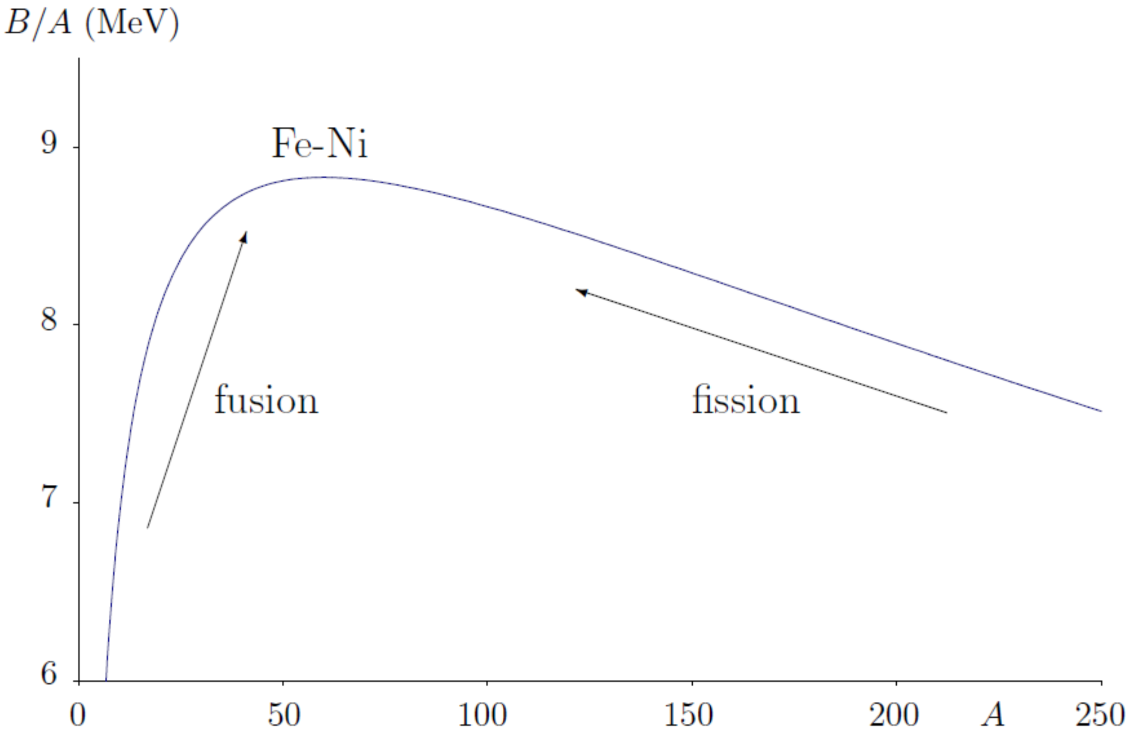
Chapter IX:
Nuclear fusion

Summary

1. General remarks
2. Basic processes
3. Characteristics of fusion
4. Solar fusion
5. Controlled fusion

General remarks (1)

- Maximum of binding energy per nucleon at about A ≈ 56 (Fe) → fission for heavier nuclei → fusion for lighter nuclei
- Fusion of 2 light nuclei produces an increase of binding energy per nucleon



General remarks (2)

- Advantages of fusion (compared to fission) as energy source:
  1. Light nuclei are abundant and easy to obtain
  2. End products are lights and usually stable
  3. Production of small quantity of short-live radioactive wastes
  4. No risk of core meltdown
  5. No proliferation
- Disadvantages of fusion:
  1. Before combination of light nuclei → overcoming of the Coulomb repulsion → enormous disadvantage
- To reach fusion → kinetic energy at least equal to energy repulsion has to be provided to nuclei
- Use of accelerators → current very low → power output very low (a few W)
- Use of thermal energy → very high temperature has to be reached → magnetic or inertial confinement

Basic processes (1)

- In controlled fusion reactors → to overcome the Coulomb barrier → choice of nuclei with small charge
- Most elementary fusion reaction → $p + p \rightarrow {}^{2}He$ → not possible (2 + charges) → attention: other possible pp reaction: positron emission → see solar fusion section
- Another elementary reaction → ${}^{2}H + {}^{2}H \rightarrow {}^{4}He + \gamma$ (γ is necessary for energy balance because ${}^{4}He$ has no excited states) → problem because $Q = 23.8$ MeV > the separation energy of both proton and neutron of ${}^{4}He$
- More likely reactions → ${}^{2}H + {}^{2}H \rightarrow {}^{3}He + n$ ($Q = 3.3$ MeV) or ${}^{2}H + {}^{2}H \rightarrow {}^{3}H + p$ ($Q = 4.0$ MeV) → deuterium-deuterium (D-D) reactions
- But more stable is the fusion product greater is the energy released → a reaction that form ${}^{4}He$ has a particularly large energy release → ${}^{2}H + {}^{3}H \rightarrow {}^{4}He + n$ ($Q = 17.6$ MeV) → deuterium-tritium (D-T) reaction

Basic processes (2)

- The D-T reaction is often used as neutron source
- The D-T reaction produces a large amount of energy (17.6 MeV) → separated into ^{4}He and n
- The D-T reaction has a large cross section (see below)
- The D-T reaction is characterized by the same Coulomb barrier as D-D reactions
- For these reasons the D-T reaction has been selected to be used in controlled fusion reactors

- Other basic processes will be explained in the solar fusion section

Characteristics of fusion: Energy release

- Calculation of the *Q* value → for controlled fusion reactors and solar processes the kinetic energy of the initial particles is small (in the 1-10 keV energy range) compared to *Q* → energy release = final total energy of the products particles →

$$A + B \to C + D \qquad \Longrightarrow \qquad \frac{1}{2}m_C v_C^2 + \frac{1}{2}m_D v_D^2 \simeq Q$$

- Again neglecting the initial motions → $m_C v_C \simeq mDv_D$
- We obtain thus →

$$\frac{1}{2}m_C v_C^2 \simeq \frac{Q}{1 + m_C/m_D} \qquad \frac{1}{2}m_D v_D^2 \simeq \frac{Q}{1 + m_D/m_C} \qquad \frac{\frac{1}{2}m_C v_C^2}{\frac{1}{2}m_D v_D^2} \simeq \frac{m_D}{m_C}$$

- For D-T reaction → 80% of energy is taken by neutron ↔ for D-D reaction → 75% of energy is taken by neutron/proton

Characteristics of fusion: Coulomb barrier

- With R_A and R_B the radii of reacting particles → the Coulomb barrier for just touching particles is

$$V_C = \frac{e^2}{4\pi\epsilon_0}\frac{Z_A Z_B}{R_A + R_B}$$

- The effect of the Coulomb barrier on the fusion reaction is similar to the effect of the Coulomb barrier on α decay → the product $Z_A Z_B$ will appear in an exponential barrier penetration probability → cross section very sensitive to its → the barrier is lowest for the hydrogen isotopes
- For D-T reaction V_C = 0.4 MeV → it is low but still far above the typical incident particle energy of 1-10 keV → tunnel effect

Characteristics of fusion: Cross section (1)

- Calculations from basic expression for nuclear reaction cross sections → use of the Breit-Wigner formula (eq. 10.102 of N. Cerf, Mécanique quantique, Première partie: Notions de base) →

$$\sigma_l = \frac{4\pi}{k^2}(2l+1)\frac{\Gamma^2/4}{(E_r - E)^2 + \Gamma^2/4}$$

- Particles reacting at thermal energies → reaction occurs far from any resonance → energy dependence of the cross section comes mainly from two terms →
  1. The k^{-2} factor → v^{-2} dependence
  2. A barrier penetration factor on the form e^{-2G} as for α decay but substituting for Q the center of mass energy E of reacting particles

