Is Dark Energy an illusion?

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**ABSTRACT**

Much evidence has accumulated that within the context of general relativistic Friedmann–Robertson–Walker (FRW) cosmology there must exist a new, and gravitationally repulsive, substance in the Universe. The effect of this new type of energy density on the expansion of the Universe is to cause its acceleration, and the name that is given to it is 'Dark Energy'. To say whether or not Dark Energy really exists, however, requires a definite model for the Universe. That is, to be sure of the existence of Dark Energy, and the cosmological acceleration it causes, we must first be sure of the cosmological model we are using to interpret our observations. This is the subject of the present contribution, which will concentrate on the observational status of the Copernican Principle, which is at the heart of the FRW model. In particular, we will outline recent progress that has been made toward answering the question 'can the observations usually requiring the existence of Dark Energy be accounted for without introducing any new and exotic types of energy density, if we are prepared to give up some of the assumptions of the standard cosmological model?', or, alternatively, 'is Dark Energy an illusion?'.
assumptions in all of these areas, and it is therefore instructive to look at the effect that deviations away from our usual models have on the evidence for Dark Energy. This allows us not only to critically evaluate the degree to which we can consider Dark Energy necessary, but also allow us to solidify (or potentially revolutionise) our models of the Universe.

2. Modifications to gravity

A vast literature has accrued on the possibility that what we interpret as Dark Energy is, in fact, a break down of General Relativity on cosmological scales. If this is the case then it may be possible to create a cosmological model that self-accelerates without having to introduce any exotic matter fields into the space-time. On a more fundamental level, studies on this subject allow us the possibility to build up a body of evidence that extends the validity of General Relativity to new scales, that will complement the very strong evidence that is available on solar system, and astrophysical scales [7].

The possibility of extending general relativity is, however, already strongly constrained. It is a theorem of relativistic field theory that General Relativity is the only tensor theory of gravity that can be constructed from a Lagrangian that is a functional of the metrical one and that simultaneously satisfies all of the following conditions (where \( A^{\mu\nu} = \delta L/\delta g^{\mu\nu} \)):

1. \( A_{\mu\nu} = A_{\nu\mu} \)
2. \( A_{\mu\nu,\nu} = 0 \)
3. \( n = 4 \)
4. \( A_{\mu\nu} = A_{\mu\nu}(g_{\alpha\beta}, g_{\alpha\beta,\gamma}, g_{\alpha\beta,\gamma\delta}) \)

up to the value of constants, or terms in \( L_g \) given by \( \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu} R_{\rho\sigma} \) or \( \sqrt{-g} (R^2 - 4R^{\mu\nu} R_{\mu\nu} + R_{\mu\nu} R_{\rho\sigma} R^{\mu\nu \rho\sigma}) \), which do not contribute to the field equations in 4 dimensions. This is known as Lovelock’s theorem. Condition (1) states that the metric is symmetric, which is always true in (pseudo) Riemannian geometry. Condition (2) states that energy–momentum is conserved. Condition (3) says that there are 4 space-time dimensions, and Condition (4) says that the field equations are no higher than second order in derivatives of \( g_{\mu\nu} \). To construct a theory of gravity that differs from General Relativity we must therefore either give up on a Lagrangian formalism or Riemannian geometry, add extra fields beyond a single metric tensor, give up on energy–momentum conservation, add extra (or fewer) dimensions to the space-time manifold, or allow our field equations to be higher than second order.

These possibilities have all been considered extensively in the literature. For a review on higher-dimensional models the reader is referred to the review by Maartens and Koyama [8], for models of theories with higher than second-derivative field equations to the review of Sotiriou and Faraoni [9], and for tests gravity in the solar system and binary pulsars to the review of Will [10]. In order to be interesting on cosmological scales while still satisfying local bounds, however, these theories must in some way hide the extra light degrees of freedom that otherwise generically cause unacceptable behaviour in the solar system, and that have been shown to exist in a number of different theories [11–14]. The reason for this is that in modifying the IR limit of General Relativity massive modes are typically introduced, so that the metric in the weak field becomes

\[
\begin{align*}
g_{00} & \sim -1 + 2G (U + Y) \\
g_{ij} & \sim [1 + 2G (U - Y)] \delta_{ij},
\end{align*}
\]

where

\[
U = \int \frac{\rho(x')}{|x - x'|} d^3x' \quad \text{and} \quad Y = \frac{1}{3} \int \frac{\rho(x')}{|x - x'|} e^{-m|x - x'|} d^3x'.
\]

General relativity is then recovered as \( m \to \infty \), but not as \( m \to 0 \). Instead one can see that the post-Newtonian (or Eddington) parameter \( \gamma \to 1/2 \) (When G is appropriately normalised). It is the latter of these limits that is generally the
one that is of most interest for deviations from general relativity on cosmological scales, and so one must find a mechanism which hides this discontinuity in the massless limit sufficiently well to pass solar system bounds which currently say that \( \gamma = 1 \) to at least 1 part in \( 10^5 \) [15]. This type of behaviour is known to exist at the linearised level in bimetric, \( f(R) \) and DGP models that try and account for accelerating expansion, and is sometimes called the ‘vDVZ discontinuity’.

