Chapter VII:
Gamma decay

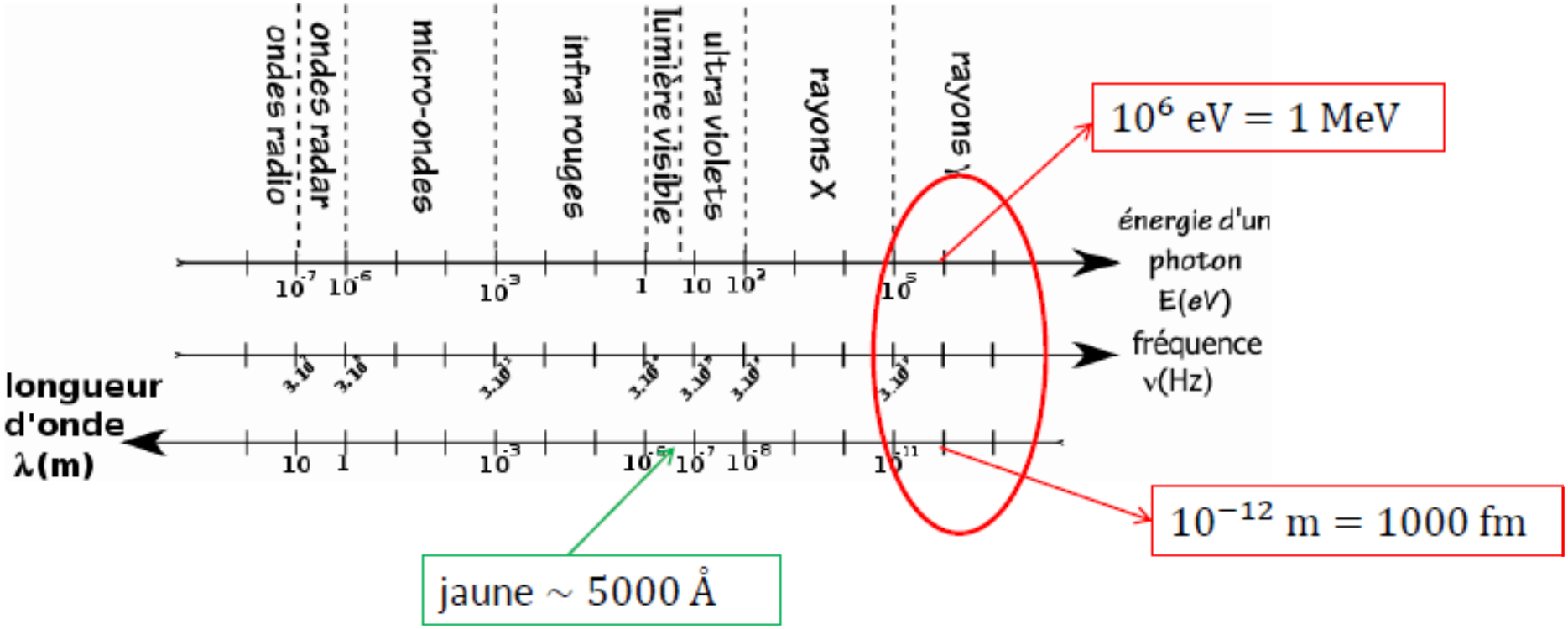
Summary

1. General remarks
2. Energy release in γ decay
3. Classical theory of radiation
4. Quantum mechanical theory
5. Selection rules
6. Internal conversion
7. Pairs creation
8. Lifetime for γ emission
9. γ-ray absorption and Mössbauer effect

General remarks (1)

- Most of α and β decays and generally most nuclear reactions leave the final nucleus in an excited state → emission of one (or more) γ
- Unlike the 2 other types of decay → no change in the element with γ decay → decay from an excited state to a lower (possibly ground) state
- γ-rays are photons (or electromagnetic radiation) like X-rays or visible light
- Due to energy difference between nuclear states → γ-rays have generally energies in the range 0.1 – 10 MeV → corresponding to λ in 10^4 – 100 fm
- The analyze of γ-ray emission (γ spectroscopy) is the standard technique for nuclei studies
- Competition with internal conversion → e^- emission

General remarks (2)



Energy release in γ decay

- We consider the decay of a nucleus of mass *M* at rest from an initial excited state E_i to a final state E_f
- To conserve linear momentum → final nucleus is not at rest → recoil momentum corresponding to recoil energy (non-relativistic) → $T_R = p_R^2/2M$
- Conservation of total energy and momentum →

$$\begin{aligned} E_i &= E_f + E_\gamma + T_R \\ 0 &= \boldsymbol{p}_R + \boldsymbol{p}_\gamma \end{aligned}$$

- Using $E_\gamma = cp_\gamma$ → $T_R = E_\gamma^2/2Mc^2 \ll E_\gamma$
- We define $\Delta E = E_i - E_f$ →

$$E_\gamma = \Delta E - \frac{E_\gamma^2}{2Mc^2} \simeq \Delta E - \frac{\Delta E^2}{2Mc^2}$$

- Very small correction → for *M* = 50 uma and E_γ = 1 MeV → $\Delta E/2Mc^2$ = 10 eV

Classical theory of radiation: introduction

- The emission of electromagnetic waves can be treated as a classical wave phenomenon or else as a quantum phenomenon
- For radiation from individual atom or nucleus → the quantum description is more appropriate but can be easily understood with classical description
- Static (= constant in time) distributions of charges and currents gives static electric fields → studied in chapter 2 in terms of multipole moments
- If the charge and current distributions vary in time (particularly with sinusoidal variation with circular frequency ω) → radiation field is produced
- Radiation field can be analyzed in terms of its multipole character

Classical theory of radiation: multipole character (1)

- Classical mechanics → the electrostatic potential of a charge distribution $\rho_e(\boldsymbol{r})$ is given by:

$$V(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\int_{V'}\frac{\rho_e(\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r'}|}d\boldsymbol{r'}$$

- When treating radiation → we are only interested in the potential outside the charge (= nucleus) which is localized → $r' \ll r$ → the denominator can be expanded in power series
- The norm $|\boldsymbol{r'} - \boldsymbol{r}| = (r^2 - r'^2 - 2rr'\cos\theta)^{1/2} = r(1+(r'/r)^2 - 2(r'/r)\cos\theta)^{1/2}$ → considering $R = r'/r$ and $\epsilon = R^2 - 2R\cos\theta$ (that is considered as small) →

$$\frac{1}{|\boldsymbol{r}-\boldsymbol{r'}|} = \frac{1}{r}\frac{1}{\sqrt{1+\epsilon}} = \frac{1}{r}\left(1 - \frac{1}{2}\epsilon + \frac{3}{8}\epsilon^2 - \frac{5}{16}\epsilon^3 + \dots\right)$$

Classical theory of radiation: multipole character (2)

- Replacing *R* in this expression →

$$\frac{1}{r}\frac{1}{\sqrt{1+\epsilon}} = \frac{1}{r}\left[1 + R\cos\theta + R^2\left(\frac{3\cos^2\theta}{2} - \frac{1}{2}\right) + R^3\left(\frac{5\cos^3\theta}{2} - \frac{3\cos\theta}{2}\right) + \dots\right]$$

- In the coefficients to the powers of *R* → Legendre polynomials $P_l(\cos\theta)$ →

$$\frac{1}{r}\frac{1}{\sqrt{1+\epsilon}} = \frac{1}{r}\sum_{l=0}^{\infty} R^l P_l(\cos\theta) = \frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{r'}{r}\right)^l P_l(\cos\theta)$$

- The potential becomes

$$V(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r}\int_{V'}\rho_e(\boldsymbol{r'})\sum_{l=0}^{\infty}\left(\frac{r'}{r}\right)^l P_l(\cos\theta)d\boldsymbol{r'}$$

Classical theory of radiation: multipole character (3)

- The various terms in the expansion are the multipoles →

$$\text{Monopole} \rightarrow \frac{1}{4\pi\epsilon_0}\frac{1}{r}\int_{V'}\rho_e(\boldsymbol{r'})d\boldsymbol{r'}$$

$$\text{Dipole} \rightarrow \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\int_{V'}\rho_e(\boldsymbol{r'})r' P_1(\cos\theta)d\boldsymbol{r'} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\int_{V'}\rho_e(\boldsymbol{r'})r'\cos\theta d\boldsymbol{r'}$$

$$= \frac{\hat{\boldsymbol{r}}\boldsymbol{d}}{4\pi\epsilon_0 r^2} \quad \text{with } d\text{, the electric dipole moment}$$

$$\text{Quadrupole} \rightarrow \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\int_{V'}\rho_e(\boldsymbol{r'})r'^2 P_2(\cos\theta)d\boldsymbol{r'}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\int_{V'}\rho_e(\boldsymbol{r'})r'^2\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)d\boldsymbol{r'}$$

$$\text{Octupole} \rightarrow \ldots$$

- This type of expansion can be carried out for magnetic potential and for electromagnetic time-dependent field

Classical theory of radiation: multipole character (4)

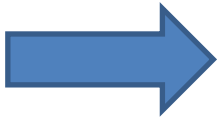
- The potential can also be written →

$$V(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_0}{r} + \frac{Q_1}{r^2} + \frac{Q_2}{r^3} + \dots\right]$$

$$Q_0 = \int_{V'} \rho_e(\boldsymbol{r'}) d\boldsymbol{r'}$$

$$Q_1 = \int_{V'} \rho_e(\boldsymbol{r'}) r' \cos\theta d\boldsymbol{r'}$$

$$Q_2 = \frac{1}{2}\int_{V'} \rho_e(\boldsymbol{r'}) r'^2 (3\cos^2\theta - 1) d\boldsymbol{r'}$$

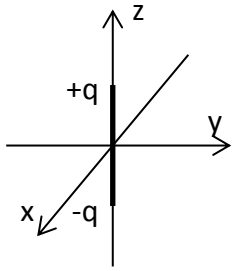


$$V(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r}\sum_l \frac{Q_l}{r^l}$$

- Q_0 is the total charge, Q_1 is the dipole moment, Q_2 is the quadrupole moment, …
- In classical theory → the higher *l*'s diminish in influence as *r* grows ↔ in quantum theory → the higher *l*'s are associated with weaker transitions

Classical theory of radiation: electric dipole field (1)

- In a static electric dipole → we have a positive charge *q* located on the z-axis at *z* = *l* and a charge of the opposite sign on the z-axis at *z* = −*l* → pure electric dipole:



$$\begin{aligned} \rho(\boldsymbol{r}) &= q[\delta(x)\delta(y)\delta(z-l) - \delta(x)\delta(y)\delta(z+l)] \\ Q_0 &= 0 \\ Q_1 &= ql + (-q)(-l) = 2ql = d \\ Q_{l\geq 2} &= 0 \end{aligned}$$

- Under a parity operation **r** → −**r** the configuration is opposite to its original configuration → the parity of the electric dipole radiation is Π(E1) = −1

Classical theory of radiation: electric dipole field (2)

- We can produce electromagnetic radiation fields by varying the dipole moments → we can allow the charges to oscillate along the z-axis → $r(t) = 2l\cos(\omega t)$ → electric dipole radiation field
- The power radiated is given by the integral of the energy flux (as given by the Poynting vector) over all solid angles → Larmor equation for a non-relativistic accelerated charge (with a the acceleration) →

$$P = \frac{q^2\langle a^2\rangle}{6\pi\epsilon_0 c^3}$$

- For an electric dipole →

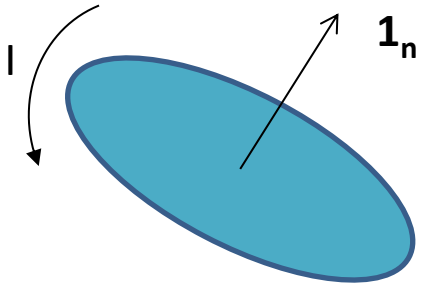
$$r(t) = 2l\cos\omega t \quad \rightarrow \quad a = -2l\omega^2\cos\omega t$$

$$\rightarrow \quad \langle a^2\rangle = \frac{4l^2\omega^4}{2}$$

$$\rightarrow \quad P = \frac{4l^2q^2\omega^4}{12\pi\epsilon_0 c^3} = \frac{d^2\omega^4}{12\pi\epsilon_0 c^3}$$

Classical theory of radiation: magnetic dipole field

- A static magnetic dipole can be represented as a circular current loop of current *I* enclosing area *S* → the magnetic moment is $\mu = IS$ and is oriented along the surface normal $\mathbf{1}_n$



- Under a parity operation $\mathbf{r} \rightarrow -\mathbf{r}$ there no change of sign → $\Pi(\mathbf{r} \rightarrow -\mathbf{r}) = \Pi(x \rightarrow -x)\ \Pi(y \rightarrow -y) \rightarrow \Pi(M1) = +1$
- To produce electromagnetic radiation → we vary the current such as $\mu(t) = IS\cos(\omega t)$ → the average power radiated is

$$P = \frac{\omega^4 \mu^2}{12\pi\epsilon_0 c^5}$$

Classical theory of radiation: generalization to multipoles (1)