➡ $$\sigma \propto \frac{1}{v^2}e^{-2G}$$

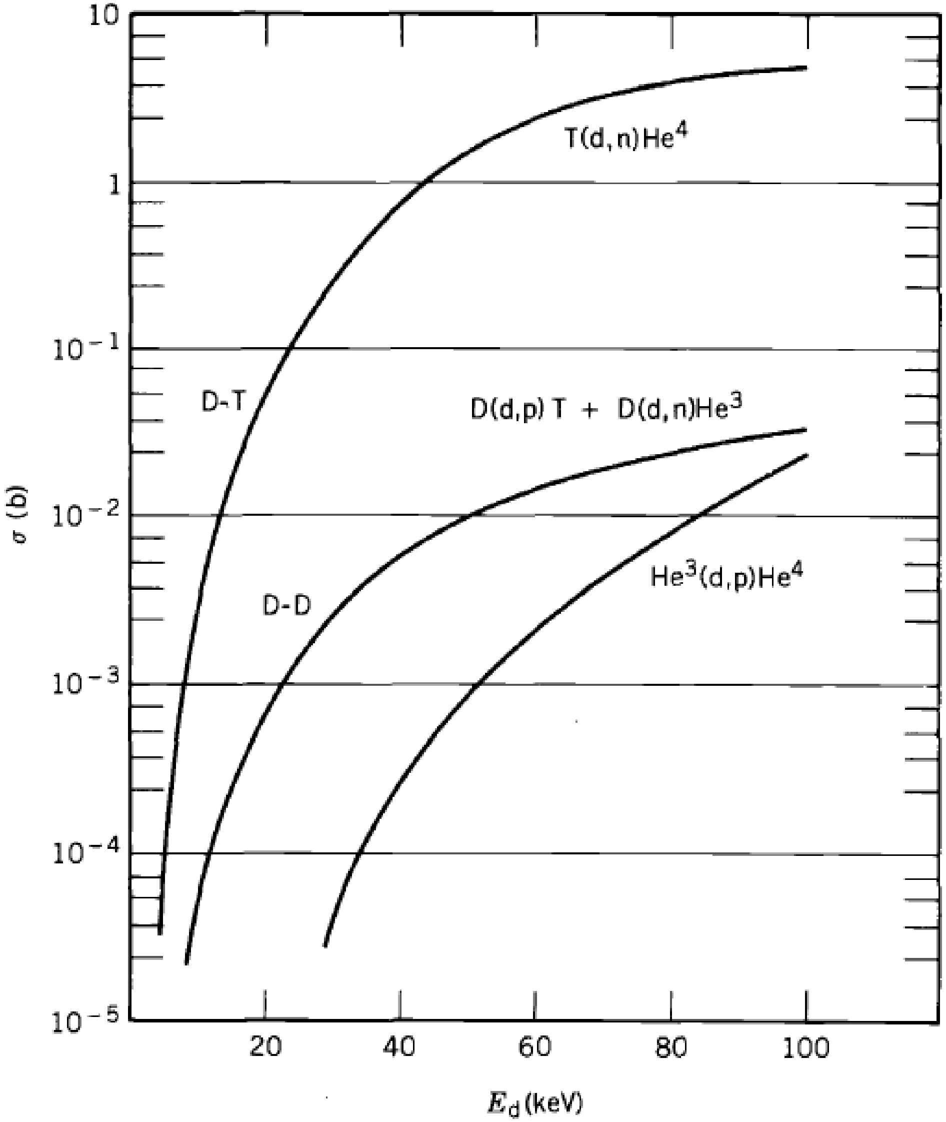
Characteristics of fusion: Cross section (2)

- *G* is the Gamow factor of α decay but substituting for *Q* the center of mass energy *E* of reacting particles → as $E \ll B$ → with *v* the relative velocity of interacting particles →

$$G \simeq \frac{e^2}{4\pi\epsilon_0}\frac{\pi Z_A Z_B}{\hbar v}$$

- In σ the proportionality factor involve nuclear matrix elements and statistical factors depending on the spins of the particles but energy dependence is correct

Characteristics of fusion: Cross section (3)



Characteristics of fusion: Reaction rate (1)

- The reaction rate is $\propto$ to $\sigma \times v$ (see Nuclear Metrology Techniques)
- In thermonuclear fusion the distribution of particle speeds is described by the usual Maxwell-Boltzmann velocity distribution for particles in thermal equilibrium (with k the Boltzmann constant and T the temperature) →

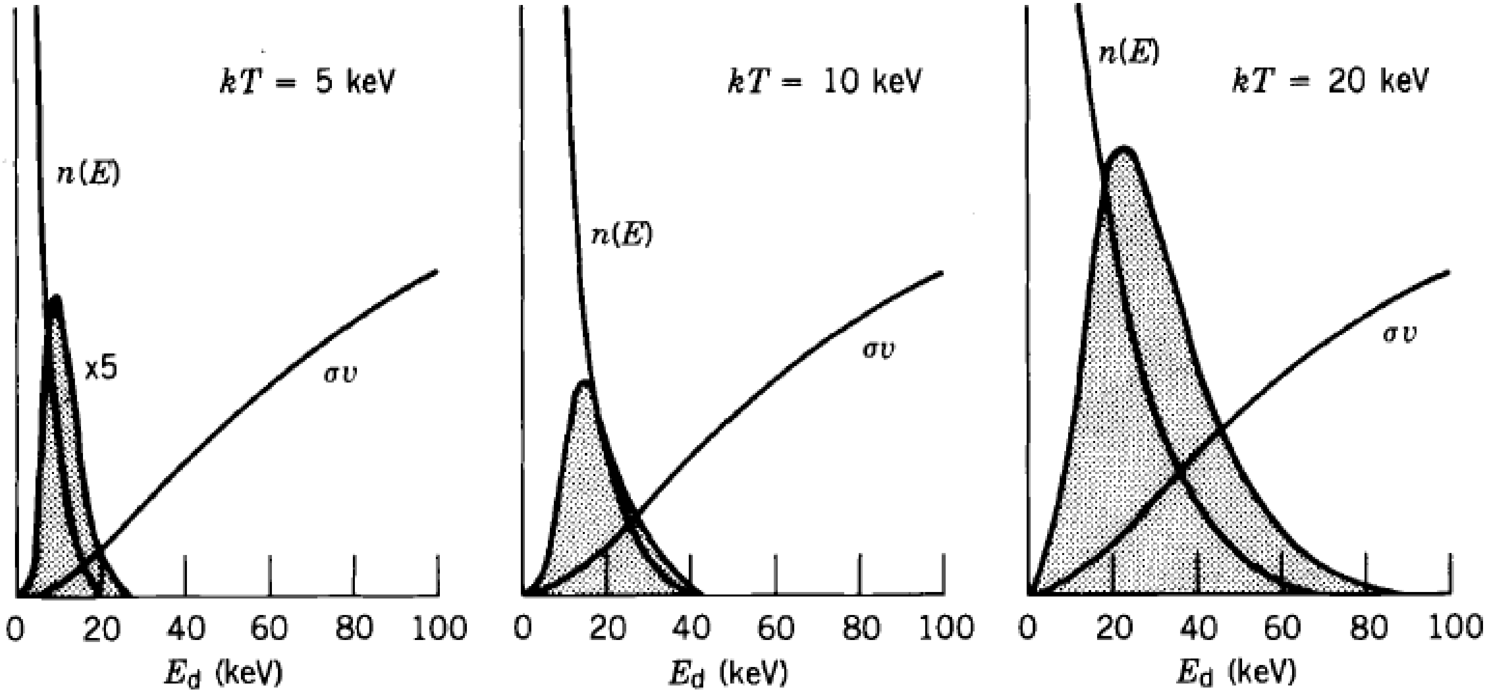
$$n(v) \propto e^{-mv^2/2kT}$$

- $n(v)v^2dv$ gives the relative probability to find a particle with speed between v and $v + dv$ in a collection of particles in thermal equilibrium at T
- For a gas of identical particles →

$$\langle \sigma v \rangle \propto \int_0^\infty \frac{1}{v} e^{-2G} e^{-mv^2/2kT} v^2 dv \quad \Longleftrightarrow \quad \langle \sigma v \rangle \propto \int_0^\infty e^{-2G} e^{-E/kT} dE$$