Known methods of dealing with the vDVZ discontinuity include the ‘Vainshtein mechanism’ where non-perturbative effects become important [16, 17], the ‘Chameleon mechanism’ where the scalar degree of freedom achieves an environment dependent mass [18, 19], or by the mutual cancellation effects from multiple extra degrees of freedom [20]. Even if such mechanisms can be found to be present for a particular theory, however, then there are still very tight cosmological constraints that must be overcome. These include observations of the CMB, baryonic acoustic oscillations and supernovae data, which can all help constrain the background evolution of the Universe, as well as weak lensing, integrated Sachs–Wolfe and matter power spectrum observations, which all help to constrain the permissible form of weak gravitational fields about an FRW background. The way in which these constraints can be applied for modified theories of gravity differs from case to case, but, as an example, one could consider the constraints implied on the higher-dimensional DGP model [21, 22], which can be seen to be very stringent. Permissible deviations from General Relativity are therefore becoming increasingly well constrained not only in the weak field, but also on cosmological scales.

We will not consider modifications to General Relativity any further here, but will instead move on the subject that is intended to be the main topic of this presentation: The Copernican Principle, and the assumption of homogeneity and isotropy that is made in FRW cosmology.

3. FRW Cosmology

The concordance cosmological model is based on linear perturbations about an otherwise perfectly homogeneous and isotropic Friedmann–Robertson–Walker (FRW) space-time. The line-element for such a space-time is given in the longitudinal (or conformal Newtonian) gauge by:

\[
ds^2 = -(1 + 2\psi)dt^2 + a^2(t)(1 - 2\Phi)g_{ij}dx^idx^j,\]

where \( \psi \) and \( \Phi \) are small fluctuations, \( g_{ij} \) is the metric of a static 3-space of constant curvature, and \( a(t) \) is the FRW scale factor, which is the only dynamical degree of freedom in (1). The assumption of FRW geometry is well motivated, with many benefits, but also comes with a number of drawbacks. In particular, we have the following positive features

**Benefits**

- Well motivated by the near isotropy of the CMB.
- Galaxy surveys are consistent with \( \sim \) homogeneous.
- It is simple & predictive.
- Consistent with a large number of other observations.

It is certainly true that an observer in an FRW space-time should expect to see an isotropic CMB, and one can in fact prove the reverse: That if all observers in a space-time see an isotropic CMB then the space-time is necessarily FRW [23]. This theorem, known as the Ehlers–Geren–Sachs theorem, can be extended to near isotropy and nearly FRW space-time, given some assumptions about what one means by these statements (see [24], and references therein). Likewise, the distribution of galaxies around us also seems to be largely consistent with an FRW universe. These are important consistency checks. The FRW models are also simple and predictive. This does not make them any more correct, of course, but it is an appealing feature for a model to have. What is more, there are now a large number of cosmological probes which all appear to show (for the most part) a large degree of consistency with the concordance \( \Lambda \)CDM model.

As well as benefits, however, there a number of drawbacks to using a linearly perturbed FRW geometry to model the Universe. Some of these are:

**Difficulties**

- It assumes a universal background.
- It assumes that we know how to average in relativity.
- It assumes the Copernican Principle is valid.
- It necessitates the introduction of Dark Energy.

The first of these is a response to the often cited justification for a global FRW description of the Universe: that with the exception of black holes and neutron stars it appears that all massive objects in the Universe are well described by a linear perturbation about an FRW background. This is true, but there is at present only a limited justification for taking this background to be the same for all gravitating systems in the entire Universe. The second point alludes to the well known problem that averaging in General Relativity is very difficult to even define, let alone carry out. Even if this basic problem can be solved, however, we are still likely to be left with a ‘back-reaction’ problem. The third point is that we assume that all observers in the Universe see observations that are the same as ours, up to statistical fluctuations. This assumption is perhaps more significant than is sometimes realised, as it is *required* in order to formulate the Ehlers–Geren–Sachs theorem, as well as its generalisations. Without it the isotropy of the CMB does not even tell us that the Universe is isotropic around us. Lastly,
of course, we have the requirement that Dark Energy needs to exist in FRW cosmology, which we have already discussed. These problems could range from anywhere between technical issues, to severe foundational problems, depending on what their resolution turns out to be.

4. Lemaître–Tolman–Bondi cosmology

In the previous section we outlined some of the difficulties that exist in the FRW model of cosmology. We expect these provide motivation, at least, for looking into other cosmological models. The benefits to such studies, besides bolstering our understanding of cosmology, are that they will place in context the need for Dark Energy, and that they offer (potentially) natural ways to avoid the fine-tuning problems associated with Dark Energy. The reason for this last hopeful statement is that the fine-tuning of the Dark Energy density is essentially a mismatch in scale between high-energy physics and the low energy Universe. We observe the effects of Dark Energy on the low energy scale, and try and interpret it on the high-energy scale. If instead there is no high-energy interpretation, and the affects we are attributing to Dark Energy are in fact due to low energy cosmological effects, then there is no fine-tuning problem (or, at least, there is not likely to be one of such problematic proportions).

