The Fourier component indicating toroidal curvature is so small that it is even not among the small ones indicated. This structure of the field strength in magnetic coordinates was named quasi-symmetry for two reasons: other constituents of the metric, e.g. the orientation of the flux surfaces relative to the magnetic axis do not exhibit this structure; the second reason is that the second largest component, which is a component of different helicity, could not be eliminated. As was later shown by Garren and Boozer [8], this is indeed impossible so that this symmetry can be realized only in very good approximation but not exactly.

The physical significance of quasi-helical symmetry becomes more obvious by inspecting the topography of the magnetic field strength on a magnetic surface, as seen in figure 3 (left) for a 4-period realization as was built in Madison, the HSX, which, e.g., indicated the elimination of
Figure 2. Top left: cross-sections of magnetic surfaces along half a period of a stellarator found to be stable with respect to stability criteria at $\langle \beta \rangle \approx 0.05$ [5]. Bottom left: cross-sections of magnetic surfaces along half a period of a quasi-helically symmetric stellarator with 6 periods and aspect ratio about 11 [7]. Right: magnetic field strength structure of this stellarator [7].

of direct orbit loss in quasi-helical symmetry [9]. The helical structure is dominant, the effect of toroidal closure nearly invisible, so that the drift of charged particles within the helical valley of the field strength will be helical as seen in figure 3 (right), a situation which had been deemed impossible in toroidal stellarators. Three simple possibilities can be imagined for such a valley to close on itself within one period of a configuration: as is the case in figure 3, satisfying the periodicity condition by running helix-like, by running toroidally, this is the case of quasi-axisymmetry, or by running poloidally, this would be the case of quasi-poloidal symmetry. So, different lines have developed. The realization of quasi-helical symmetry, of course, triggered the computational search for quasi-axisymmetry, because this allows introduction of external rotational transform into the tokamak concept [10]. In view of the problems with disruptions and current drive this has a significant potential of augmenting the tokamak concept. Projects with two periods (CHS-qa) as shown in figure 4 (left) and 3 periods (NCSX) have been developed. The third symmetry, for which the field strength would not depend on the poloidal magnetic variable, does not exist in any approximation, because in a place where the plasma column is curved the field on the outside is necessarily smaller than on the inside. This problem was solved in connection with Wendelstein 7-X, the project under construction in IPP, see figure 1 (right).

With the discovery of quasi-helical symmetry and the wish for a project succeeding the experiment Wendelstein 7-AS—a stellarator realized with modular coils and reducing the parallel current density first predicted by Pfirsch and Schlüter—the most obvious choice would have been quasi-helical symmetry for this successor, in particular, since the European fusion community would not accept a stellarator design without acceptable $\alpha$-particle confinement in a fusion-size device. The reason for the further research effort then undertaken is the bootstrap current generally occurring in any toroidal device.
Figure 3. Left: perspective view of a magnetic surface of a 4-period, aspect ratio about 8, quasi-helically symmetric stellarator with the field strength on this surface color coded. Right: trajectory of a reflected particle in this stellarator.

Figure 4. Left: side view of a magnetic surface of a 2-period quasi-axisymmetric stellarator with the field strength on this surface color coded. Right: magnetic surfaces and drift surfaces of barely passing particles in magnetic coordinates for quasi-axisymmetry, quasi-helical symmetry and Wendelstein 7-X from left to right.

In figure 4 (right), three characteristic situations for the bootstrap current are illustrated. Cross-sections of magnetic surfaces are shown in magnetic coordinates, so that they become circles, in color drift surfaces of barely passing particles, parallel and antiparallel to the magnetic field, which—together with density and temperature gradients across the magnetic surfaces—lead to the bs current: in quasi-axisymmetry increasing the external rotational transform, which is one of the virtues of quasi-axisymmetry; in quasi-helical symmetry, because of the reversed deviations from the surfaces, decreasing the rotational transform. So, it could be conjectured that an appropriate mixture of quasi-axisymmetry and quasi-helical symmetry should allow a vanishing bs current density; this is indeed the case and the drift surface deviations are not only small but even to both sides of the flux surfaces. So, passing particles behave well, but with mixing quasi-symmetries, this is a truly three-dimensional structure of the strength of the magnetic field so that one expects the reflected particles to be lost because there is no quasi-symmetry. This was indeed the case but only in the vacuum field of the bs-eliminated preliminary Wendelstein 7-X design and not at finite $\langle \beta \rangle$ of about 5% [11] at which the design aimed anyway.