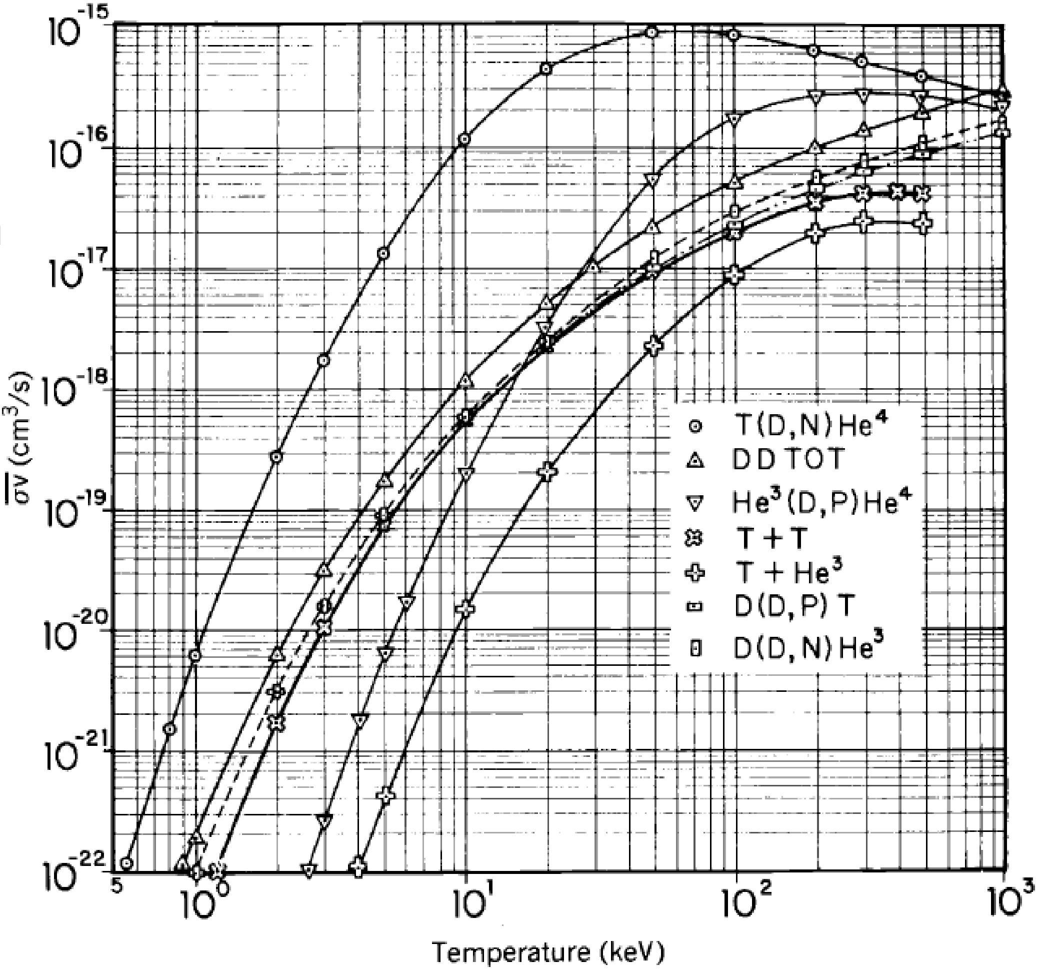
Characteristics of fusion: Reaction rate (2)

- At low T → little overlap between $n(E)$ and σv → the average is small
- AT very high T → the area of the Maxwell-Boltzmann distribution becomes small → the average value of σv is small
- At intermediate T → $\langle \sigma v \rangle$ reaches a maximum



Characteristics of fusion: Reaction rate (3)

- At extremely high *T* ≈ 10^{10} K (corresponding to MeV energies) → the D-T reaction may become less favorable than others
- In the temperature region achievable in a thermonuclear fusion reactor (1-10 keV or *T* ≈ 10^7-10^8 K) → the D-T reaction is clearly favored



Characteristics of fusion: Reaction rate (4)

- The simple theory used here is appropriate only for the D-D reaction (only 1 type of nuclei)
- For reactions involving two different nuclei (D-T, …) → the velocities of the different species have to be considered
- The cross section and reaction rate involve a relative velocity → $\sigma(v_{rel})$ and $\langle \sigma(v_{rel})v_{rel} \rangle$ have to be determined → average is done over the Maxwell-Boltzmann distribution of both species
- More complicated calculations but previous general conclusions about fusion reaction rates remain valid

Solar fusion: General remarks

- Sun = extremely successful prototype of a self-sustaining thermonuclear reactor at (in the core) $T \approx 15 \times 10^6$ K (output constant during 10^9 years)
- Basic process in the Sun (and in most other stars) → fusion of H into He
- Abundance of atoms in universe → about 92% of the atoms in the universe are hydrogen ↔ 7% are He (formed during the early stages of universe) ↔ less than 1% are other ones
- All reactions in any fusion cycle must be two-body reactions because the simultaneous collision of three particles is too improbable

Solar fusion: pp chain (1)

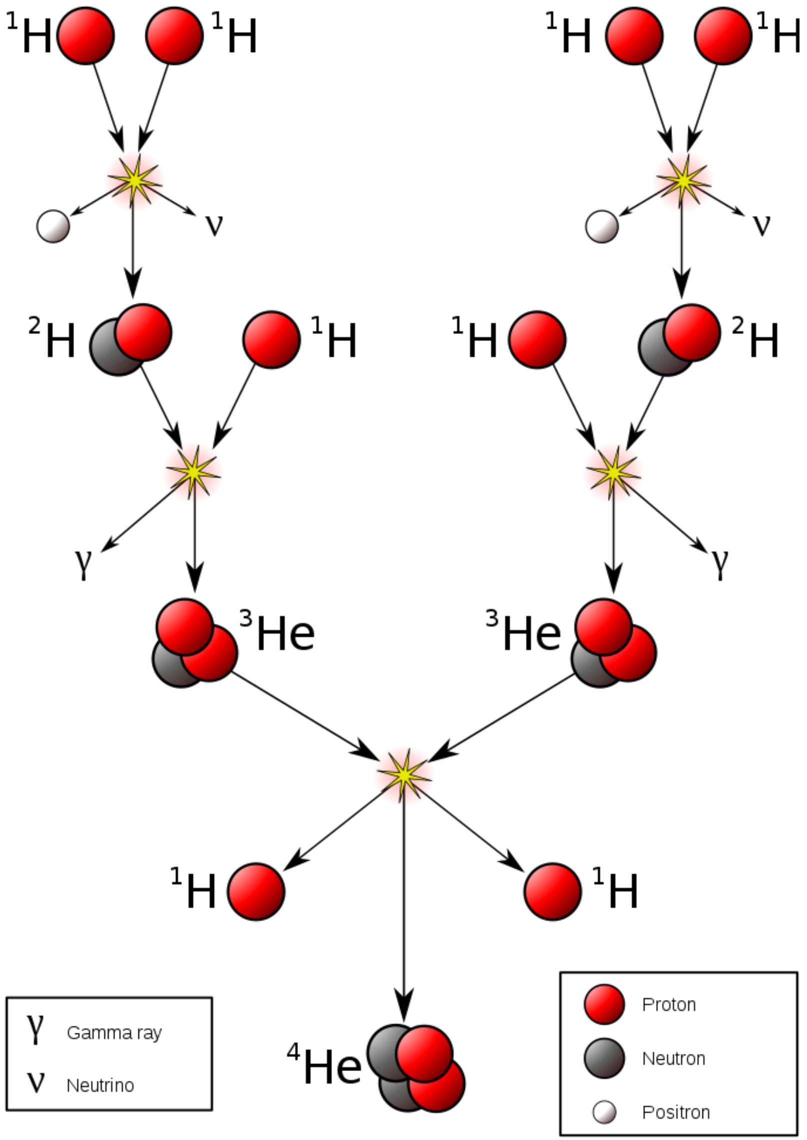
- First step: $p + p \rightarrow {}^2H + e^+ + \nu$ ($Q = 0.42$ MeV) → very small reaction rate (5×10^{-18} s^{-1} per proton) → bottleneck process → but number of protons in the Sun ≈ 10^{56}
- Second step: $p + {}^2H \rightarrow {}^3He + \gamma$ ($Q = 5.49$ MeV) → D-D reaction is very unlikely because the number of deuterons is small (1 ^2H for ≈ 10^{18} ^1H)
- Third step: ${}^3He + {}^3He \rightarrow {}^4He + 2{}^1H + \gamma$ ($Q = 12.86$ MeV) → ^3He-p reaction is not possible (^4Li does not exist as a bound system) and ^3He-D is unlikely because density of D is very low and because D is rapidly converted to T
- The net reaction called the proton-proton (pp) chain is thus the conversion of 4 protons to helium : $4p \rightarrow {}^4He + 2e^+ + 2\nu$ with a total $Q = 2 \times (0.42 + 5.49) + 12.86 + 4 \times 0.51 = 26.72$ MeV