There are a number of ways that one could go about evaluating other, non-FRW models of the Universe, and these have, of course, been studied in great detail in the literature. Here we focus on the assumption of the Copernican Principle, and what it would mean for cosmology and Dark Energy if we were to do away with it. We will briefly comment on other approaches later on. One might state the Copernican Principle as: We, on Earth, are a typical subset of all observers. This is a separate assumption from the Cosmological Principle, that the Universe is (approximately) homogeneous and isotropic around every point in space. The Copernican Principle is weaker, and theorems of the type given by Ehlers, Geren and Sachs are needed to extrapolate from the weaker to the stronger statement.

As already mentioned, the Ehlers–Geren–Sachs theorem, and its generalisations, are not valid without the Copernican Principle. Without this principle we are therefore free to consider less symmetric space-times than FRW. In particular, we can consider an observable Universe that is isotropic around us (on Earth) alone. In general we then have only three Killing vectors, instead of the six that exist in FRW. Such a space-time is sufficient to produce an isotropic CMB for the observers at the centre of symmetry, and for a space-time filled with dust only can be given in its most general form by the line-element

$$\text{d}s^2 = -\text{d}t^2 + \frac{a^2(t, r)}{1 - k(r)r^2} \text{d}r^2 + a^2(t, r) \text{d}\Omega^2.$$  (2)

The analogue of the FRW scale factor is now given by $a'(t, r)$ in the radial direction (where the prime denotes partial differentiation wrt $t$), and $a(t, r)/r$ in the azimuthal direction. The dot here denotes a partial derivative with respect to $t$. This is the well known Lemaître–Tolman–Bondi solution of General Relativity.

The equivalent of the Friedmann equation in such a space-time is then given by

$$\frac{\dot{a}^2}{a^2} = \frac{m(r)}{a^{3}} - \frac{k(r)}{a^2}$$  (3)

and the energy density is

$$8\pi G \rho = \frac{m'}{a'a^2}.$$  (4)

In Eqs. (2) and (3) we have two free functions of the radial coordinate $r$: the curvature $k(r)$ and the mass $m(r)$. In the FRW limit (when the space-time approaches homogeneity, and in the usual coordinates) $k(r)$ becomes constant, and $m(r)$ becomes proportional to $r^3$. Further more, we can see that when integrating (2) we will get a further free function $t_0(r)$ which will appear in the function $a$ in the form $a = a(t - t_0(r), r)$. This gives us three free functions in our general solution, which still has one coordinate freedom in how $r$ is defined, of the form $r \rightarrow f(r)$. There are therefore two 'physical' independent free functions with which we can uniquely specify the space-time, once we have chosen our radial coordinate. In what follows we will choose these to be the curvature function $k(r)$, and the 'bang time' $t_0(r)$ (chosen such that the mass $m$ within a sphere of radius $r$ is proportional to $r^3$).

As an example of what this discussion of space-time geometry means, let us consider the energy density profiles that result for a specific choice of $t_0 = \text{constant}$ and $k(r) = -k_0 e^{-r^2/r_0^2}$, where $k_0$ and $r_0$ are positive constants. The affect this has on the energy density in the Universe is shown in Figs. 2 and 3.

It can be seen that the negative curvature at the origin causes the energy density to drop there, as the expansion rate is faster than the asymptotic regions where the spatial curvature is zero. An over-dense region can be seen to form around the edge of the under-density due to expansion in the radial direction slowing at late times in this region, while the azimuthal rate continues to be higher than the asymptotic value. These solutions represent inhomogeneous generalisations of FRW geometry which are spherically symmetric around their central point and can appear as an under-dense 'void', as shown in Fig. 2.
Fig. 2. Energy density of the Universe at some late time when \( t_0(r) = \text{constant} \) and \( k(r) = -k_0 e^{-r^2/r_0^2} \), with one space dimension suppressed. Energy density increases in the vertical direction.

Fig. 3. Energy content of the Universe evolving with time when \( t_0(r) = \text{constant} \) and \( k(r) = -k_0 e^{-r^2/r_0^2} \), with two space-like dimensions suppressed.