- Without considering a detailed discussion of electromagnetic theory (see J. D. Jackson, *Classical Electrodynamics*) → these properties can be extended to multipole radiation in general
- We define the order of the radiation 2^L (L = 1 for dipole, L = 2 for quadrupole,…) and we note E for electric and M for magnetic
- The parity of the radiation field is

$$\begin{aligned} \Pi(EL) &= (-1)^L \\ \Pi(ML) &= (-1)^{L+1} \end{aligned}$$

- Electric and magnetic multipoles of the same order have opposite parity

Classical theory of radiation: generalization to multipoles (2)

- The radiated power is (with σ = *E* or *M*):

$$P(\sigma L) = \frac{2(L+1)c}{\epsilon_0 L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+2}[m(\sigma L)]^2$$

- m(σL) is the amplitude of the time-varying electric or magnetic multipole moment and the double factorial (2*L*+1)!! indicates (2*L*+1) $\times$ (2*L*-1) $\times$ … $\times$ 3 $\times$ 1
- The generalized multipole moment m(σL) differs for *L* = 1 from the electric dipole moment *d* and the magnetic dipole moment μ through some relatively unimportant numerical factors of order unity

Quantum mechanical theory: transition from classic (1)

- Classical theory to quantum theory → quantization of the sources of radiation field = the multipole elements
- In previous equation → replacement of the multipole moments by appropriate multipole operators changing nucleus from its initial state ψ_i to final state ψ_f
- As for β emission → decay probability is governed by a matrix element → the matrix element of the multipole operator →

$$m_{fi}(\sigma L) = \int \psi_f^* m(\sigma L) \psi_i dv$$

- The integration is performed over the volume of the nucleus
- The function of m_{fi} is to change the nuclear state ψ_i into ψ_f while simultaneously creating a photon of proper energy, parity and multipole order

Quantum mechanical theory: transition from classic (2)

- If we consider the classical radiated power $P(\sigma L)$ as the energy radiated per unit time in form of the photon (each which energy $\hbar\omega$) → the probability per unit time for photon emission (= decay constant) is:

$$\lambda(\sigma L) = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L+1)}{\epsilon_0 \hbar L[(2L+1)!!]^2} \left(\frac{\omega}{c}\right)^{2L+1} [m_{fi}(\sigma L)]^2$$

- To evaluate this expression → evaluation of m_{fi} is needed → thus knowledge of initial and final wave functions
- Simplification → the transition is due to a **single** proton that changes its state (from one shell-model state to another for instance) = Weisskopf assumption

Quantum mechanical theory: electric transitions (1)

- For electric transitions → the multipole operator includes a term of the form $er^L Y_{LM}(\theta, \phi)$ (with Y_{LM} the spherical harmonics) → for $L = 1$ it reduces to $er\cos\theta$ (dipole) and for $L = 2$ to $er^2(3\cos\theta - 1)$ (quadrupole) as expected
- For the radial part of the wave functions → ψ_i and ψ_f are constant for $r < R$ (nucleus radius) and equal to 0 for $r > R$ → the radial part of the transition probability becomes

$$\frac{\int_0^R r^2 r^L dr}{\int_0^R r^2 dr} = \frac{3}{L+3} R^L$$

where the denominator is included for normalization

Quantum mechanical theory: electric transitions (2)

- By assuming that the angular integrals can be replaced by unity →

$$\lambda(EL) \cong \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{e^2}{4\pi\epsilon_0\hbar c} \left(\frac{E}{\hbar c}\right)^{2L+1} \left(\frac{3}{3+L}\right)^2 cR^{2L}$$

- Considering $R = R_0 A^{1/3}$ (with R_0 = 1.2 fm by convention) → we obtain the following estimates for the lower multipole orders (with λ in s^{-1} and E in MeV) →

$$\begin{aligned}
\lambda(E1) &= 1.0 \times 10^{14} A^{2/3} E^3 \\
\lambda(E2) &= 7.3 \times 10^{7} A^{4/3} E^5 \\
\lambda(E3) &= 34 A^2 E^7 \\
\lambda(E4) &= 1.1 \times 10^{-5} A^{8/3} E^9
\end{aligned}$$

Quantum mechanical theory: magnetic transitions (1)

- For magnetic transitions → the radial integral includes a term r^{L-1} → it becomes:

$$\frac{\int_0^R r^2 r^{L-1} dr}{\int_0^R r^2 dr} = \frac{3}{L+2} R^{L-1}$$

- The magnetic operator also includes a factor depending on the nuclear magnetic moment of the proton (μ_p) → if we also neglect several factors of order unity (m_p being the proton mass)→

$$\lambda(ML) \cong \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \left(\mu_p - \frac{1}{L+1}\right)^2 \left(\frac{\hbar}{m_p c}\right)^2 \times \frac{e^2}{4\pi\epsilon_0 \hbar c} \left(\frac{E}{\hbar c}\right)^{2L+1} \left(\frac{3}{2+L}\right)^2 c R^{2L-2}$$

Quantum mechanical theory: magnetic transitions (2)

- It is usual to consider:

$$\left(\mu_p - \frac{1}{L+1}\right)^2 \cong 10$$

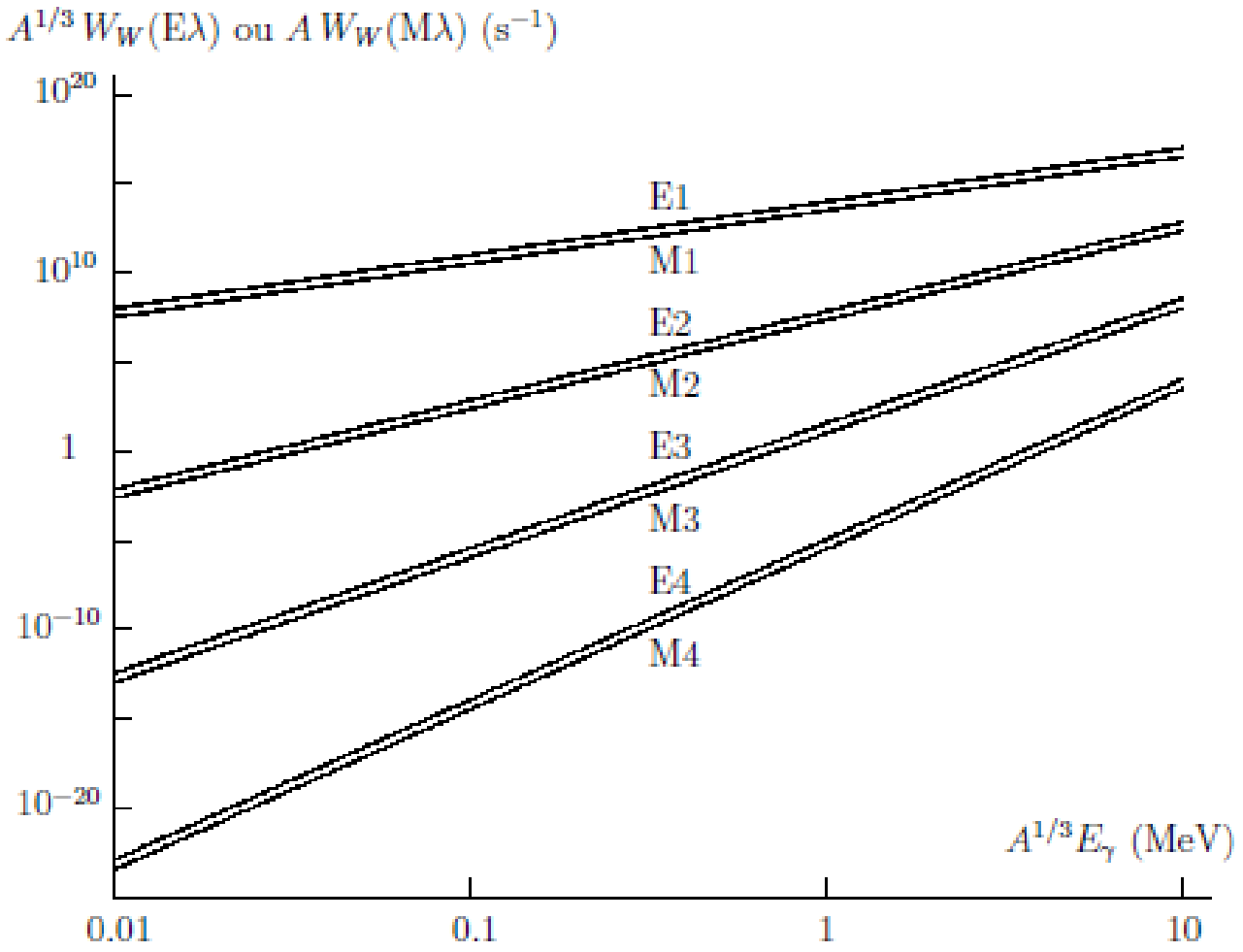
- In these conditions the following estimates for the lower multipole orders are obtained →

$$\begin{aligned}
\lambda(M1) &= 5.6 \times 10^{13} E^3 \\
\lambda(M2) &= 3.5 \times 10^{7} A^{2/3} E^5 \\
\lambda(M3) &= 16 A^{4/3} E^7 \\
\lambda(M4) &= 4.5 \times 10^{-6} A^2 E^9
\end{aligned}$$

Quantum mechanical theory: Weisskopf estimates (1)

- These estimates for E and M transitions are known as Weisskopf estimates
- They can be not very accurate (compared with measured results) but they provide reasonable relative comparison of the transition rates
- If an observed γ decay rate is many orders of magnitude smaller than the Weisskopf estimates → poor match-up of initial and final wave functions slows the transition
- If the transition rate is much greater than Weisskopf estimates → more than one single nucleon is responsible for the transition
- Some observations → lower multipolarities are dominant (L ↘ by 1 → λ ↘ by 10^{-5}) and for a given L → λ(E)/λ(M) ≈ 100 for heavy nuclei)

Quantum mechanical theory: Weisskopf estimates (2)



Quantum mechanical theory: « full quantum » (1)

- Use of the Fermi Golden Rule →

$$\lambda = \frac{2\pi}{\hbar} |\langle f;1| V_{int} |i;0\rangle|^2 \rho(E)$$

- $\rho(E) = dn/dE$ (with n the number of states) is the density of states per unit of energy of the emitted photon et per unit of solid angle
- V_{int} is the operator of interaction between the nucleus and its electromagnetic field
- The $|j;n\rangle$ state characterizes systems formed of a nuclear state $|j\rangle$ of energy E and of a photon state $|n\rangle$ → in the initial state $|\mathrm{i}\rangle$ there is no photon → in final state $|\mathrm{f}\rangle$ one photon is emitted with an energy →

$$E_\gamma = \hbar c k \approx E_i - E_f$$

Quantum mechanical theory: « full quantum » (2)

- By considering nucleus + radiation to be enclosed in a cavity of volume $V = L^3$ and quantization of the momentum p (or equivalently the wave number k) in all 3 directions → $k_i = (2\pi/L)n_i$ with n_i being an integer → similar calculation as made for β decay allows to write for all space →

$$dn = 4\pi k^2 dk \frac{L^3}{(2\pi)^3}$$

- If we consider just a small solid angle $d\Omega$ instead of 4π and since the photon energy $E = \hbar ck = \hbar\omega \approx E_i - E_f$ →

$$\rho(E) = \frac{dn}{dE} = \frac{L^3}{(2\pi)^3}\frac{k^2}{\hbar c}d\Omega = \frac{L^3}{(2\pi)^3}\frac{\omega^2}{\hbar c^3}d\Omega$$

Quantum mechanical theory: « full quantum » (3)

- Then we have to look for the potential of the interaction → the interaction of a nucleus formed of *Z* protons and *N* neutrons with the electromagnetic field can be expressed in terms of vector potential **A** →

$$V_{int} = -\frac{e}{m_p}\sum_{j=1}^{A}[g_{lj}\boldsymbol{A}(\boldsymbol{r}_j)\boldsymbol{p}_j + \frac{1}{2}g_{sj}\boldsymbol{B}(\boldsymbol{r}_j)\boldsymbol{S}_j]$$

- In this equation → ***B*** is the magnetic induction, ***S*** and g_{sj} are the spin and the gyromagnetic ratio of the particle, g_{lj} = 1/0 for proton/neutron
- The first term characterize the electric transitions and the second one the magnetic transitions

Quantum mechanical theory: « full quantum » (4)

- Considering the quantization of the operators ***A*** and ***B*** → we obtain the second quantization of the V_{int} operator →

$$V_{int} = \frac{1}{2\pi} \sum_q \int d\boldsymbol{k} \sqrt{\frac{\hbar c}{4\pi\epsilon_0 k}} (H_a a_{\boldsymbol{k}q} + H_e a^\dagger_{\boldsymbol{k}q})$$

- a_k and $a^\dagger_k$ are called annihilation and creation operators → they annihilates/create one photon of wave number ***k***
- H_a and H_e are the absorption and emission operators
- It is possible to show that →

$$\langle f; 1 | V_{int} | i; 0 \rangle = \frac{1}{2} \left(\frac{\hbar c}{4\pi\epsilon_0 k} \right)^{1/2} \langle f | H_e | i \rangle$$

Quantum mechanical theory: « full quantum » (5)