Solar fusion: pp chain (2)

$\tau \sim 10^9$ years

$\tau \sim 1$ s

$\tau \sim 10^6$ years



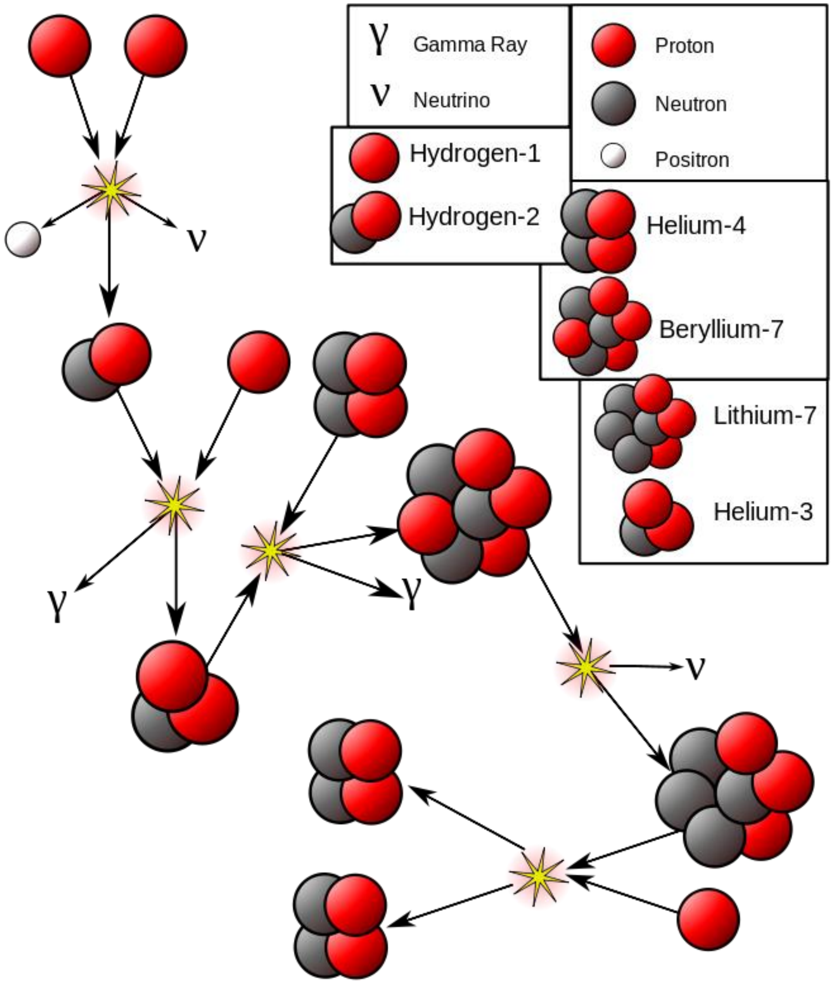
Solar fusion: pp chain (3)

- More precisely this chain is called pp1 and is dominating for T ≈ 10-14 10^6 K
- An alternative chain (pp2) is dominating for T ≈ 14-23 10^6 K → $^3He + {}^4He \rightarrow {}^7Be + \gamma$ → $^7Be + e^- \rightarrow {}^7Li + \nu$ → $^7Li + p \rightarrow 2\,{}^4He$
- pp3 is dominating for T > 23 10^6 K → $^3He + {}^4He \rightarrow {}^7Be + \gamma$ → $^7Be + p \rightarrow {}^8B + \gamma$ → $^8B \rightarrow {}^8Be + e^+ + \nu$ → $^8Be \rightarrow 2\,{}^4He$

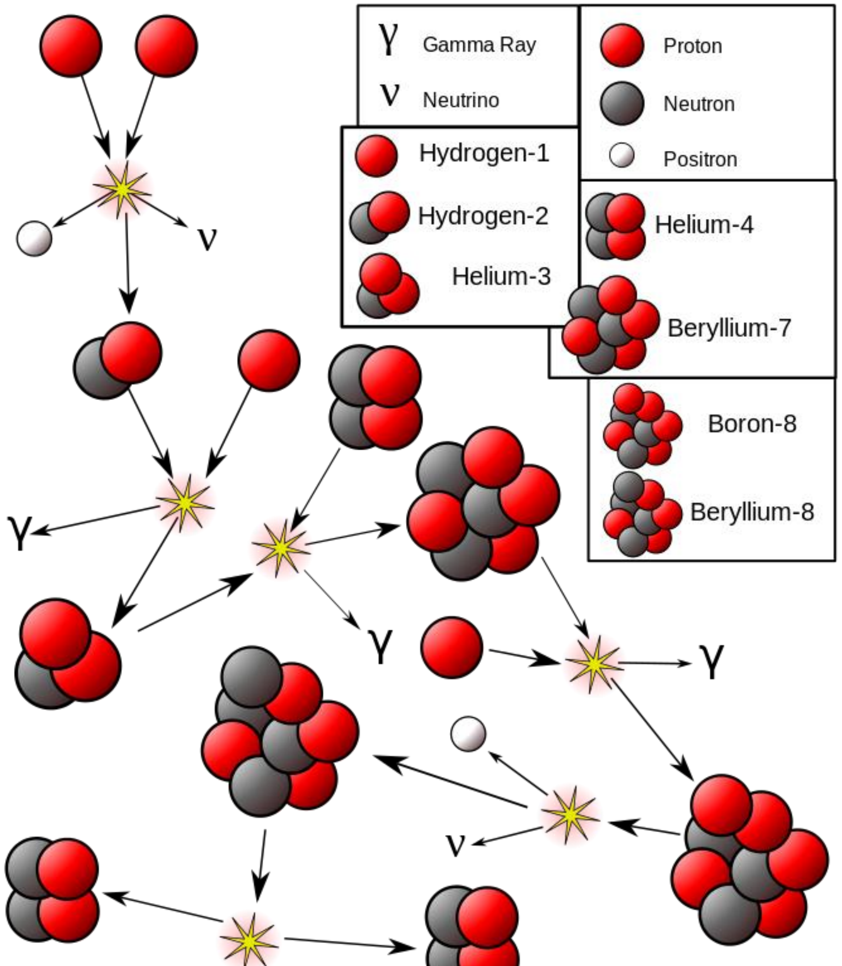
- The net reaction and the net *Q* value are the same for these three possible paths