5. Hubble diagrams in a void

In order to understand cosmological observables we must now understand distance measures and redshifts in LTB voids. The usual relations for these quantities, derived in FRW cosmology, are no longer necessarily valid. First of all, let us consider the ‘angular diameter distance’, which for an observer at the centre of symmetry can be read off from the line-element (2) as

\[
d_A = \frac{dA}{d\Omega} = a(t_e, r_e),
\]

where subscript \( e \) denotes the coordinate at the moment a photon is emitted. Now, we have from Etherington’s reciprocity theorem that the ‘galactic diameter distance’ is always related to the angular diameter distance by \( d_G = (1 + z) d_A \) (the galactic diameter distance is like the angular diameter distance, but measured up the future light cone of the emitter to the observer, rather than down from the observer to the emitter). Now, using these relations one can straightforwardly calculate the luminosity distance using the following:

\[
\frac{L_e}{4\pi d_L^2} \equiv F = \frac{L_0}{4\pi d_G^2} = \frac{L_e}{4\pi (1 + z)^2 d_G^2}.
\]

The first equality here defines the luminosity distance, where \( L_e \) is the emitted luminosity of the source and \( F \) is the observed flux. The second equality gives \( F \) in terms of the observed luminosity of the object and the galactic angular distance, and the third relates the observed luminosity to the emitted luminosity by two factors of redshift (these can be heuristically considered as being one for the redshift of the photons, and one for the time dilation of the source). The luminosity distance is then read off as being

\[
d_L = (1 + z)^2 a(t_e, r_e).
\]

This expression can be seen to reduce to the usual FRW expression \( d_{FRW}^{FRW} = (1 + z) r \) when \( a(t, r) = a(t) r \) and \( (1 + z) = a(t_0) / a(t_e) \). In LTB cosmology, however, the redshift is no longer simply given as a ratio of scale factors, but is instead given
by an integration of expansion rates along null geodesics, such that
\[ 1 + z = \exp \int_0^{r_e} \frac{\dot{a}}{\sqrt{1 - k(r)r^2}} \, dr \neq \frac{a_o}{a_e}, \]
(7)
along radial geodesics. Again, this reduces to the FRW expression when \( a(t, r) = a(t)r \), but is otherwise different. The derivation of these quantities here has been somewhat heuristic, and the reader is referred to [25] for a more thorough derivation. The results are the same.

Now that we are in possession of distance measures and redshift in these space-times, we can calculate the Hubble diagrams that an observer at the centre of symmetry should expect to observe. These diagrams are most conveniently given here in terms of the ‘distance modulus’, \( \Delta d_m \), which can be defined as
\[ \Delta d_m(z) \equiv c + 5 \log_{10} \left( \frac{d_L(z)}{d_{\text{Milne}}(z)} \right), \]
(8)
where \( d_{\text{Milne}}(z) = z + z^2/2 \) is the luminosity distance as a function of redshift in an open empty Milne universe, and \( c \) is a constant added such that \( \Delta d_m(0) = 0 \). This definition is equivalent to the description of distance modulus as being given by the difference in ‘magnitude’ of an astronomical object at a given redshift to the magnitude it would have at the same redshift in a Milne universe (an empty FRW universe, with negative spatial curvature).

Using this definition, we can give some example Hubble diagrams for some example void profiles. This is done in Fig. 4. The three different curvature profiles given in this figure can be shown to realise density profiles with approximately the same qualitative shape as \( k(r) \). This is as expected, as the expansion rate where \( k(r) \) is allowed to be negative increases with decreasing \( k \), causing a corresponding decrease in \( \rho \). It can also be seen that this shape is reflected in the resulting distance moduli: At low \( z \) the shape of \( \Delta d_m(z) \) is quite similar to \( k(r) \). This means that in order to have \( d(\Delta d_m)/dz > 0 \) at \( z = 0 \) we require \( dk/dr > 0 \) at \( r = 0 \), which can be seen to correspond to a void with a cusp shaped density profile, as was pointed out in [26].

Now, within the context of FRW cosmology a distance modulus that increases with \( z \) corresponds to accelerating expansion. Hence the line corresponding to de Sitter space goes upward in the lower right plot of Fig. 4 (as this space–time accelerates), while the line corresponding to Einstein–de Sitter space goes down (as this space-time decelerates). With this interpretation it is plain to see that an observer at the centre of such a void who mistakenly interpreted their observations within an FRW cosmological model would think that they lived in an accelerating universe, while, in fact, they did not. The reason for this confusion is that in FRW the variation in the expansion rate along our past light cone can only be due to a change in expansion with time, as all points in space are the same by construction. In the LTB model, however, things are more complicated. The change in the rate of expansion along our past light cone can be due to the change in expansion rate
at different spatial positions, as well as different times. The implication of acceleration is then due to the expansion rate that a photon ‘experiences’ increasing as it falls into the centre of the void, due to the increasing expansion rate as radial position decreases (due to the negative spatial curvature at the centre).