- The transition probability per unit time is thus →

$$\lambda = \frac{1}{4\pi\epsilon_0}\frac{k}{2\pi\hbar}|\left\langle f\right|H_e\left|i\right\rangle|^2 d\Omega$$

- The H_e operator is $\propto$ to the electromagnetic multipolar operator $m_{fi}(\sigma L)$ → as defined previously
- To calculate the transition probability for a given σL → it is necessary to evaluate

$$\left\langle f\right|m_{fi}(\sigma L)\left|i\right\rangle$$

- The easiest way to evaluate previous quantity is to consider that only one nucleon participate to the transition → we come back to the Weisskopf approximation

Quantum mechanical theory: Weisskopf units

- Very often true transitions are expressed in Weisskopf units → allows to estimate the difference between simple values resulting from the Weisskopf approximation and the true result (generally obtained from experiment)
- The Weisskopf unit for *Eλ* is

$$B_W(EL) = \frac{1}{4\pi}\left(\frac{3}{L+3}\right)^2 (1.2)^{2L} A^{2L/3} e^2 (\mathrm{fm})^{2L}$$

- The Weisskopf unit for *Mλ* is

$$B_W(ML) = \frac{10}{\pi}\left(\frac{3}{L+3}\right)^2 (1.2)^{2L-2} A^{(2L-2)/3} (\mu_p/c)^2 (\mathrm{fm})^{2L-2}$$

Selection rules (1)

- A classical electromagnetic field produced by oscillating charges and currents transmits energy and also angular momentum → in quantum limit each photon carries a definite angular momentum
- As written above the multipole operator includes a factor $Y_{LM}(\theta, \phi)$ which is associated with angular momentum L → a multipole of order L transfers an angular momentum of $L\hbar$ per photon
- We first consider a γ transition from an initial state of angular momentum I_i and parity π_i to a final state I_f and π_f (with for the moment $I_i \neq I_f$)
- Conservation of angular momentum implies →

$$\boldsymbol{I_i = L + I_f}$$

Selection rules (2)

- In other words →

$$\left|I_i - I_f\right| \leq L \leq I_i + I_f$$

- For instance → I_i = 3/2 and I_f = 5/2 → L = 1, 2, 3, 4 → mixture of dipole, quadrupole, octupole and hexadecapole
- Now if we are looking for the parity → electric or magnetic type is determined by the relative parity of initial and final levels
  → if no change in parity ($\Delta\pi$ = no) → the radiation has even parity
  → if parity change during transition ($\Delta\pi$ = yes) → the radiation has odd parity
- As written above →

$$\begin{aligned} \Pi(EL) &= (-1)^L \\ \Pi(ML) &= (-1)^{L+1} \end{aligned}$$

- Electric transitions have even parity if *L* = even ↔ magnetic transitions have even parity if *L* = odd

Selection rules (3)

- Consequently $\Delta\pi$ = no transitions consists of even electric multipoles and odd magnetic multipoles $\leftrightarrow$ $\Delta\pi$ = yes transitions consists of odd electric multipoles and even magnetic multipoles
- In previous example → let us assume that $\pi_i = \pi_f$ → $\Delta\pi$ = no → L = 1 must be magnetic, L = 2 must be electric, L = 3 must be magnetic and L = 4 must be electric → allowed transitions are M1, E2, M3 and E4
- In conclusion → angular momentum and parity selection rules are →

$$|I_i - I_f| \leq L \leq I_i + I_f \quad \text{no } L = 0$$

$$\Delta\pi = \text{ no} \rightarrow \text{ even electric; odd magnetic}$$

$$\Delta\pi = \text{ yes} \rightarrow \text{ odd electric; even magnetic}$$

Selection rules: particular cases

- There is no monopole transition ($L = 0$) → indeed the monopole moment is just the electric charge → no variation in time → if $I_i = I_f$ the lowest multipole order is $L = 1$
- If either I_i or $I_f = 0$ → simple case → only a pure multipole is emitted → example $2^+ \rightarrow 0^+$ → pure electric quadrupole E2 transition
- If $I_i = I_f = 0$ → selection rules gives $L = 0$ which is forbidden → only internal conversion is permitted (see below)
- Usually several multipoles may be emitted → Weisskopf estimations permit to determine which multipole is likely to be emitted → in our previous example → assuming a medium-weight nucleus ($A = 125$ → $A^{2/3} = 25$) and E = 1 MeV → Weisskopf equations give λ in the ratio → $\lambda(M1)$: $\lambda(E2)$: $\lambda(M3)$: $\lambda(E4) = 1: 1.4\times 10^{-3}: 2.1\times 10^{-10}: 1.3\times 10^{-13}$ → principally M1 transition + small mixture of E2

Internal conversion: generalities

- Internal conversion (internal-conversion-electron emission or ce) is an electromagnetic process in competition with γ emission
- The electromagnetic field interacts with an atomic electron → emission of this electron from the atom
- No electron creation during this process but emission of an existing e^- → ce decay is (slightly) altered by a modification of the chemical environment and of the atomic orbits
- Attention → it is **not** a two-step process → no emission of photon knocking loose an orbiting electron → such process has a negligible probability of occurrence

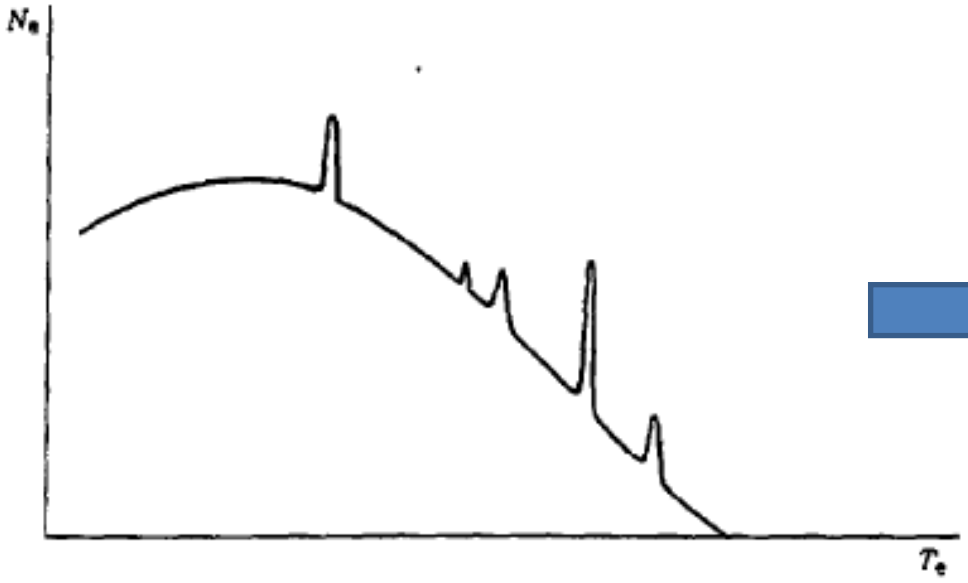
Internal conversion: energy release in ce (1)

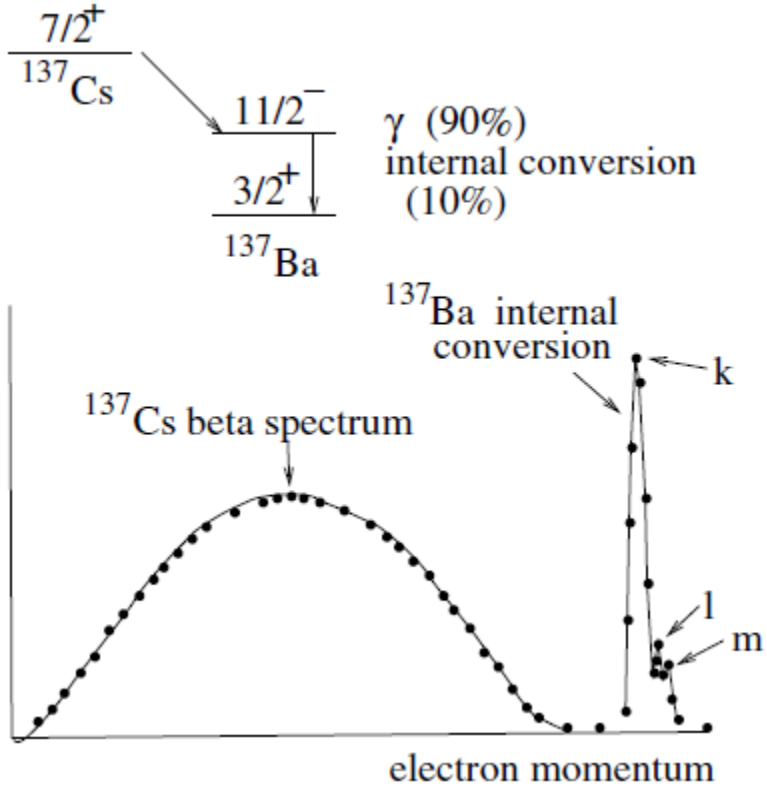
- The transition energy ΔE is converted as kinetic energy of the emitted electron T_e less its binding energy B_i

$$T_e = \Delta E - B_i$$

- The ce spectrum thus consists of a number or discrete individual components corresponding to each orbital or B_i
- The ce electrons are labelled according to the electron shell from which they come: K, L, M,…
- If very high resolution observations are made → the substructure can be seen → for instance for *L* (*n* = 2): L_I, L_{II}, L_{III}
- Following a ce → atom is left with a vacancy → filling of this vacancy by electron from higher shells → emission of characteristic X-rays

Internal conversion: energy release in ce (2)





Internal conversion: shell nomenclature

DISCONTINUITE D'ABSORPTION X	ÉLECTRON ARRACHE
K	$1s$ (j = 1/2)
L_I	$2s$ (j = 1/2)
L_{II}	$2p$ ($j = l - s$ = 1/2)
L_{III}	$2p$ ($j = l + s$ = 3/2)
M_I	$3s$ (j = 1/2)
M_{II}	$3p$ ($j = l - s$ = 1/2)
M_{III}	$3p$ ($j = l + s$ = 3/2)
M_{IV}	$3d$ ($j = l - s$ = 3/2)
M_V	$3d$ ($j = l + s$ = 5/2)
N_I	$4s$ (j = 1/2)
N_{II}	$4p$ ($j = l - s$ = 1/2)
N_{III}	$4p$ ($j = l + s$ = 3/2)
N_{IV}	$4d$ ($j = l - s$ = 3/2)
N_V	$4d$ ($j = l + s$ = 5/2)
N_{VI}	$4f$ ($j = l - s$ = 5/2)
N_{VII}	$4f$ ($j = l + s$ = 7/2)

Binding energies (1)

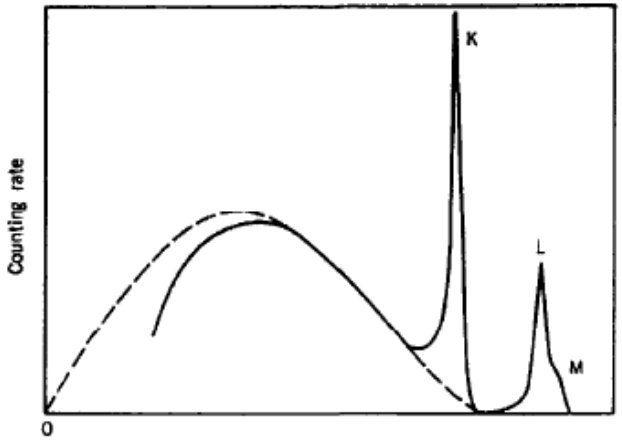
Z	Élément	*K*	*LI*	*LII*	*LIII*	*MI*	*MII*	*MIII*	*MIV*	*MV*
1	H	0.014								
2	He	0.025	0.001							
3	Li	0.055	0.003	0.001	0.001					
4	Be	0.111	0.006	0.002	0.002					
5	B	0.188	0.009	0.004	0.004					
6	C	0.284	0.013	0.005	0.005					
7	N	0.4	0.018	0.007	0.007					
8	O	0.533	0.024	0.009	0.009					
9	F	0.687	0.032	0.012	0.012					
10	Ne	0.867	0.045	0.018	0.018	0.001				
11	Na	1.0721	0.063	0.032	0.032	0.002				
12	Mg	1.305	0.088	0.05	0.05	0.003				
13	Al	1.5596	0.118	0.073	0.073	0.005				
14	Si	1.8389	0.151	0.099	0.1	0.007	0.0011	0.001		
15	P	2.1455	0.188	0.1301	0.13	0.01	0.0021	0.002		
16	S	2.472	0.227	0.1651	0.165	0.014	0.0041	0.004		
17	Cl	2.8224	0.27	0.203	0.202	0.018	0.0071	0.007		
18	Ar	3.2029	0.32	0.247	0.245	0.025	0.0121	0.012		
19	K	3.6074	0.377	0.296	0.294	0.034	0.0181	0.018		
20	Ca	4.0381	0.438	0.35	0.346	0.044	0.0251	0.025		
21	Sc	4.4928	0.5	0.406	0.401	0.053	0.0321	0.032		
22	Ti	4.9664	0.563	0.462	0.456	0.06	0.0351	0.035		
23	V	5.4651	0.628	0.521	0.513	0.066	0.0381	0.038		
24	Cr	5.9892	0.696	0.584	0.575	0.074	0.0421	0.042	0.0011	0.001
25	Mn	6.539	0.769	0.651	0.64	0.084	0.0471	0.047	0.0021	0.002
26	Fe	7.112	0.846	0.721	0.708	0.093	0.0531	0.053	0.0031	0.003
27	Co	7.7089	0.926	0.794	0.779	0.101	0.0601	0.06	0.0041	0.004
28	Ni	8.3328	1.0081	0.871	0.845	0.111	0.0671	0.067	0.0051	0.005
29	Cu	8.9789	1.0961	0.953	0.933	0.122	0.0741	0.074	0.0071	0.007
30	Zn	9.6586	1.1936	1.0428	1.0197	0.138	0.0881	0.087	0.0101	0.01
31	Ga	10.367	1.2977	1.1423	1.1154	0.158	0.106	0.103	0.0171	0.017
32	Ge	11.103	1.4143	1.2478	1.2167	0.18	0.126	0.121	0.0281	0.028
33	As	11.867	1.5265	1.3586	1.3231	0.204	0.146	0.14	0.0411	0.041