Solar fusion: pp chain (4)

pp2



pp3



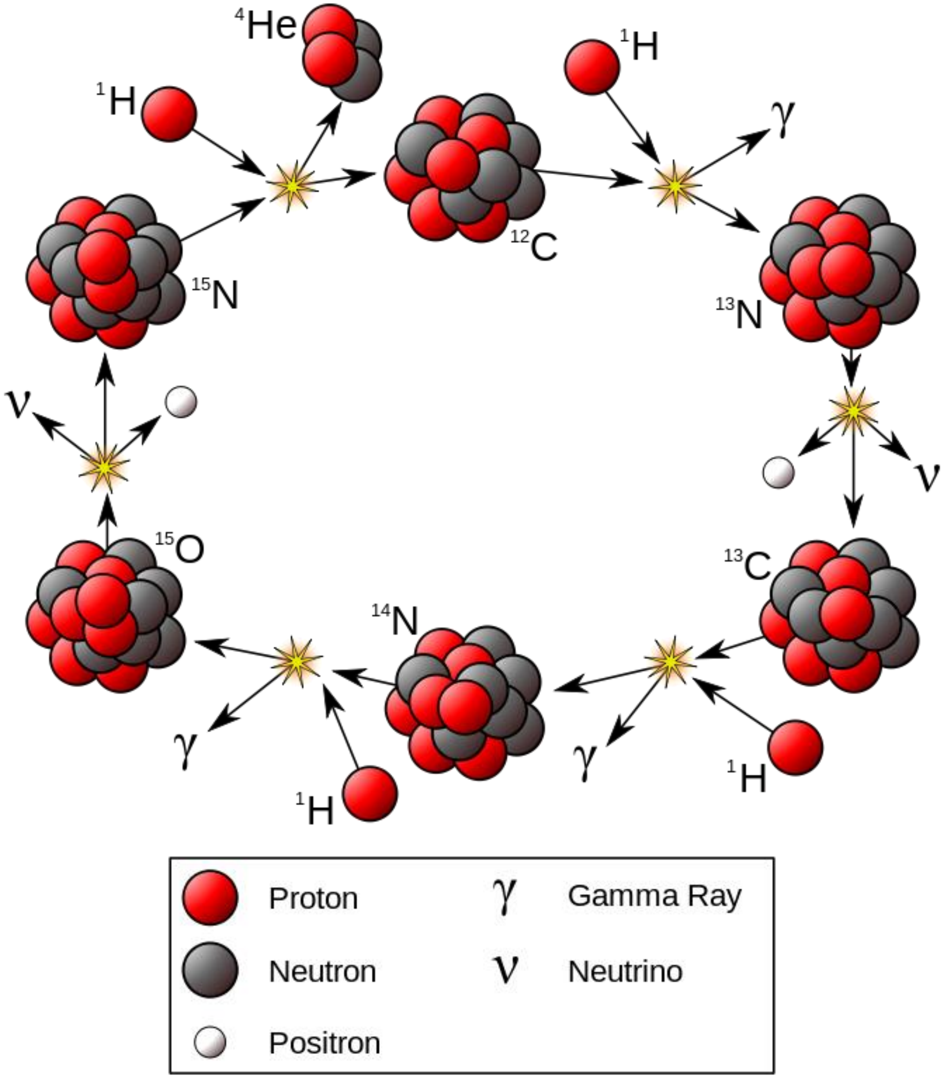
Solar fusion: CNO cycle (1)

- Heavier elements are present in a star → different series of fusion reactions can occur
- One important process is the carbon or CNO cycle →

$$\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}$$

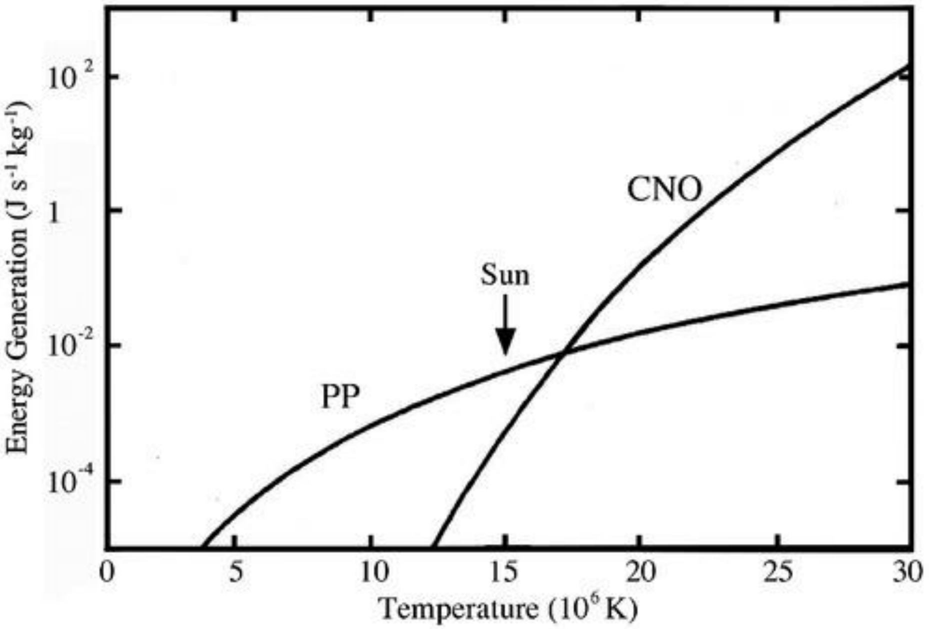
- ^{12}C is neither created nor destroyed → acts as a catalyst to aid in the fusion process
- The net process is 4p $\rightarrow$ ^{4}He + 2e^{+} + 2ν as in pp chain and *Q* is the same

Solar fusion: CNO cycle (2)



Solar fusion: CNO cycle (3)

- The CNO cycle proceeds more rapidly than the pp cycle (no deuterium bottleneck)
- However the Coulomb barrier is 6 or 7 times higher for proton reactions with carbon and nitrogen than for proton-proton reactions
- The CNO cycle is thus dominant at higher *T* (additional thermal energy is needed to increase the probability to penetrate the Coulomb barrier)



Solar fusion: After…

- Once a star has consumed its hydrogen → helium fusion reactions takes place with $3\,^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C}$ at the higher T needed to penetrate the Coulomb barrier
- Other reactions involving fusion of light nuclei and α-particle capture continues to release energy
- The process ends near $^{56}\mathrm{Fe}$ → beyond it there is no energy gain in combining nuclei

Controlled fusion: General remarks

- High T ($\sim 10^8$ K → mean particle kinetic energies of 10 keV) → the atoms are ionized → fuel is a hot mixture of clouds of positive ions and negative electrons (overall electrically neutral) → plasma (see Y. Louis, Statistical physics and plasma physics)
- In a star → gravitational force confine the plasma → allows high temperature and pressure → fusion
- In Earth → gravitational confinement is impossible → magnetic or inertial confinement (electrostatic confinement is also possible but not mature)

Controlled fusion: Lawson criterion (1)