This simple illustration shows that the implication of cosmological acceleration, and hence Dark Energy, does indeed depend on the cosmological model we are using to interpret our observations. With this is mind the pertinent question becomes, how can we tell which is the correct cosmological model? The general problem here requires a large number of cosmological observables (see [24], and references therein). A reasonable starting point, however, would be to first ask how can we tell the difference between LTB and FRW, before moving onto more general space-times. In this regard we can start from the observation made above that in order to have ‘apparent acceleration’ at low redshift in an LTB universe (as occurs in $\Lambda$CDM), we would require a cusp shaped void. Using the data from supernovae surveys we can therefore try and distinguish a realistic smooth void from $\Lambda$CDM: Each of these produces apparent acceleration, but only $\Lambda$CDM produces it at $z \lesssim 0.1$. In fact, performing such an analysis we find that the Bayesian evidence for $\Lambda$CDM over a smooth LTB void is only very minimal: Both can fit the existing data almost equally well [26] (in fact, using most data sets a Gaussian shape void provides a considerably better fit to the data than $\Lambda$CDM, and it is only when the Bayesian evidence is considered that $\Lambda$CDM is sometimes marginally favoured, due to it having less parameters). To distinguish a smooth void model from $\Lambda$CDM we must therefore wait for many hundreds more supernovae to be collected (see [26] for estimates of numbers), or consider other cosmological observables. This latter option will be the subject of the following section, where we will primarily be concerned with voids with constant bang time, $t_0$.

6. Constraints on a void

There are a number of cosmological probes that can be applied to constrain LTB models that try and explain observations without Dark Energy. Some of these are:

- The CMB dipole, which constrains our distance from the centre of symmetry.
- The CMB angular power spectrum.
- Large-scale structure observations.
- The kinetic Sunyaev–Zeldovich effect.

The first of these is due to an off-centre observer seeing an anisotropic universe, due to different expansion rates when they look through either the faster expanding centre of the void, or the slowing expanding regions away from the centre. This anisotropy induces a dipole in the CMB which can be constrained using observations to imply that we must be within $\leq 15$ Mpc of the centre [27] ($\sim 1\%$ of the distance to the edge of the structure). Such a constraint is strong, and questions the naturalness of such a scenario, but cannot rule it out entirely. To do this we must consider other observables.

6.1. The CMB angular power spectrum

The CMB angular power spectrum provides notoriously strong constraints on the curvature of space in FRW cosmology, when supplemented with a local measurement of $H_0$, and so one may also suspect it to place strong constraints on hypothesized voids with large amounts of negative spatial curvature locally.

The reason for such strong constraints on $k$ in FRW is eluded to by the illustrations in Fig. 5. The outer circle here is supposed to depict the last scattering surface, and the lines connecting the centre of the circle to the edge are supposed to represent null geodesics, as followed by CMB photons. In the illustration on the left space is flat, and so the null geodesics are straight lines. On the illustration on the right, however, space is negatively curved and so the null geodesics are distorted (the curvature of these lines in the illustration is supposed to signify the effect of projecting from a negatively curved space onto the flat page). Now, even though the length scale on the last scattering surface may be the same, the observers in the two space-times can observe this scale as corresponding to different angular separations. This fact is supposed to be represented by the two null geodesics meeting with a different angle at the centre of the circles in each case. The affect of spatial curvature on the CMB in FRW cosmology is therefore to shift the scale of the angular power spectrum, as it is seen by the observer on their sky. This effect allows one to place strong constraints on the curvature of the Universe in FRW cosmology.

Let us now consider what happens in an LTB void model. To do this it is useful to define the angular power spectrum on the sky as

$$\hat{P}_{\Omega^2}(q) \equiv \int d\Omega^2 \langle \Delta T(\hat{n}) \Delta T(\hat{n}') \rangle e^{-iq\cdot\theta}, \quad (9)$$

where $\Delta T(\hat{n})$ is the temperature fluctuation in the direction specified by the vector $\hat{n}$, and $q$ is a two-dimensional wave-number on the sky. One can then show that the power spectrum of the small angle CMB in a void can be related to that in an FRW universe (which is easily calculated using known methods) by [28]

$$\hat{P}_{\Omega^2}^{\text{void}}(q) = S_{\Omega^2}^2 \hat{P}_{\Omega^2}^{\text{FRW}}(Sq), \quad (10)$$
where \( S = \frac{d_{FRW}^A}{d_{void}^A} \) is known as a “shift factor”. It should be noted that this approach should not be expected to reproduce the large angle CMB for an observer in a void. Such an understanding would require knowledge of the growth of structure in these models, which we will return to later. In practice this is not a problem, as the information in the CMB peak positions and heights can be retained, and the large angle effects set aside, by introducing a cut-off in our CMB analysis at a scale of \( l \sim 100 \). We can then retain the \( l < 100 \) power spectrum to analyse later, when we understand effects such as the integrated Sachs–Wolfe effect, which will require a knowledge of linear perturbations. For now the shift factor, \( S \), together with the known power spectrum in an FRW universe, provide us with all we want to know about the positions and relative heights of the acoustic peaks in the CMB angular power spectrum, as seen by our observer in a void.

Now, the standard FRW result is that spatial curvature of \( \Omega_k = 0.7 \) produces a shift in the power spectrum from a spatially flat universe of around \( S \sim 1.7 \). As the depth of void required to reproduce the supernovae observations usually attributed to Dark Energy is also around 70% under-dense at its centre we might naively expect a similar shift in the CMB power spectrum. Such a shift would be completely unacceptable observationally, as the peak positions are now known to very high accuracy [3]. In fact, this does not happen, for the following reason: The majority of the shift introduced by spatial curvature in FRW cosmology is due to curvature at high redshifts. This is a somewhat counter-intuitive result, as spatial curvature is sometimes only thought to be important in the late Universe, when it can dominate the cosmological expansion. Here, however, we find that it is important in the early Universe, due to the optical effects it introduces there. This is illustrated in Fig. 6, where the shift parameter accumulated by looking out of a void is given as a function of \( z \), where \( z \) represents the radius of the void in redshift. It can be seen that \( \sim 95\% \) of the shift is contributed in the first 5% of the Universe’s history.