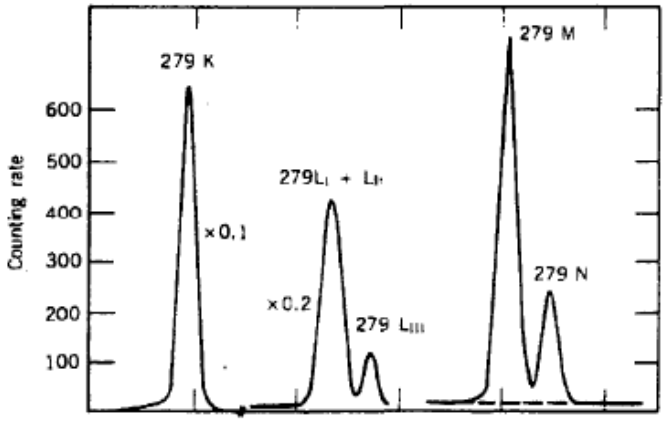
Energies are given in keV

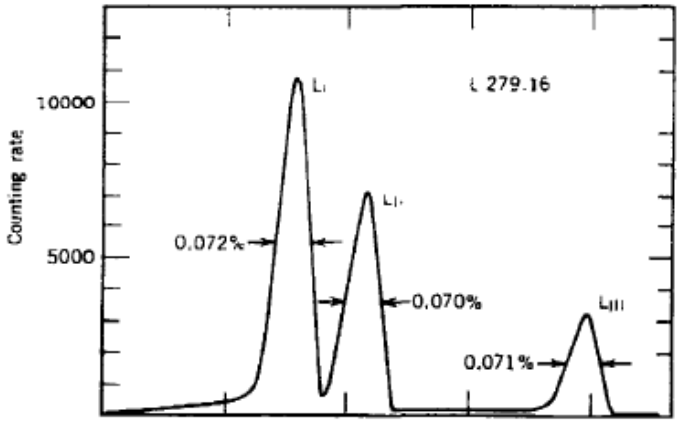
Binding energies (2)

Z	Elément	K	LI	LII	LIII	MI	MII	MIII	MIV	MV
34	Se	12.658	1.6539	1.4762	1.4358	0.23	0.168	0.161	0.0551	0.055
35	Br	13.474	1.782	1.596	1.5499	0.257	0.191	0.184	0.072	0.071
36	Kr	14.326	1.921	1.7272	1.6749	0.288	0.219	0.21	0.091	0.09
37	Rb	15.2	2.0651	1.8639	1.8044	0.322	0.248	0.239	0.112	0.11
38	Sr	16.105	2.2163	2.0068	1.9396	0.358	0.28	0.269	0.135	0.133
39	Y	17.038	2.3725	2.1555	2.08	0.394	0.312	0.299	0.158	0.156
40	Zr	17.998	2.5316	2.3067	2.2223	0.43	0.344	0.33	0.182	0.18
41	Nb	18.986	2.6977	2.4647	2.3705	0.467	0.377	0.361	0.206	0.204
42	Mo	20.0	2.8655	2.6251	2.5202	0.505	0.41	0.392	0.23	0.228
43	Tc	21.044	3.0425	2.7932	2.6769	0.545	0.445	0.426	0.256	0.253
44	Ru	22.117	3.224	2.9669	2.8379	0.585	0.483	0.461	0.284	0.28
45	Rh	23.22	3.4119	3.1461	3.0038	0.627	0.521	0.496	0.312	0.307
46	Pd	24.35	3.6043	3.3303	3.1733	0.67	0.559	0.532	0.34	0.335
47	Ag	25.514	3.8058	3.5237	3.3511	0.718	0.602	0.571	0.373	0.367
48	Cd	26.711	4.018	3.727	3.5375	0.77	0.651	0.616	0.41	0.404
49	In	27.94	4.2375	3.938	3.7301	0.826	0.702	0.664	0.451	0.443
50	Sn	29.2	4.4647	4.1561	3.9288	0.884	0.756	0.714	0.493	0.485
51	Sb	30.491	4.6983	4.3804	4.1322	0.944	0.812	0.766	0.537	0.528
52	Te	31.814	4.9392	4.612	4.3414	1.006	0.87	0.819	0.583	0.572
53	I	33.169	5.1881	4.8521	4.5571	1.0721	0.931	0.876	0.633	0.619
54	Xe	34.561	5.4528	5.1037	4.7822	1.149	0.997	0.936	0.686	0.672
55	Cs	35.985	5.7143	5.3594	5.0119	1.2171	1.065	0.998	0.74	0.726
56	Ba	37.441	5.9888	5.6236	5.247	1.2928	1.1367	1.0622	0.794	0.78
57	La	38.925	6.2663	5.8906	5.4827	1.3613	1.2044	1.1234	0.848	0.832
58	Ce	40.443	6.5488	6.1642	5.7234	1.4366	1.2728	1.1854	0.901	0.883
59	Pr	41.991	6.8348	6.4404	5.9643	1.511	1.3374	1.2422	0.951	0.931
60	Nd	43.569	7.126	6.7215	6.2079	1.5753	1.4028	1.2974	1.005	0.978
61	Pm	45.184	7.4279	7.0128	6.4593	1.653	1.4714	1.3569	1.0515	1.0269
62	Sm	46.834	7.7368	7.3118	6.7162	1.7228	1.5407	1.4198	1.106	1.0802
63	Eu	48.519	8.052	7.6171	6.9769	1.8	1.6139	1.4806	1.1606	1.1309
64	Gd	50.239	8.3756	7.9303	7.2428	1.8808	1.6883	1.544	1.2172	1.1852
65	Tb	51.996	8.708	8.2516	7.514	1.9675	1.7677	1.6113	1.275	1.2412
66	Dy	53.788	9.0458	8.5806	7.7901	2.0468	1.8418	1.6756	1.3325	1.2949
67	Ho	55.618	9.3942	8.9178	8.0711	2.1283	1.9228	1.7412	1.3915	1.3514
68	Er	57.485	9.7513	9.2643	8.3579	2.2065	2.0058	1.8118	1.4533	1.4093
69	Tm	59.39	10.116	9.6169	8.648	2.3068	2.0898	1.8845	1.5146	1.4677
70	Yb	61.332	10.486	9.9782	8.9436	2.3981	2.173	1.9498	1.5763	1.5278
71	Lu	63.314	10.87	10.349	9.2441	2.4912	2.2635	2.0236	1.6394	1.5885
72	Hf	65.351	11.271	10.739	9.5607	2.6009	2.3654	2.1076	1.7164	1.6617
73	Ta	67.416	11.681	11.136	9.8811	2.708	2.4687	2.194	1.7932	1.7351
74	W	69.525	12.1	11.544	10.207	2.8196	2.5749	2.281	1.8716	1.8092
75	Re	71.676	12.527	11.959	10.535	2.9317	2.6816	2.3673	1.9489	1.8224
76	Os	73.871	12.968	12.385	10.871	3.0485	2.7922	2.4572	2.0308	1.9601
77	Ir	76.111	13.419	12.824	11.215	3.1737	2.9087	2.5507	2.1161	2.0404
78	Pt	78.395	13.88	13.273	11.564	3.296	3.0265	2.6454	2.2019	2.1216
79	Au	80.725	14.353	13.734	11.919	3.4249	3.1478	2.743	2.2911	2.2057
80	Hg	83.102	14.839	14.209	12.284	3.5616	3.2785	2.8471	2.3849	2.2949
81	Tl	85.53	15.347	14.698	12.657	3.7041	3.4157	2.9566	2.4851	2.3893
82	Pb	88.004	15.861	15.2	13.035	3.8507	3.5542	3.0664	2.5856	2.484
83	Bi	90.526	16.387	15.711	13.419	3.9991	3.6963	3.1769	2.6876	2.5796
84	Po	93.105	16.939	16.244	13.814	4.1494	3.8541	3.3019	2.798	2.683
85	At	95.73	17.493	16.785	14.213	4.317	4.008	3.426	2.9087	2.7867
86	Rn	98.404	18.049	17.337	14.619	4.482	4.159	3.538	3.0215	2.8924
87	Fr	101.14	18.639	17.907	15.031	4.652	4.327	3.663	3.1362	2.9999
88	Ra	103.92	19.237	18.484	15.444	4.822	4.4895	3.7918	3.2484	3.1049
89	Ac	106.76	19.84	19.083	15.871	5.002	4.656	3.909	3.3702	3.219

Z	Elément	K	LI	LII	LIII	MI	MII	MIII	MIV	MV
90	Th	109.65	20.472	19.693	16.3	5.1823	4.8304	4.0461	3.4908	3.332
91	Pa	112.6	21.105	20.314	16.733	5.3669	5.0009	4.1738	3.6112	3.4418
92	U	115.61	21.757	20.948	17.166	5.548	5.1822	4.3034	3.7276	3.5517
93	Np	118.68	22.427	21.6	17.61	5.7232	5.3662	4.4347	3.8503	3.6658
94	Pu	121.82	23.097	22.266	18.057	5.9329	5.5412	4.5566	3.9726	3.7781
95	Am	125.03	23.773	22.944	18.504	6.1205	5.7102	4.667	4.0921	3.8869
96	Cm	128.22	24.46	23.799	18.93	6.288	5.895	4.797	4.227	3.971
97	Bk	131.59	25.275	24.385	19.452	6.556	6.147	4.977	4.366	4.132
98	Cf	135.96	26.11	25.25	19.93	6.754	6.359	5.109	4.497	4.253
99	Es	139.49	26.9	26.02	20.41	6.977	6.574	5.252	4.63	4.374
100	Fm	143.09	27.7	26.81	20.9	7.205	6.793	5.397	4.766	4.498







Internal conversion: example

- $^{203}Hg \rightarrow {}^{203}Tl \rightarrow \beta$ emission followed by emission of a γ of 279.19 keV
- Electron spectrum is the sum of β spectrum + ce lines
- The energy of the lines corresponds to 279.19 - B_i(Tl)

High resolution

Very high resolution

Internal conversion: lines intensities (1)

- Variable intensity for each conversion electron → depends on the multipole character
- Probability of ce have to be considered when calculating γ emission → the total decay probability λ_t has two components: one from γ emission (λ_γ) and another arising form ce (λ_e) →

$$\lambda_t = \lambda_\gamma + \lambda_e$$

- From this we define the internal conversion coefficient → $\alpha = \dfrac{\lambda_e}{\lambda_\gamma}$
- The total decay probability becomes →

$$\lambda_t = \lambda_\gamma(1 + \alpha)$$

- We also define partial coefficients for the individual atomic shells →

$$\lambda_t = \lambda_\gamma + \lambda_{e,K} + \lambda_{e,L} + \lambda_{e,M} + \dots$$

$$\lambda_\gamma(1 + \alpha_K + \alpha_L + \alpha_M + \dots)$$

- Of course subshell coefficients can also be defined (α_{LI}, α_{LII},…)

Internal conversion: lines intensities (2)

- Precise calculation of α coefficients is beyond the level of the course → only some ingredients are given here
- We use as usually the Fermi Golden Rule →

$$\lambda_{ce} = \frac{2\pi}{\hbar} |\left\langle \Psi^f \right| V_{int} \left| \Psi^i \right\rangle|^2 \rho_{ce}(E_e)$$

- The density of state $\rho(E_e)$ is expressed as (T_e is e$^-$ kinetic energy) →

$$\rho_{ce} = \frac{d\boldsymbol{k_e}}{dT_e} = k_e^2 \left(\frac{dT_e}{dk_e}\right)^{-1} d\Omega_e = k_e^2 \left(\frac{\hbar p_e c^2}{E_e}\right)^{-1} d\Omega_e = \frac{k_e E_e}{\hbar^2 c^2} d\Omega_e$$

- The interaction potential is the difference between the potentials for nucleus with finite radius and zero radius and also couples the nuclear and atomic Hamiltonians

Internal conversion: lines intensities (3)