- In a plasma → high agitation of ions and electrons → many collisions between → to obtain fusion 3 quantities have to be considered →
  1. Temperature *T*
  2. Density *N*
  3. Confinement time τ
- Considering a D-T plasma → we assume that →
  - densities of D and T are each equal to *n*/2
  - *Q* is the energy released per reaction (17.6 MeV for D-T)
  - τ is the confinement time during which reactions occur
- The energy released per unit volume from fusion reactions in the plasma is →

$$E_f = \frac{1}{4} n^2 \langle \sigma v \rangle Q \tau$$

Controlled fusion: Lawson criterion (2)

- The thermal energy per unit volume needed to raise both ions and electrons to temperature T is $3/2nkT$ (with $n = n_e$) → the total thermal energy is →

$$E_{th} = 3nkT$$

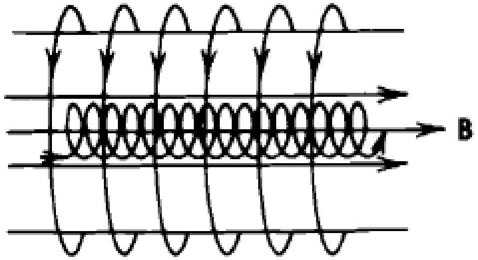
- Energy E_{th} is supplied to heat the plasma → during time τ plasma is confined and energy E_f can be extracted
- The reactor shows a net energy gain if →

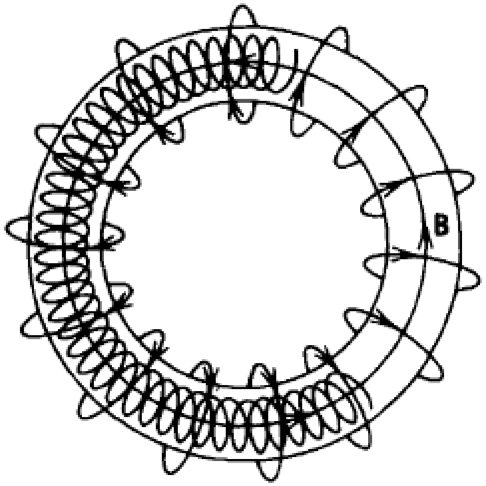
$$\begin{aligned} E_f &> E_{th} \\ \frac{1}{4}n^2\langle\sigma v\rangle Q\tau &> 3nkT \\ n\tau &> \frac{12kT}{\langle\sigma v\rangle Q} \end{aligned}$$

- This is called the Lawson criterion → represents the goal of reactor designers → for D-T reaction → $n\tau > 10^{20}$ s/m^3

Controlled fusion: Magnetic confinement (1)

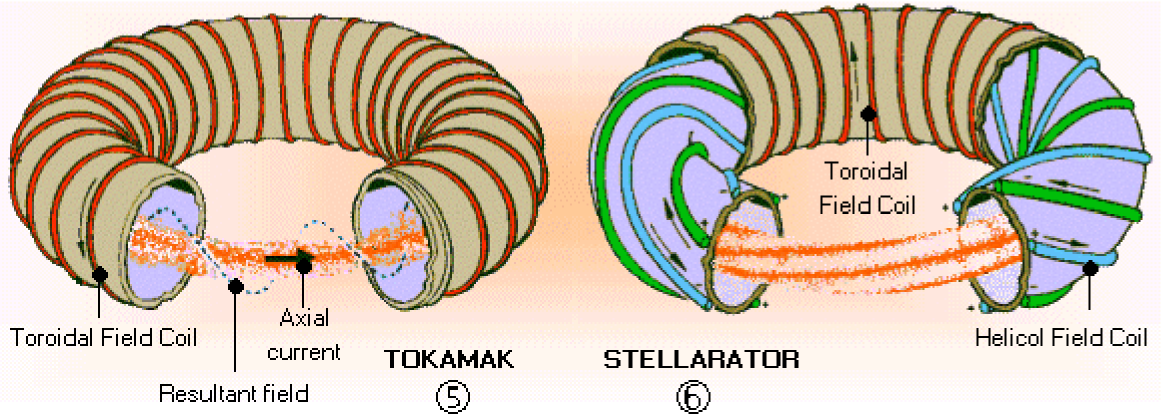
- In a magnetic confinement → the plasma is trapped and maintained at high temperature
- It must be confined in a limited immaterial box far from any material wall → the simplest magnetic confinement is a uniform magnetic field → charged particles spiral about the field direction
- To prevent the loss of particles along the axis → the line forms a torus → the spiral is kept in a ring





Controlled fusion: Magnetic confinement (2)

- In a toroidal geometry → the field is weaker at larger radii → as a particle spirals it sees a region of lower field which lets the spiral radius become larger → the particle approach the outer wall
- To reduce this effect a magnetic field component along the surface of the toroid is introduced → the poloidal field → helical path
- It can be achieved using
  - a set of external coils → stellarator
  - a current along the axis of the toroid through the plasma itself → tokamak



Controlled fusion: Plasma heating (1)

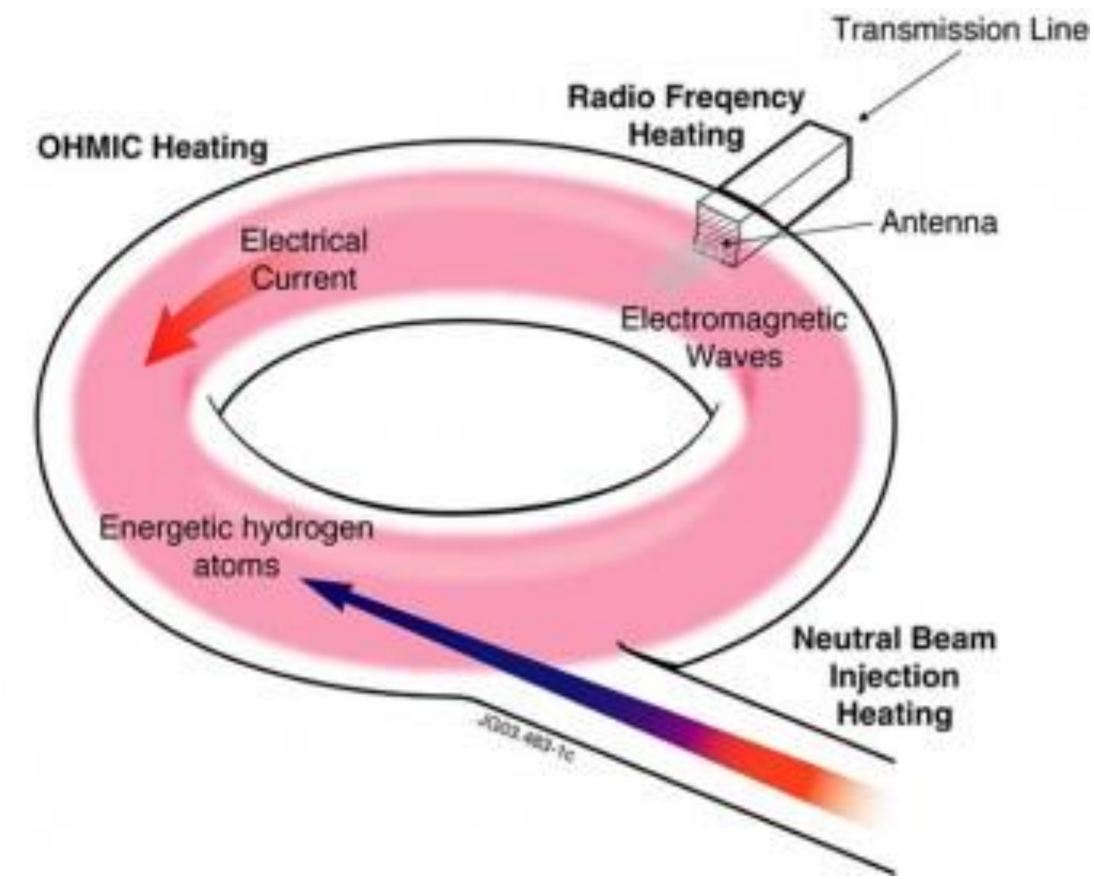
- The plasma is created inside the confinement structure at temperature too low to reach fusion → 3 methods are possible to heat the plasma up