These results show that the spatial curvature of the voids in these models of the Universe do not necessarily rule them out due to the shift they produce in the CMB angular power spectrum. Instead, we find that rather than having too much
shift, we have to struggle to produce enough shift in these void models, in order to produce the \( \sim 10\% \) shift that is needed from the Einstein–de Sitter predictions to what is observed (in \( \Lambda \)CDM just this amount of shift is contributed by \( \Lambda \) through a slightly different mechanism, due to its late-time acceleration). This difficulty of fitting the small angle CMB in void models was found first of all in [29], who suggested adding regions that were locally positively curved at intermediate redshifts. The interpretation provided above suggests adding a relatively small amount of positive spatial curvature to the background in which the void is embedded does the same job, as was found in [28]. In this way one can produce any shift one wishes by adding a suitable amount of background curvature.

Now, while it turns out that we can achieve the desired shift parameter this comes at the expense of adding at least one further parameter to the model: asymptotic spatial curvature. However, even if this is done, the Hubble rate that it required to get the correct relative heights of the first two peaks in the CMB spectrum is unacceptably low, around 55 km s\(^{-1}\) Mpc\(^{-1}\) at best. Again, this was first identified in [29], who used it as evidence that these models are not observationally viable. This is not necessarily true, however, as one can adjust the locally measured value of \( H_0 \) by adjusting the bang time, \( t_0 \). Until now, we have been considering the case \( t_0 = \text{constant} \). By allowing the centre of the void to have a bang time that is later than the asymptotic regions, however, the situation can be rescued, and an acceptable \( H_0 \) can be achieved [28]. The expense of this is at adding another 2 parameters to the model: The width and depth of the bang time fluctuation. We therefore must add 3 extra parameters in total to explain the small angle CMB observations, as well as local measurements of the Hubble rate.

### 6.2. Large-scale structure observations

In order to study the formation of large-scale structure, and the observations associated with it, we must first understand linear perturbation theory about an LTB background. This proceeds differently to the FRW case, and particular there is no longer a decoupling of scalar, vector and tensor modes at the linear level. This means that gravitational potentials can, in principle, source gravitational waves, as well as a number of other possibilities.

The reason for this lack of decoupling is due to the lack of a homogeneous 3-space. In LTB, however, we have homogeneous 2-spaces, and so we can decouple two different modes which transform differently under coordinate transformations on these spheres. These are called polar and axial modes. The equations that govern the evolution of the perturbations, as well as how they can be transformed into quantities that reduce to the usual scalar, vector and tensor modes familiar from FRW, are given in [30]. Here we will present only some of the relevant equations, which remain to be solved in full generality. For details the reader is referred to [30].

One can show that the polar perturbations contain what reduces to scalar perturbations in the limit of an FRW background. The most general polar perturbations to the LTB line-element can be written in Regge–Wheeler gauge as

\[
ds^2 = -[1 + (2\eta - \chi - \varphi)Y] \, dt^2 - \frac{2a_\perp \varsigma Y}{\sqrt{1 - \kappa r^2}} \, dt \, dr + [1 + (\chi + \varphi)Y] \frac{a_\parallel^2 \, dr^2}{(1 - \kappa r^2)} + a_\perp^2 r^2 (1 + \varphi Y) \, d\Omega^2,
\]

where \( Y \) are spherical harmonic functions, and the scale factors here can be written in terms of the function \( a \) from Eq. (2) as \( a_\parallel = a/\tau \) and \( a_\perp = a' \). To linear order, the field equations governing these perturbations can then be written

\[
-\ddot{\chi} + \chi'' - 3H_\parallel \dot{\chi} - 2W \dot{\chi}' + \left[ 16\pi \rho - \frac{6M}{a_\parallel^3} - 4H_\parallel (H_\parallel - H_\perp) - \frac{(\ell - 1)(\ell + 2)}{a_\perp^2 r^2} \right] \chi
\]

\[
= -2(H_\parallel - H_\perp) \zeta' - 2 \left[ H_\parallel^2 - 2 (H_\parallel - H_\perp) W \right] \zeta + 4(H_\parallel - H_\perp) \dot{\phi} - 2 \left[ 8\pi \rho - \frac{3M}{a_\parallel^2} - 2H_\parallel (H_\parallel - H_\perp) \right] \varphi,
\]

and

\[
\ddot{\varphi} + 4H_\parallel \dot{\varphi} - 2 \left( \frac{1}{a_\perp^2 r^2} - W^2 \right) \varphi = -H_\perp \dot{\chi} + W \chi' - \left[ 2W^2 - \frac{\ell(\ell + 1) + 2}{2a_\perp^2 r^2} \right] \chi + 2W (H_\parallel - H_\perp) \zeta,
\]

together with

\[
\ddot{\zeta} + 2H_\parallel \dot{\zeta} = -\chi',
\]

and the constraint \( \eta = 0 \). These equations are a set of three coupled second-order PDEs in two variables, and are the system of equations that need to be solved in order to understand perturbations about LTB. In the limit \( H_\perp = H_\parallel \) one finds FRW is recovered, and the perturbation modes decouple from one another, as should be expected. In general, however, this does not happen, although analytic solutions exist when one ignores the modes that reduce to tensor perturbations [30].