- The initial state includes a bound electron and the final state includes a free electron → the initial and final total wave functions accounts for nuclear and electron wave functions:

$$\Psi^{i,f} = \Psi_N^{i,f}\,\psi_e^{i,f}$$

  with ψ_e^f the free-particle wave function = exp(-i$\boldsymbol{k_e r_e}$)
- The $\Psi_N^{i,f}$ are proper wave functions of the nuclear Hamiltonian and ψ_e^i is the proper wave function of the electron Hamiltonian corresponding to an energy -B_i(*nl*)
- All of the specifically nuclear information is contained in $\Psi_N^{i,f}$ → the same electromagnetic operator $m_{fi}(\sigma L)$ governs both γ emission and internal conversion → the nuclear part of the matrix element is identical for both processes →

$$\lambda_{\gamma,ce}(\sigma L) \propto |m_{fi}(\sigma L)|^2$$

Internal conversion: lines intensities (4)

- Consequently the α coefficients are independent of the details on nuclear structure → the α depend on the atomic number *Z* of the atom, on the energy of the transition *E* and on the multipolarity *L* (hence indirectly on nuclear structure)
- Nonrelativistic calculations give (with *n* the principal quantum number of the bound electron wave function) →

$$\alpha(EL) \cong \frac{Z^3}{n^3}\left(\frac{L}{L+1}\right)\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^4\left(\frac{2m_ec^2}{E}\right)^{L+5/2}$$

$$\alpha(ML) \cong \frac{Z^3}{n^3}\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^4\left(\frac{2m_ec^2}{E}\right)^{L+3/2}$$

Internal conversion: lines intensities (5)

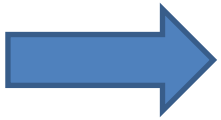
- We note that the factor $(Z/n)^3$ comes from the term $|\psi_e^i|^2$ that appears in the equation ↔ the hydrogenic wave function shows a factor $(Z/n)^{3/2}$ in its normalization constant
- $(e^2/4\pi^2\epsilon_0\hbar c) \simeq 1/137$ is the fine structure constant
- Two approximations in previous results

  → generally in nuclear physics electrons must be treated relativistically

  → a simple hydrogenic wave function does not properly take account the effect that occurs when the electron penetrates the nucleus
- However these expressions illustrate some features of α coefficients
  1. Increase as Z^3 → ce process is more important for heavy nuclei than for light nuclei → example: the 1.27-MeV *E2* transition in $^{22}_{10}$Ne has $\alpha_K = 6.8 \times 10^{-6}$ and the 1.22-MeV *E2* transition in $^{182}_{74}$W has $\alpha_K = 2.5 \times 10^{-3}$ → ratio of 0.0027 ≈ $(10/74)^3 = 0.0025$

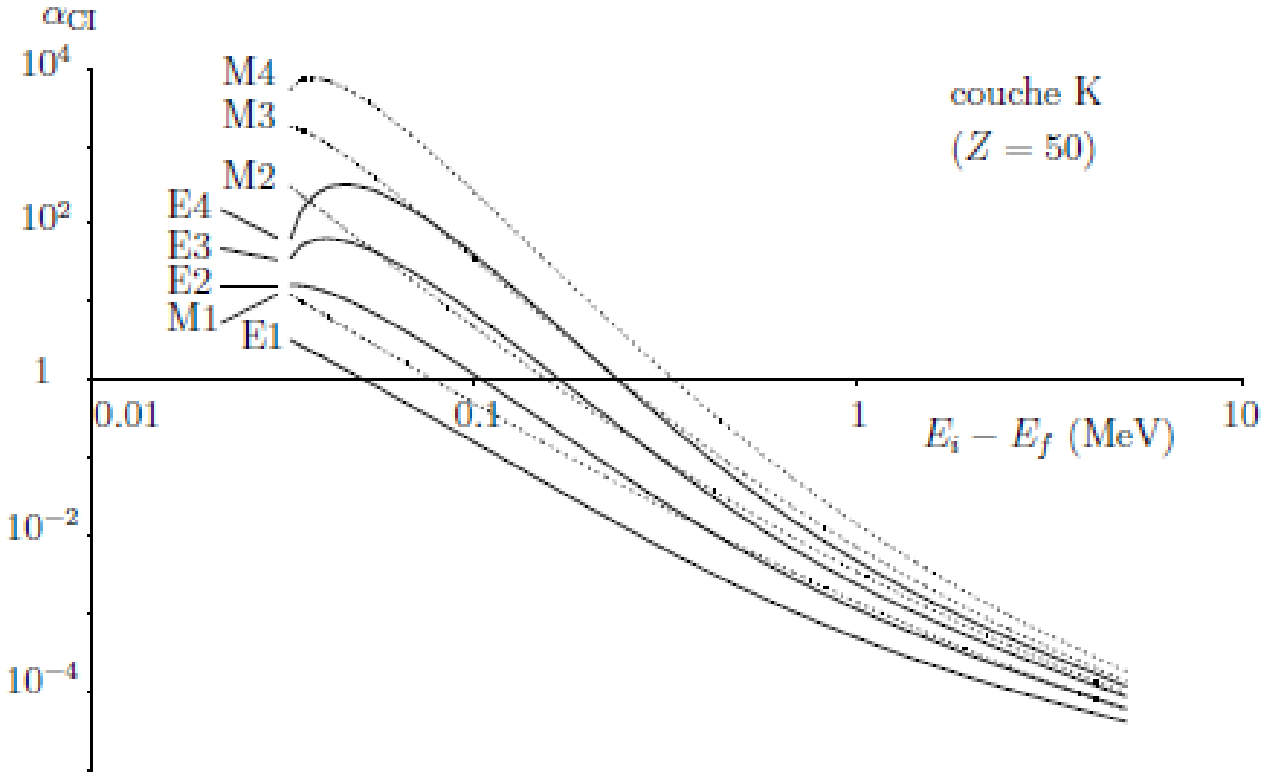
Internal conversion: lines intensities (6)

2. The α coefficients ↘ rapidly with E ↗ ↔ on the opposite the probability for γ emission ↗ rapidly with E ↗ → example: in ^{56}Co there are 3 *M1* transitions with E = 158 keV (α_K = 0.011), 270 keV (α_K = 0.0034), 812 keV (α_K = 0.00025) → decrease in about $E^{-5/2}$
3. The α coefficients ↗ rapidly for L ↗ (for the higher L → ce emission is more probable than γ emission) → example: in ^{99}Tc the *M1* transition (E = 141 keV) has α_K = 0.10 while the *M4* transition (E = 143 keV) has α_K = 30 → ratio of 300 compared to the theoretical ratio of $(2m_ec^2/E)^3 \approx 370$
4. The α coefficients for higher atomic shells (n > 1) ↘ like $1/n^3$ → for a given transition we expect $\alpha_K/\alpha_L \simeq 8$ but using correct electronic wave functions causes this estimate to vary considerably ↔ however many experiments show α_K/α_{KL} in [3,6] → useful estimation



We expect relatively large K-shell α coefficients for low-energy, high-multipolarity transitions in heavy nuclei

Internal conversion: examples of lines intensities (tin)

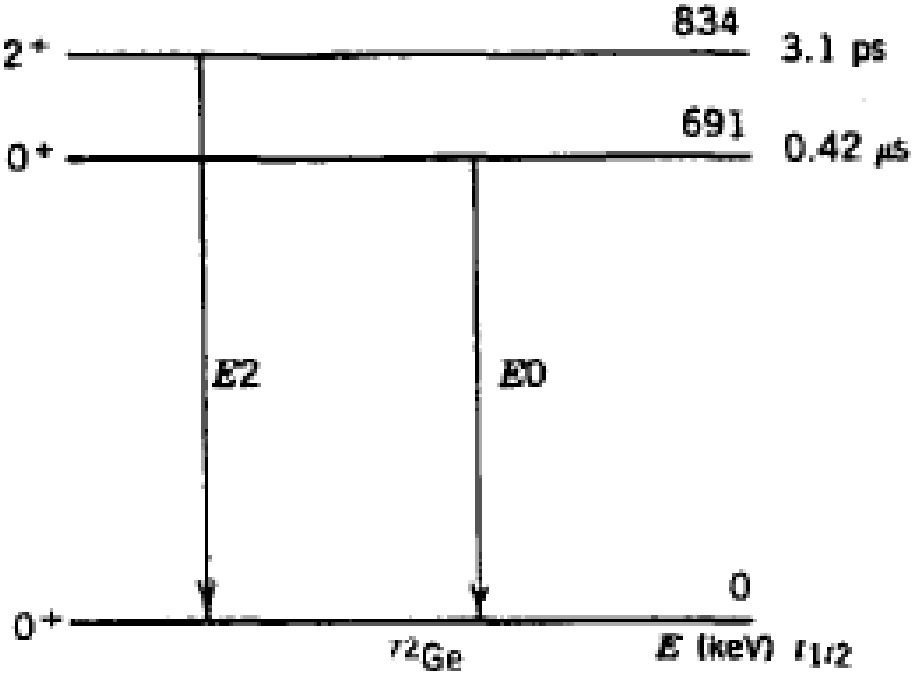


Results obtained with correct atomic wave functions → more precise

Internal conversion: *E*0 emission

- *E0* is forbidden by electromagnetic radiation because the nuclear monopole moment cannot radiate to points external to the nucleus
- *E0* is important for $0^+ \rightarrow 0^+$ transition
- In this particular case → the nucleus can be seen as a spherically symmetric ball of charge → only possible motion is pulsation → no alteration of the electric field at points external to the sphere → no radiation
- But electronic orbits (the s states) that do not vanish near $r = 0$ (i.e. the electron is inside the nucleus) can sample the varying potential within the pulsating nucleus → transfer of energy to the electron is possible → small probability but important when other modes are forbidden
- As no γ is emitted → impossible to define α coefficient

Internal conversion: example of *E*0 emission



Energy levels of ^{72}Ge

Pair creation (1)

- For large enough transition energy → decay occurs by emission of an electron-positron pair →

$$X^* \rightarrow X + e^- + e^+$$

- Conservation of energy implies →

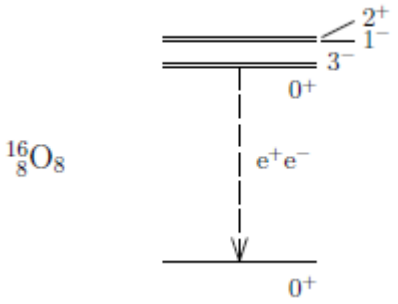
$$\Delta E = E_i - E_f - 2m_e c^2 \approx T_{e^-} + T_{e^+}$$

- The threshold for this process is obviously →

$$\Delta E = E_i - E_f > 2m_e c^2 \approx 1.022 \text{ MeV}$$

Pair creation (2)

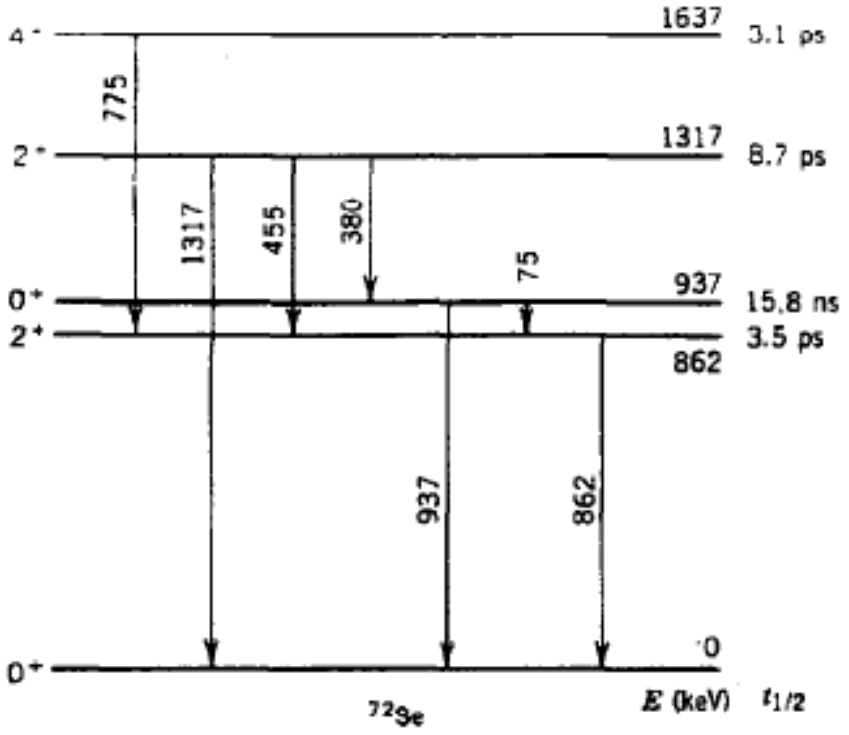
- Pair creation process is generally negligible compared to electromagnetic transitions (not a first-order effect in electromagnetic field)
- It can become principal mode transition when other transitions are forbidden or very inefficient → $0^+ \rightarrow 0^+$ transition for light nuclei and large energy transition
- For $0^+ \rightarrow 0^+$ ce is in competition with pair creation → for light nuclei and large energy transition ce is very inefficient → pair creation