1. The current in the plasma itself is used to heat the plasma by Joule effect → effective to T ≈ 10^7 K → beyond plasma resistivity becomes too weak → effectiveness ↘ (impossible in stellarator ↔ no central current)
2. Heating by injection of neutrals → a beam of ions (outside the machine) is created → neutralized before entering the plasma → collisions transfer energy → T ↗
3. Absorption of energy from electromagnetic waves → heating is transmitted to the plasma by antennas covering part of the confinement area

Controlled fusion: Plasma heating (2)

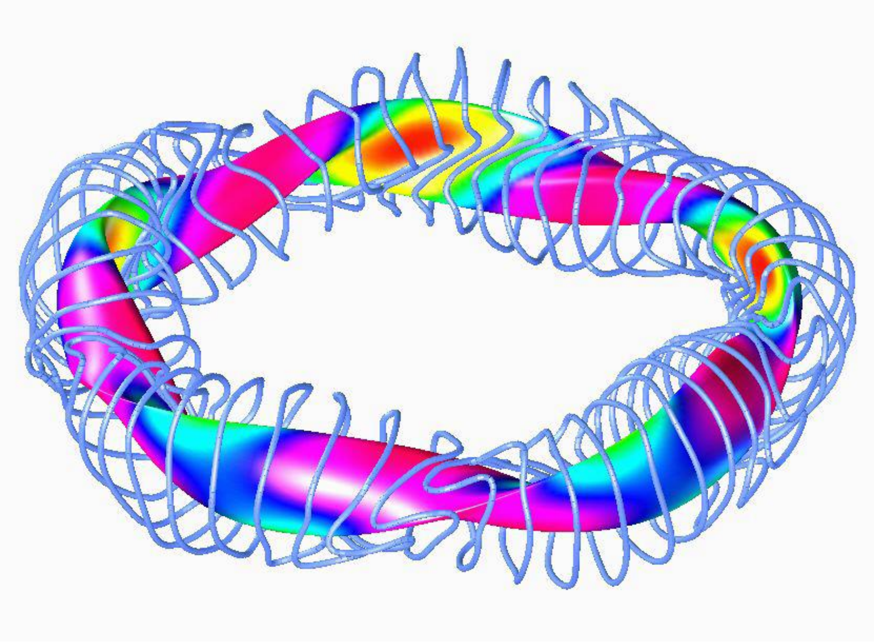
- Generally in fusion reactor with magnetic confinement → temperature is raised by a combination of the 3 methods
- When the number of fusion reactions ↗ → the energy carried by the helium nuclei is confined in the plasma → heating
- If this contribution becomes equal to the energy lost by the plasma → previous heating methods are no more necessary → plasma is self-maintained → ignition process
- We define the amplification factor Q as the ratio between the power generated by the plasma and the heating power injected into the plasma → If $Q = \infty$ → plasma is self-maintained → If $Q = 1$ → plasma supplies as much energy as injected → « break even »
- European tokamak JET (Joint European Torus) is close to break even ($Q \approx 0.65$)

Controlled fusion: JET heating system



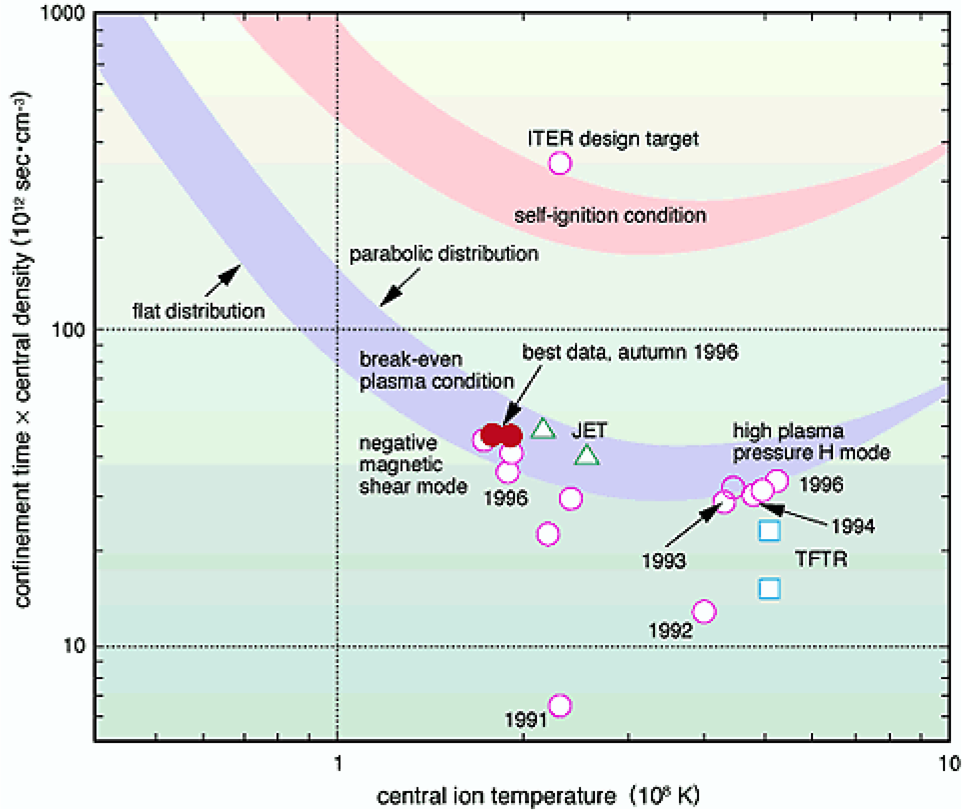
Controlled fusion: Stellarator

- The stellarator attempts to create a natural twist plasma path using external magnets
- In first tests → stellarator suffers from instabilities and plasma leakage
- Now → development of computers → new calculations possibilities → building of the largest stellarator (Wendelstein 7-X) at the Max Planck Institute of Plasma Physics in 2015



Controlled fusion: Tokamak

- Imagined by Tamm and Sakharov in 1950 → tokamak = **to**roïdalnaïa **ka**mera s **ma**gnitnymi **k**atushkami = toroidal chamber with magnetic coils
- Actually → the Lawson criterion is not reached (JET) → most interesting way to achieve controlled fusion for energy production → ITER project = International Thermonuclear Experimental Reactor