The situation at finite $\beta$ is seen in figure 5 (left). In the area where the helical and the toroidal curvatures are opposite to each other the remaining curvature is so small that the $\beta$ effect, i.e. the diamagnetic effect (the average field strength increases to the outside), overwhelms the residual curvature so that the reflected particles move in a poloidally closed valley. Since there is no quasi-symmetry they do not have a local invariant except their magnetic moment but because of their quasi-periodic reflections have the parallel action integral as a second adiabatic invariant, so that their confinement relies on the poloidal closure of the surfaces of constant second adiabatic invariant. Later, when this situation had been explored
more systematically [12], this case was named quasi-isodynamicity, following Palumbo [13] who called the case of guiding center drifts purely in magnetic surfaces in poloidal direction ‘isodynamic’ so that quasi-isodynamic just refers to the drift of the substitute particle with the invariant magnetic moment and parallel action.

A view at the complete torus, figure 5 (right), shows that particles with sufficiently large parallel energy will be able to leave a period on the outside and transit to the next. This leads to slow collisionless stochastic diffusion and loss. So, a further refinement of the concept of quasi-isodynamicity is to eliminate these transiting particles by forming a poloidally closed line of maximum strength of $B$. This leads to quasi-isodynamic stellarators with poloidally closed contours of the magnetic field strength [14] whose consistency with respect to neoclassical physics is briefly reviewed in the next section.

3. Neoclassical consistency

First, the evidence for vanishing bs current is seen in figure 6 (left) in the limit of long mean free path: virtually vanishing bs current density; also, for comparison, quasi-axisymmetric cases with bs current increasing the rotational transform and a quasi-helically symmetric case with bs current decreasing the rotational transform are shown.
Figure 7. Left: contours of the second adiabatic invariant in magnetic coordinates for deeply trapped particles (top left) to shallowly trapped ones (bottom right). In each case the red color corresponds to its maximum (from [14]). Right: diffusion coefficients versus mean free path at half the plasma radius in the configuration of [14] for 10^4 m^3 volume and 5 T for electrons (red, ◦) and ions (black, ◦+) with about 33 keV temperature (from [15]).

Second, figure 6 (right) shows the collisionless α-particle behavior indicating about 2% loss after 1 s (0.1 s is indicative of the slowing-down time in stellarators) if the particles are started at half the plasma radius, and about 8% if started at 2/3 of the plasma radius. Also indicated is a case with a reduced magnetic field of 3 T, with about 3% if started at half the plasma radius.

Third, figure 7 (left) shows contours of the second adiabatic invariant for deeply trapped particles to shallowly trapped particles which coincide quite well with the magnetic surfaces which are circles in this representation and so justify the notion quasi-isodynamic.

Fourth, figure 7 (right) shows neoclassical particle transport coefficients over a wide range of mean free paths [15] increasing to the left in figure 7 (right), with collisionality decreasing to the left. The interesting regime is the so-called 1/ν regime for the electrons in which the transport increases with decreasing collisionality. Because of collisionless confinement of the particles, long-mean-free-path regimes, in which the transport decreases with decreasing collisionality, exist for ions and electrons so that equal transport coefficients necessary for quasi-neutrality exist without radial electric field, so that even a radially outward pointing electric field, a so-called electron root, can be envisaged.

4. Conclusions

The set of computational tools now existing begins to justify the notion computational stellarator but it is, of course, not complete. Turbulence, e.g. its role in the core of quasi-symmetric stellarators [16], and plasma edge simulations, e.g. concerning properties of island divertors [17], are important areas now developing. But all this research would be in vain, at least for the foreseeable future, without corresponding devices. So such devices must be completed and exploited to justify a future for stellarator research.

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