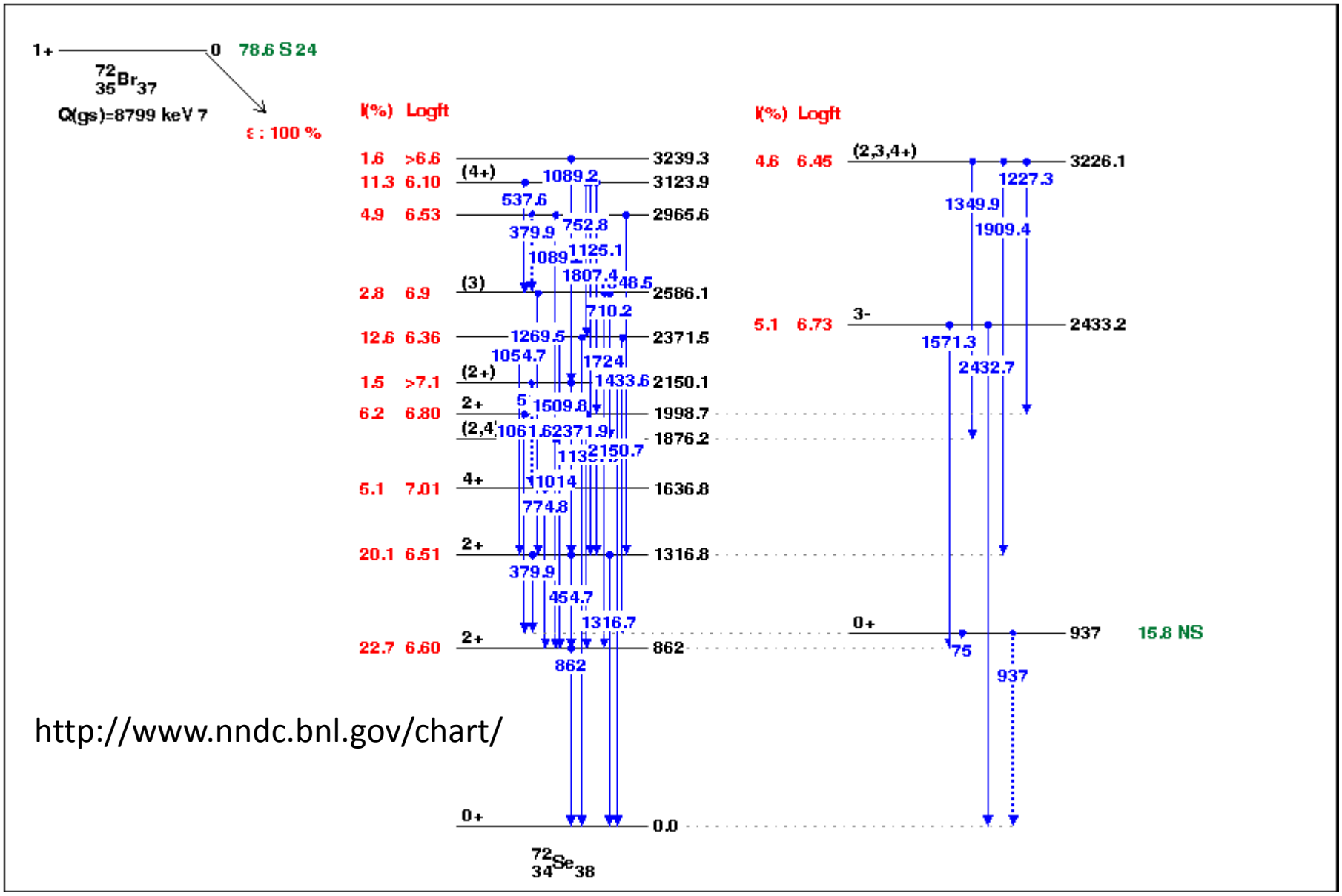
ΔE = 6.05 MeV → 100% pair creation

Lifetime for γ emission

- Evaluation of partial decay rate for γ emission → example of 72-Se



Lifetime for γ emission: full disintegration spectrum of 72-Br



http://www.nndc.bnl.gov/chart/

Lifetime for γ emission: example of 72-Se (1)

- We consider the 1317-keV level → measured half-life $T_{1/2}$ = 8.7 ps → the total decay $\lambda_t = \ln(2)/T_{1/2} = 8.0 \times 10^{10}$ s^{-1}
- This decay rate is the sum of the decay rates of the 3 transitions that depopulate the 1317-keV state → $\lambda_t = \lambda_{t,1317} + \lambda_{t,455} + \lambda_{t,380} = \lambda_{\gamma,1317}(1 + \alpha_{1317}) + \lambda_{\gamma,455}(1 + \alpha_{455}) + \lambda_{\gamma,380}(1 + \alpha_{380})$
- From standard data → the α coefficients can be neglected (< 0.01) → $\lambda_t = \lambda_{\gamma,1317} + \lambda_{\gamma,455} + \lambda_{\gamma,380}$
- The relative intensities have been measures to be $\lambda_{\gamma,1317} : \lambda_{\gamma,455} : \lambda_{\gamma,380} = 51 : 39 : 10$ → the partial decay rates are →

$$\begin{aligned}
\lambda_{\gamma,1317} &= 0.51(8.0 \times 10^{10}\ \mathrm{s}^{-1}) = 4.1 \times 10^{10}\ \mathrm{s}^{-1} \\
\lambda_{\gamma,455} &= 0.39(8.0 \times 10^{10}\ \mathrm{s}^{-1}) = 3.1 \times 10^{10}\ \mathrm{s}^{-1} \\
\lambda_{\gamma,380} &= 0.10(8.0 \times 10^{10}\ \mathrm{s}^{-1}) = 0.80 \times 10^{10}\ \mathrm{s}^{-1}
\end{aligned}$$

Lifetime for γ emission: example of 72-Se (2)

- For the 937-keV level → measured half-life $T_{1/2}$ = 15.8 ns → the total decay $\lambda_t = \ln(2)/T_{1/2} = 4.39 \times 10^7\ s^{-1}$
- This decay rate is the sum of the decay rates of 2 transitions → $\lambda_t = \lambda_{t,937} + \lambda_{t,75} = \lambda_{e,937} + \lambda_{\gamma,75}\,(1 + \alpha_{75}) \leftrightarrow$ indeed the 937-keV transition is a $0^+ \rightarrow 0^+$ transition
- From standard data → the α coefficient for the 75-keV transition is 2.4
- Experimentally we have $\lambda_{\gamma,75} : \lambda_{e,937} = 73 : 27 \rightarrow \lambda_{e,937} = 4.3 \times 10^6\ s^{-1}$ and $\lambda_{\gamma,75} = 1.16 \times 10^7\ s^{-1}$
- For the 862-keV transition → $\lambda_{\gamma,75} = 2.0 \times 10^{11}\ s^{-1}$

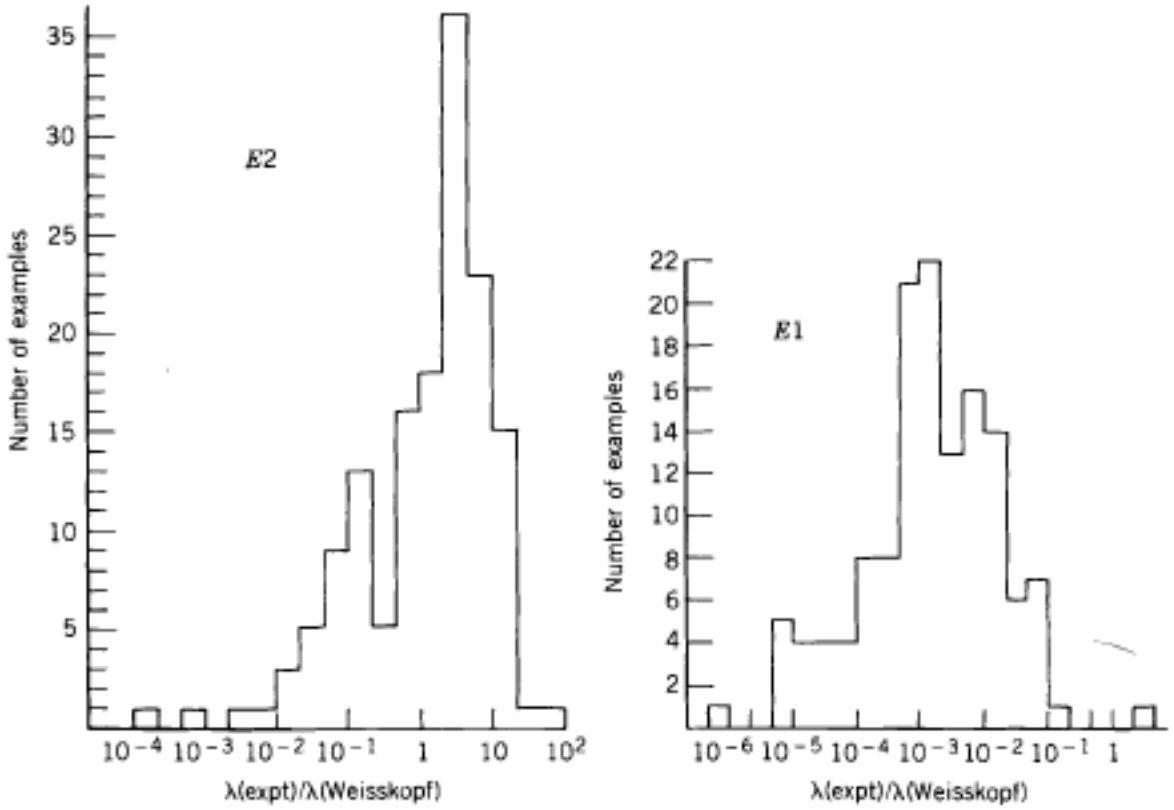
Lifetime for γ emission: example of 72-Se (3)

- From selections rules → we consider the Weisskopf estimates for the E2 transition →

$$\begin{aligned}
\lambda_{E2,1317} &= = 8.7 \times 10^{10}\ \mathrm{s}^{-1} \\
\lambda_{E2,455} &= = 4.3 \times 10^{8}\ \mathrm{s}^{-1} \\
\lambda_{E2,380} &= = 1.7 \times 10^{8}\ \mathrm{s}^{-1} \\
\lambda_{E2,75} &= = 5.2 \times 10^{4}\ \mathrm{s}^{-1} \\
\lambda_{E2,862} &= = 1.0 \times 10^{10}\ \mathrm{s}^{-1}
\end{aligned}$$

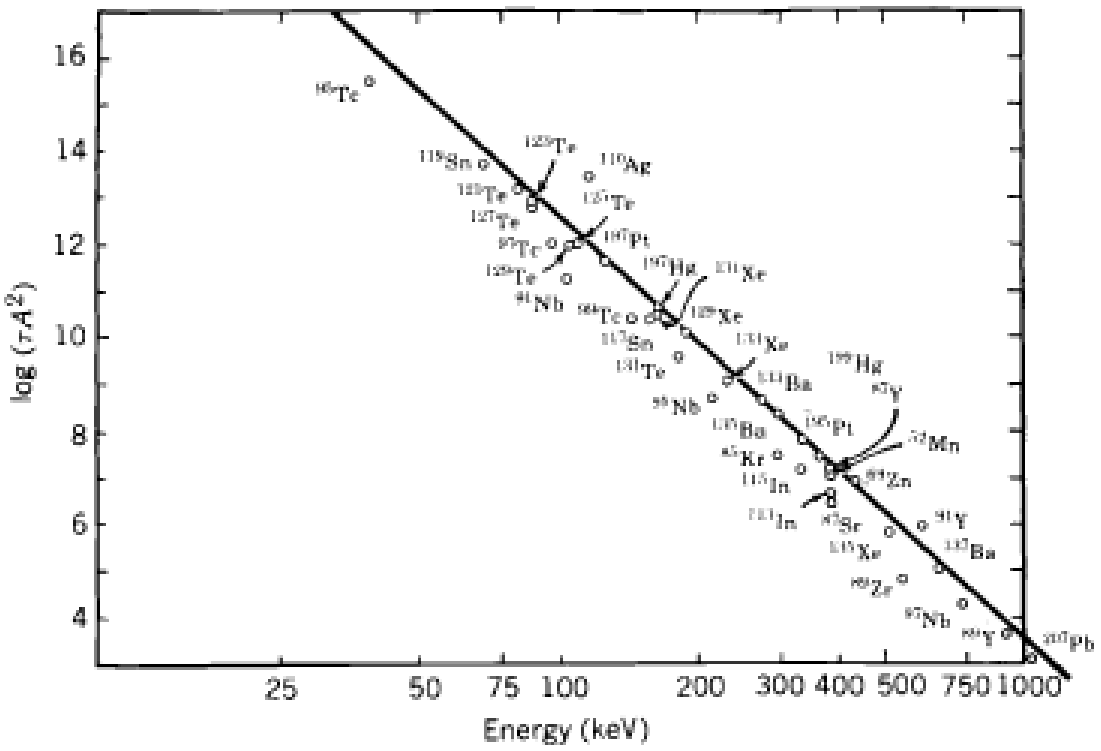
- The measured value are frequently at least an order of magnitude larger than the Weisskopf estimates → evidence of collective aspects of nuclear structure

Lifetime for γ emission: comparison Weisskopf $\leftrightarrow$ exp. (1)