Various authors have tried to make progress in applying constraints on these models from observations attributed to structure formation, in particular the Baryon Acoustic Oscillations [5]. We will not do this here, as we prefer to understand the linear perturbation theory that is required to understand structure formation before applying these observations to try and constrain the model. Once the solutions to the equations presented above are known and understood then one can use a variety of different observational probes to constrain these models, including the integrated Sachs–Wolfe effect, weak lensing, matter power-spectra, and peculiar velocity surveys.
6.3. The kinetic Sunyaev–Zeldovich effect

The kinetic Sunyaev–Zeldovich is a very powerful tool for constraining inhomogeneity, and particularly the LTB models we have been discussing above. The effect itself was originally formulated in terms of perturbed FRW, and in this context it is due to the peculiar velocity of distant clusters of galaxies. It works in the following way: photons from the CMB are scattered off the gas within a cluster. If the cluster has any peculiar velocity with respect to the background FRW cosmology, then an observer within the cluster will see a dipole in the CMB radiation. The size of this dipole is evident from the re-scattered light that other observers see when they look at this cluster. It is therefore possible to determine the peculiar velocity of clusters by looking at the CMB photons that scatter off them.

Now, within the context of FRW this effect is expected to be quite small. In the LTB models, however, it can be large. The reason for this is the result that we have already discussed, that observers off-centre in an LTB void model see a CMB with a large dipole. The re-scattered light, if interpreted within an FRW model, will then look like the distant cluster has an anomalously high peculiar velocity, even if it is comoving with the LTB background. This effect has been studied in the context of LTB cosmology in [31] and [32]. The basic point here is that an observer at the centre can use the clusters away from centre to ‘see’ what the environment in their locale is like, by observing the reflected photons. Therefore, even if the observer is exactly at the centre, and sees an isotropic universe around them, they can still distinguish the inhomogeneity by inferring anisotropy around the off-centre points.

Constraints imposed in [31,32] were significant, but due to lack of available data did not manage to rule out the void models entirely, as one may have hoped for. These studies did appear to suggest, however, that future data sets may well be able to do this. [In fact, during this conference a paper was made public in which recent observations were used to claim that void models with a simultaneous big bang are ruled out [33]].

6.4. Status of the void models

We have discussed here a number of observational probes of the LTB void cosmology. The result of the activity in this area is now the following: In order to consider a void as a viable alternative to Dark Energy one must:

- Give up the Copernican Principle.
- Consider a cusp shaped void, to reproduce $\Lambda$CDM. (+2 parameters)
- Add asymptotic curvature. (+1 parameter)
- Add an inhomogeneous big bang. (+2 parameters)
- Hope that large-scale structure and kSZ are compatible with LTB.

The first of these points is an intrinsic part of the model, made clear by the constraints on the distance we must be from the centre. The second point assumes that future supernova observations will be compatible with $\Lambda$CDM. If this is the case, a cusp shape void would be required to replace the need for Dark Energy (note, current supernova observations are not yet able to do this). The third and fourth points are required for compatibility with the small angle CMB. The last point reflects the current state of affairs that structure formation and kSZ effects are likely to place strong constraints on inhomogeneity of this type in the future [Ref. [33] already claim to have done this for voids with constant bang time]. To be compatible with observations we see from the above that we must therefore have quite a complicated void model, with a total of 5 parameters, compared to 1 in $\Lambda$CDM. So, while these models are still compatible with observations, they are now strongly disfavoured in the sense of Bayesian evidence due to their complicated nature.

7. Other approaches

We have so far considered primarily the observational consequences of a very drastic, but tractable, deviation from the concordance $\Lambda$CDM model in order to try and investigate to degree to which Dark Energy must be considered necessary. Even this quite simple, yet very different model of cosmology, however, has proven to be quite hard to dismiss with the observations we currently have at hand. Of course, if it turned out that future observations favoured the LTB void model then this would be quite a surprising, and certainly revolutionary, result for cosmology. The more expected result to the contrary, however, hardly proves the validity of $\Lambda$CDM, although it does improve the amount of evidence we have for the existence of Dark Energy. To improve this evidence further we must consider other potential explanations of Dark Energy, and eventually we can then hope to either find a satisfactory explanation of the observations that have led to the inference of Dark Energy, or build up a body of evidence that is compelling enough to remove all doubt of its genuine existence. In this section we will briefly mention some other approaches to this eventual goal.