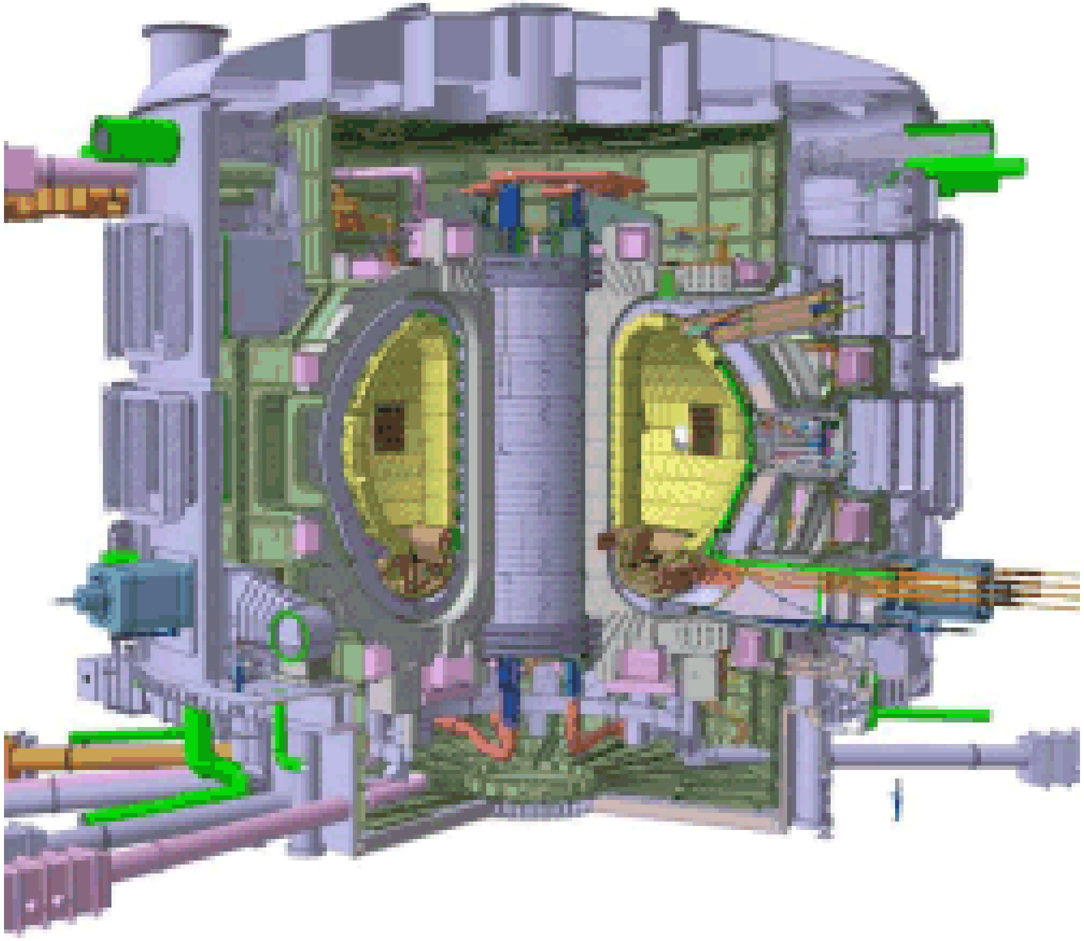


Controlled fusion: JET

- First plasma production: 1983
- Pulse duration: 20 s
- Maximum magnetic field (toroidal): 3.45 T
- Maximum plasma current: 5 MA
- Maximum injected power: 25 MW
- Mass (nucleus) : 2 800 tons

Controlled fusion: ITER (1)

- First expected test → 2025
- Electricity production → 2050



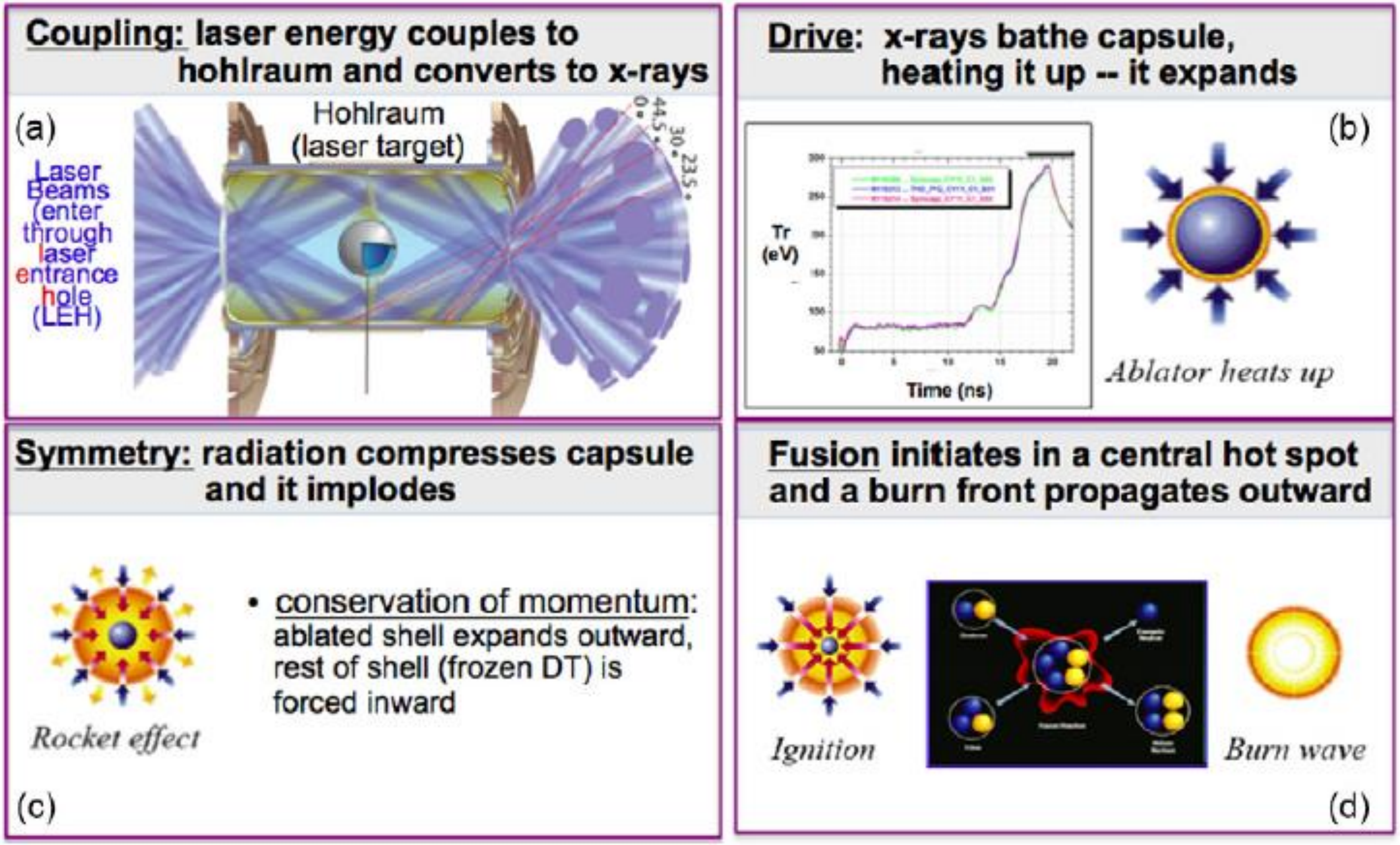
Controlled fusion: ITER (2)

- Plasma volume: 840 m^3
- Plasma mass: $\sim$ g
- Maximum plasma current: 15 MA
- Maximum magnetic field (toroidal): 5.3 T
- Pulse duration: 6 min to 1 h
- Expected Q: $\sim$ 10

Controlled fusion: Inertial confinement (1)

- We consider a microball generally in gold (« hohlraum ») enclosing a pellet of gaseous deuterium and tritium encased in ablator material
- Laser energy enters the hohlraum target → strikes the hohlraum walls → is converted to X-radiation
- X-radiation fills the hohlraum → creating a radiation oven that bathes the capsule → the ablator heats up
- The ablated shell expands outward → the remainder of the capsule is compressed inward
- Fusion initiates in a central hot spot where the ion temperature is high → a burn front propagates outward

Controlled fusion: Inertial confinement (2)



Coupling: laser energy couples to hohlraum and converts to x-rays

Drive: x-rays bathe capsule, heating it up -- it expands

Symmetry: radiation compresses capsule and it implodes

- conservation of momentum: ablated shell expands outward, rest of shell (frozen DT) is forced inward

Fusion initiates in a central hot spot and a burn front propagates outward