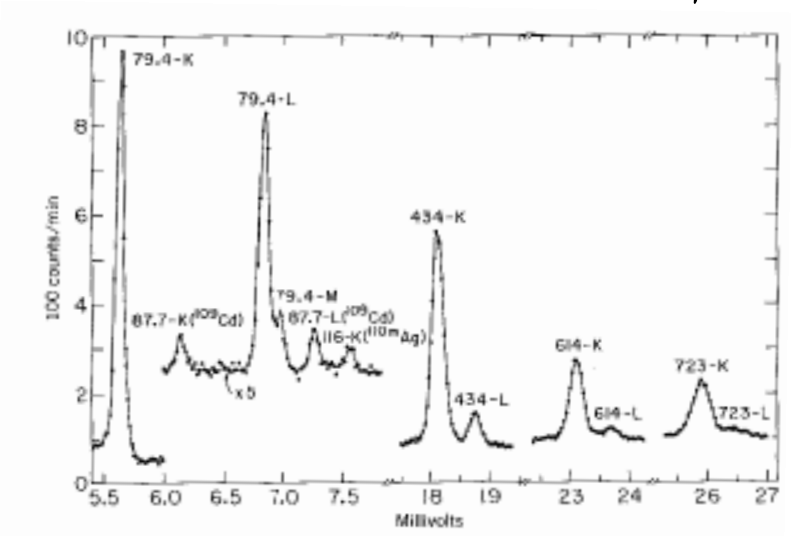
- Number of cases versus the ratio between the observed decay rate and the value calculate form Weisskopf formulas for *E*2 and *E*1

Lifetime for γ emission: comparison Weisskopf ↔ exp. (2)



- Mean life (= $1/\lambda$) versus the energy for *M*4 transition → good agreement between experiments (points) and theory (line) → especially the E^{-9} dependence is well observed

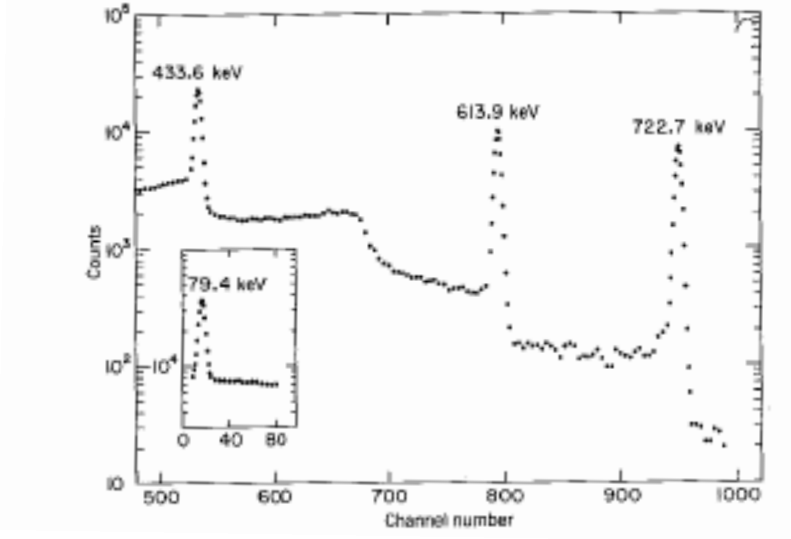
Lifetime for γ emission: γ-spectroscopy



electron spectrum

Transition Energy (keV)	Relative γ Intensity	Relative Electron Intensity	Conversion Coefficient (units of 10^{-3}) Experimental	Theoretical
79.2	7.3 ± 0.8	204 ± 10 (K)	220 ± 26	270 (*E*1), 710 (*M*1), 2400 (*E*2)
		25 ± 2 (L)	27 ± 4	33 (*E*1), 88 (*M*1), 777 (*E*2)
434.0	= 100	= 100 (K)	= 7.89	7.89 (*E*2)
		14.8 ± 2.3 (L + ···)	1.17 ± 0.18	1.02 (*E*2)
614.4	103 ± 3	37 ± 3 (K)	2.83 ± 0.24	1.03 (*E*1), 3.01 (*M*1), 2.92 (*E*2)
		5.1 ± 1.6 (L + ···)	0.39 ± 0.12	0.12 (*E*1), 0.35 (*M*1), 0.36 (*E*2)
632.9	0.16 ± 0.02			
723.0	102 ± 3	25.0 ± 1.2 (K)	1.93 ± 0.11	0.72 (*E*1), 2.06 (*M*1), 1.91 (*E*2)
		4.6 ± 0.8 (L + ···)	0.35 ± 0.06	0.08 (*E*1), 0.24 (*M*1), 0.23 (*E*2)

Source: Experimental data from O. C. Kistner and A. W. Sunyar, *Phys. Rev.* **143**, 918 (1966).



γ-ray spectrum

- For more explanations about γ-spectroscopy → see « Nuclear Metrology Techniques »

γ-ray absorption: principles (1)

- Inverse process of γ-ray emission is γ-ray absorption → a nucleus at rest and in its ground state absorb a photon of energy E_γ → jumps to an excited state at an energy ΔE above the ground state →

$$\Delta E = E_\gamma - \frac{E_\gamma^2}{2Mc^2}$$

- $E_\gamma^2/2Mc^2 = E_R$ is the recoil energy
- We assume a source of γ-rays of continuously variable energy → if the energy of the excited state was sharp → the absorption takes place only for a γ energy equal to the resonant value $\Delta E + E_R$
- However the energy of the excited state is not sharp ↔ any state that has a mean live τ has a width $\Gamma = \hbar/\tau$ → the absorption takes place even when the γ energy differs somewhat from the resonant value

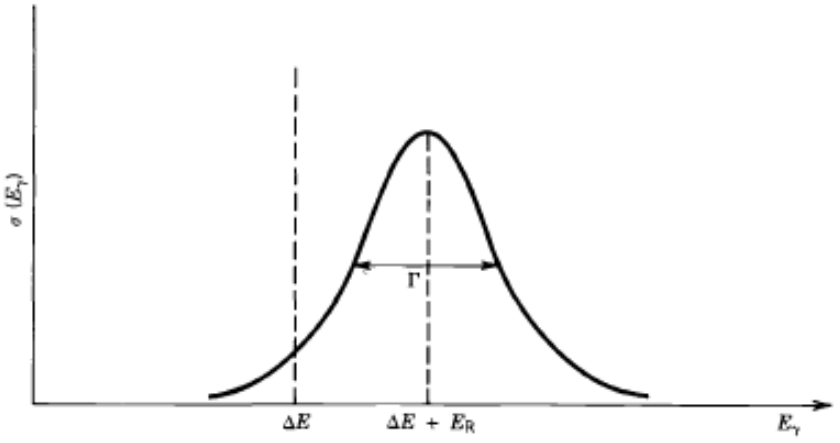
γ-ray absorption: principles (2)

- We consider a beam of γ-rays through a cloud of bare nuclei (to avoid scattering and absorption processes due to atomic electrons) → the resonant absorption cross section is

$$\sigma(E_\gamma) = \sigma_0 \frac{(\Gamma/2)^2}{[E_\gamma - (\Delta E + E_R)]^2 + (\Gamma/2)^2}$$

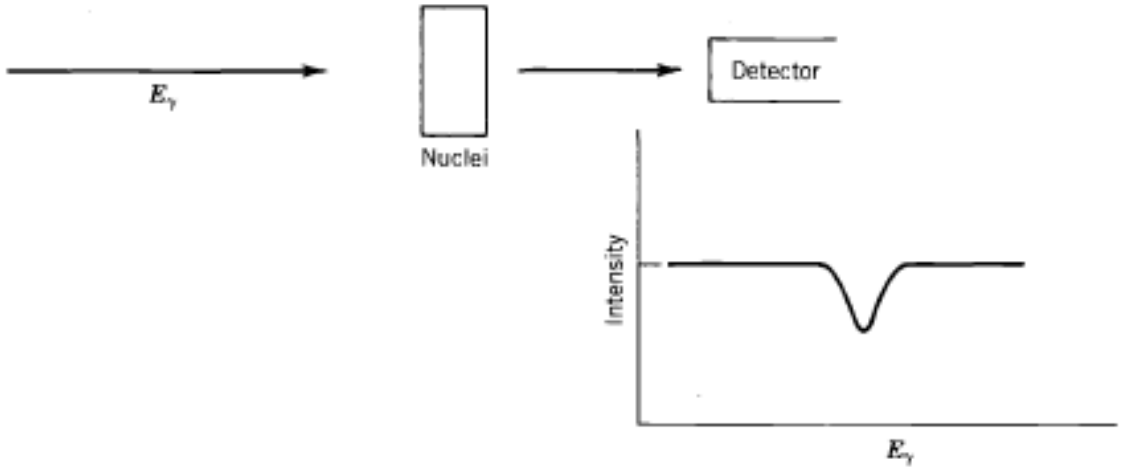
with σ_0 the cross section calculated from fundamental principles

- For typical nuclear states of mean lives ns to ps → Γ is in the range of 10^{-6} to 10^{-3} eV



γ-ray absorption: principles (3)

- Schematic view of a resonant absorption experiment →



- E_γ is varying → resonance curve

  → at energies far from the resonance nuclei are transparent to the radiation → no absorption

  → at the resonance transmitted intensity reaches a minimum

γ-ray absorption: Doppler broadening (1)

- In practice it is unlikely to observe natural linewidth Γ → additional contribution to the observed linewidth is the Doppler broadening Δ
- Indeed nuclei are not at rest → they are in thermal motion at any temperature *T* → photons emitted or absorbed in the lab frame are Doppler shifted with energies

$$E'_\gamma = E_\gamma(1 \pm v_x/c)$$

  where v_x is the velocity component along the photon direction
- The motion of nuclei is usually represented by a Maxwell distribution → we obtain the distribution of energies →

$$e^{-[(1/2)Mv_x^2]/kT} \Rightarrow e^{-(Mc^2/2kT)(1-E'_\gamma/E_\gamma)^2}$$

- This gives a Gaussian distribution of width $\Delta = 2\sqrt{\ln 2}E_\gamma\sqrt{\frac{2kT}{Mc^2}}$

γ-ray absorption: Doppler broadening (2)

- For a 100 keV-transition and for a medium-weight nucleus → Δ = 0.1 eV (at room T → kT ≈ 0.025 eV) → Doppler broadening dominates natural linewidth (even cooling at T = 4 K → Δ = 0.01 eV) → the width observed in experiment as previously shown is a combination of natural linewidth plus additional dominating Doppler broadening

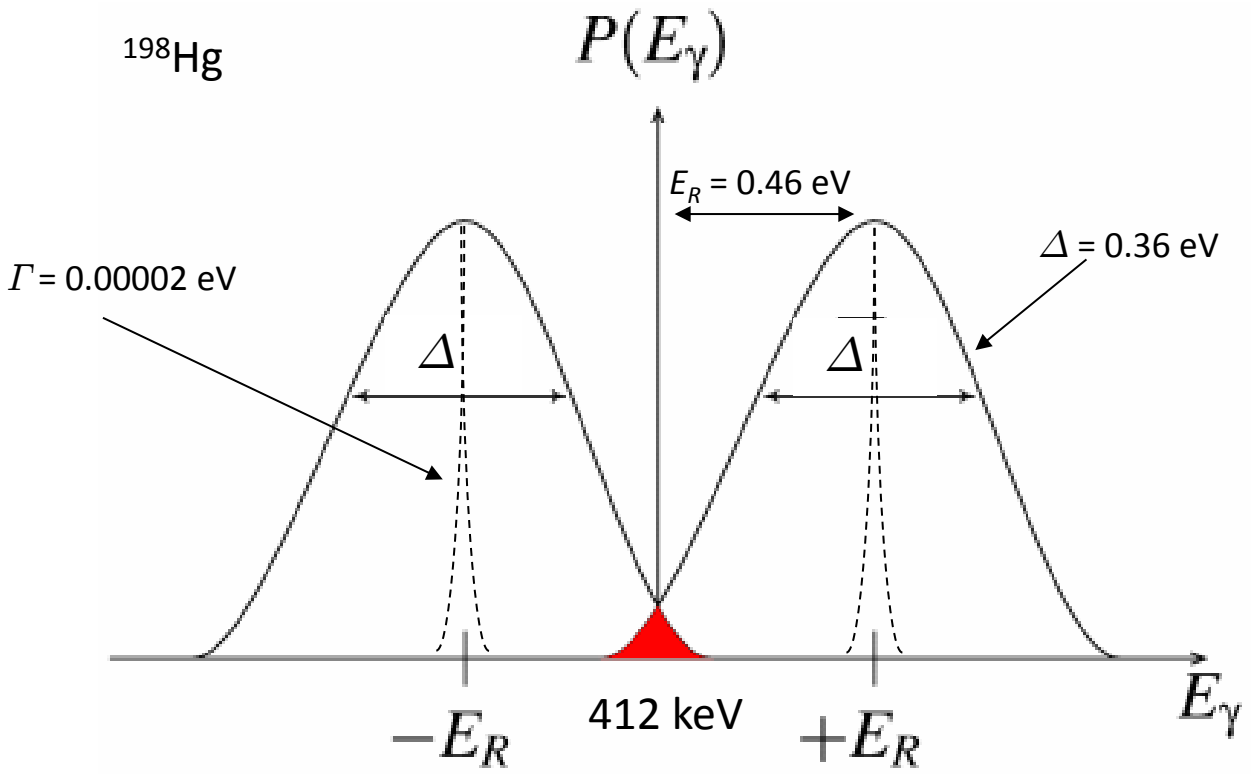
γ-ray absorption: practical experiment (1)