One area of research that has generated a lot of interest in this area recently is that of ‘back-reaction’ [34]. The problem here involves the averaging of quantities in General Relativity. As mentioned in Section 3, such a procedure is very hard to define [35,36]. However, even if one can unambiguously define a relativistically valid averaging procedure, then there is almost certainly still going to remain a problem with the commutativity of this procedure with evolution under Einstein’s
equations. This effect is called back-reaction, and works in the following way. Let us try and define an average in the following way:

$$\langle \Psi \rangle \equiv \frac{1}{V} \int \Psi J d^3x,$$

where $V$ is the volume of the domain being averaged over, and $J = \sqrt{-\det(g^{(3)})}$ is the square root of the determinant of the metric on the 3-space. Now, if one applies such a definition to try and obtain averaged Friedmann-like equations from Einstein’s equations with dust and $\Lambda$ then we get

$$\frac{\ddot{a}_D}{a_D^2} = \frac{8\pi G}{3} \langle \rho \rangle + \frac{\Lambda}{3} - \frac{\langle R \rangle}{6} - \frac{\langle Q \rangle}{6},$$

and

$$\frac{\ddot{a}_0}{a_0} = -\frac{4\pi G}{3} \langle \rho \rangle + \frac{\Lambda}{3} + \frac{\langle Q \rangle}{3},$$

where $a_0$ corresponds to the scale of an averaging domain, $R$ is the Ricci curvature of the domain and $Q$ is given by (ignoring shear) $Q = \frac{2}{3} (\theta - \langle \theta \rangle)^2$, and where $\theta$ is the expansion scalar. In the usual approach to cosmology we use averaged quantities in Einstein’s equations directly, treating them as if they were local quantities. This is the equivalent of setting $Q = 0$ in the equations above. In general, however, it can be seen that $Q \neq 0$, and so we should use the modified ‘Buchert equations’ above, rather than the Friedmann equations. While very compelling, however, the details of how to apply this in an unambiguous way to the real Universe, and what it means for observational probes of cosmology, is not yet fully understood. Some authors have claimed this effect could be responsible for the observations attributed to Dark Energy, while others claim it is observationally insignificant. For a summary of this area, recent progress, and challenges, the reader is referred to [37], and references therein.

Another approach to investigating possible deviations from FRW cosmology caused by inhomogeneity is to try constructing cosmological models that do not involve averaging at all, and working out observables in these un-averaged space-times directly. This has recently been attempted in [38]. The point here is that photons travel largely through the empty spaces between astrophysical objects as they propagate from the point of emission to us, on Earth. The space-time geometry of these regions is different to the ‘average’ density that is used in FRW cosmology, and so the redshift and focusing of bundles of null geodesics need no longer be the same. It also provides us with a route whereby we can test not only the consequences of averaging on the large-scale expansion of the Universe, but also on observational quantities directly: It is not necessarily true that observables calculated in an averaged space-time are the same as the average of observables in an un-averaged space-time. This is sometimes referred to as ‘weak back-reaction’. The consequences of this found in [38] were only enough to explain around 10% of the observed amount of Dark Energy, but further study and more refined models are required in order to properly understand this problem.

Beyond this, some authors have attempted to construct non-FRW cosmological models that take advantage of the non-linear aspects of General Relativity in order to try and understand Dark Energy. Notable recent studies in this area have been performed by Wiltshire [39], Mattsson [40] and Räsanen [41]. The general idea in some of these studies is that at late times in the evolution of the Universe it is the case that over-dense region virialise, and stop collapsing, while under-dense regions can continue to expand indefinitely. It is therefore thought to be under-densities that should become the dominant contribution to the expansion of the Universe at late times, which should cause some speed up from the predictions of FRW cosmology. Again, these models, while very interesting, require further study before they can be properly understood.

8. Conclusions

We have attempted to present here an outline of ways in which the observations usually attributed to Dark Energy could be accounted for without adding any extra energy content into the Universe. We have focused, in particular, on the LTB void models in which we are at the centre of a large under-dense region. Such models can account for what appears to be an accelerating universe, when interpreted within an FRW framework, without any part of them undergoing actual accelerating expansion at any place or time.

We conclude simply with the remark that constraints on deviations from the concordance $\Lambda$CDM model are tightening, both from an observational perspective with new and better observations of cosmological phenomena becoming available, but also from a theoretical perspective, where the affect of deviating away from FRW are becoming better understood. It is now the case that many of the simplest models that would account for Dark Energy without any extra energy density are now ruled out. We have shown that this is particularly true of the LTB void models, which while still compatible with observations, are required to have a significant number of extra parameters in order to be observationally viable, above and beyond those required in $\Lambda$CDM. This situation is likely to become clearer still in the future, as observations improve and models become better understood.

For now, it still looks like $\Lambda$ is still the simplest and most compelling explanation for the apparent accelerating expansion of the Universe, although more work is required in order to establish the absolute necessity of Dark Energy.
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