- Tunable source of photons does not exist in practice (only continuous electromagnetic spectrum from Bremsstrahlung produced by charged-particle acceleration → see « Nuclear Metrology Techniques »)
- Practically → ordinary sources of γ radiation emitting at discrete energies → to obtain resonant absorption → radioactive source has to emit a γ ray of an energy within at most 0.1 eV of the desired resonant energy $\Delta E + E_R$ → almost impossible
- It make sense to use a source in which the γ ray is emitted in the same downward transition that it must be excited upward by resonant absorption

γ-ray absorption: practical experiment (2)

- Example → β decay of ^{198}Au to ^{198}Hg emitting a 412 keV-energy γ ray that interacts with a target of stable ^{198}Hg → possibility of absorption from the ground state to the 412 keV excited state
- For this 412 state τ = 32 ps → $\Gamma = 2 \times 10^{-5}$ eV ↔ E_R = 0.46 eV → attention: the recoil affects both the emitted and absorbed transitions → the emitted radiation has energy ΔE - E_R and for the absorption energy of ΔE + E_R must be supplied ↔ at room T → Doppler width Δ = 0.36 eV
- From these data → minimal overlap between emission and absorption lines → very little probability of resonant excitation
- Contrast with atomic radiations → optical transitions have energies of a few eV → $E_R \approx 10^{-12}$ eV (E_R(Hg) = 2.7×10^{-12} eV) → complete overlap → easy to perform

γ-ray absorption: practical experiment (3)



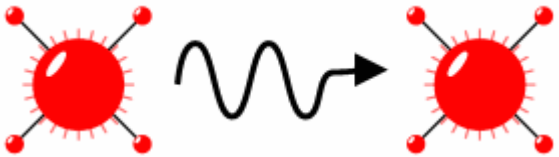
γ-ray absorption: practical experiment (4)

- Several techniques exist to overcome the energy difference $2E_R$ between source and absorption transitions

1. Raising the T → increasing Doppler broadening and the overlap
2. Move the source toward the absorber at high speed v to Doppler shift the emitted energy by $2E_R$ → the Doppler shifted energy being $E_\gamma(1+v/c)$ → the required speed is $v = 2cE_R/E_\gamma$ → for ^{198}Hg v = 670 m/s (realized by attaching the source to the tip of a rotor)
3. Use the Mössbauer effect: recoilless γ ray emission

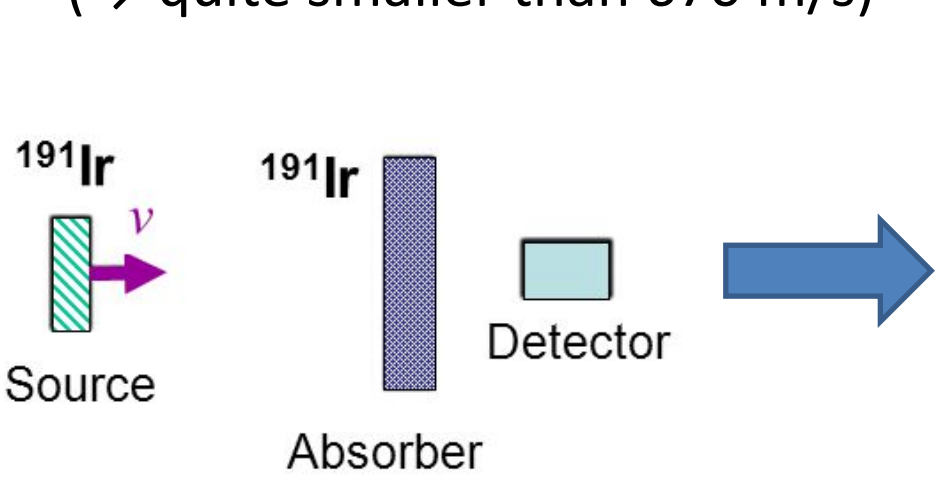
Mössbauer effect: principles

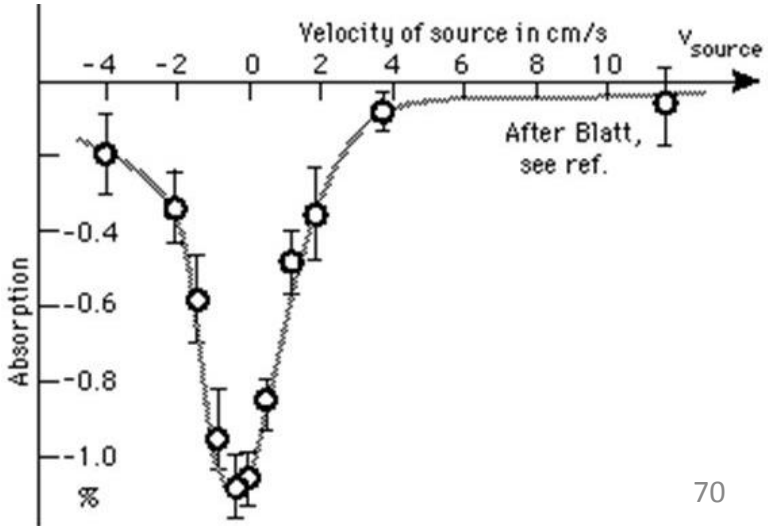
- Emitting and absorbing nuclei are bound in a crystal lattice → typical binding energies of an atom in a lattice ≈ 1 - 10 eV → not enough recoil energy available for the atom to leave its lattice site → the entire solid lattice absorbs the recoil momentum → the mass appearing in recoil energy formula becomes the mass of the entire solid making E_R very small → recoil-free event
- Moreover a certain fraction of the atoms in a lattice is in the vibrational ground state of thermal motion → very little thermal broadening effect
- It results very narrow and overlapping emission and absorption lines characterized by natural linewidth



Mössbauer effect: experimental demonstration

- To demonstrate Mössbauer effect → we move the source and absorber relative to one another at low speed → if the speed is such that the Doppler shift is greater than Γ → resonance is destroyed
- Mössbauer experiments this in 1958 for ^{191}Ir (E_γ = 129 keV / $\Gamma = 3 \times 10^{-6}$ eV) → total linewidth for absorber and emitter is 6×10^{-6} eV → the necessary speed to destroy resonance is $\approx 5 \times 10^{-11}$ c ≈ 15 mm/s (→ quite smaller than 670 m/s)





Mössbauer effect: choice of the source (1)

- The effectiveness of the Mössbauer effect is more complex that the simple question of lattice binding energy exceeding the recoil energy → indeed solids can absorb energy in many other ways than removing atoms from their lattice sites → at low energies and *T* it is made through lattice vibrations called phonons (see « Collective and cooperative phenomena in solids » → propagation of phonons is responsible for mechanical and acoustic waves) → the energy in a decay can be taken up by phonons
- Mössbauer effect occurs because there is a finite probability of a decay involving no phonon
- Definition of the recoil-free fraction *f* (or Mössbauer-Lamb factor) = fraction of nuclei in the lattice that emits (or absorbs) with no recoil (thus involving no phonons) → entire crystal acts as the recoiling body → due to the large mass → recoil-free

Mössbauer effect: choice of the source (2)

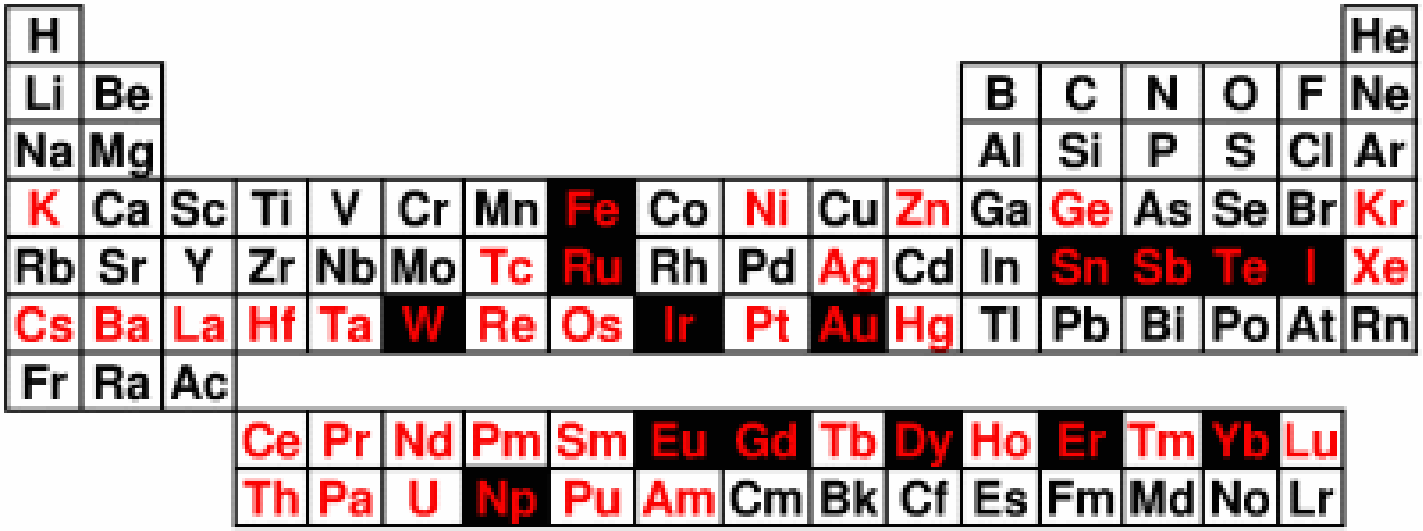
- This recoil-free fraction is

$$f = \exp\left[-\frac{\langle x^2 \rangle}{(\lambda/2\pi)^2}\right]$$

- In this expression → $\langle x^2 \rangle$ is the mean-square vibrational amplitude of the emitting nucleus and λ is the wavelength of the γ-ray
- if E_γ = ↘ → f ↗ → for the 14.4 keV transition of ^{57}Fe → f = 0.92 ↔ for the 129 keV transition of ^{191}Ir → f = 0.10
- Recoilless processes are needed for both in source and absorber → total fraction is obtained by the products of the 2 factors → Fe shows a much larger effect than Ir
- Mössbauer effect is particularly detected in isotopes with low lying excited states

Mössbauer effect: choice of the source (3)

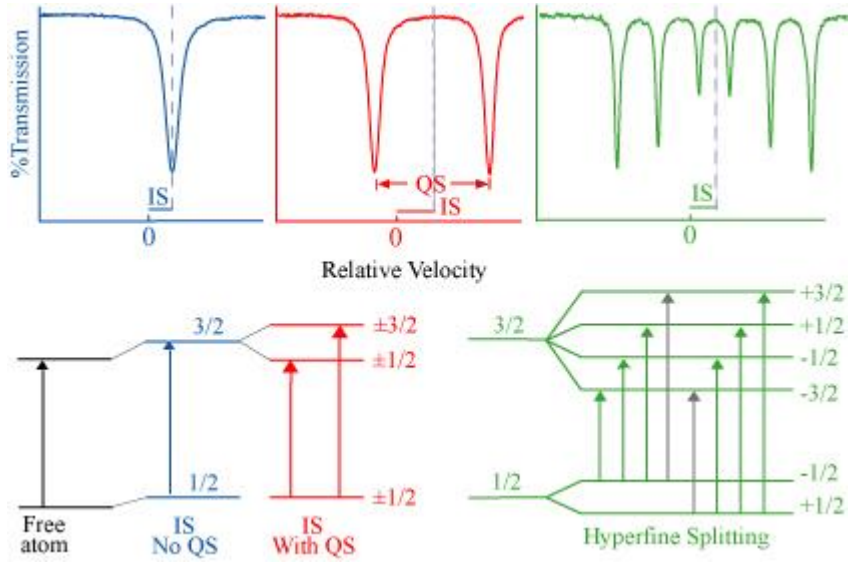
- Similarly the resolution is dependent upon the lifetime of the excited state
- Energy and resolution limit the number of isotopes for which Mössbauer effect is detected



- In red → elements which have known Mössbauer isotopes ↔ in black boxes → elements used in practice → principally ^{57}Fe

Mössbauer effect: Mössbauer spectroscopy

- When source and absorber atoms are in different local environments → their nuclear energy levels are different
- For ^{57}Fe → $\Gamma = 5 \times 10^{-9}$ eV → compared to the Mössbauer γ ray energy (14.4 keV) → resolution of 1 in 10^{12} or(i.e. the equivalent of one sheet of paper in the distance between the Sun and the Earth) → detection of the hyperfine interactions in the nucleus



Mössbauer effect: typical energies

Physical phenomenon	*Energy range*, eV
Mössbauer γ-ray emission/ absorption	10^{3}–10^{5}
Chemical binding energies	1–10
'Free' atom recoil energies	10^{-4}–10^{-1}
Lattice vibration phonon energies . .	10^{-3}–10^{-1}
Natural linewidths, 2Γ	10^{-9}–10^{-6}
Isomer shift	0–10^{-6}
Quadrupole splitting	0–10^{-6}
Magnetic hyperfine splitting	0–10^{-6}