

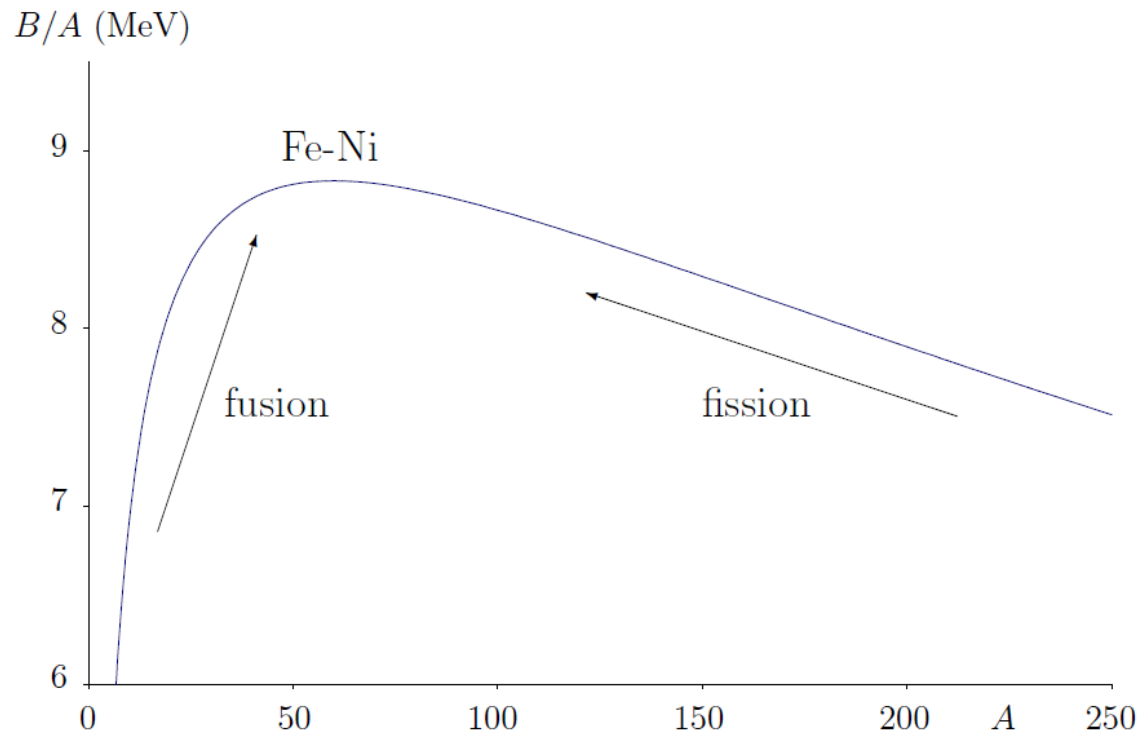
Chapter IX: Nuclear fusion

Summary

1. General remarks
2. Basic processes
3. Characteristics of fusion
4. Solar fusion
5. Controlled fusion

General remarks (1)

- Maximum of binding energy per nucleon at about $A \approx 56$ (Fe)
→ fission for heavier nuclei → fusion for lighter nuclei
- Fusion of 2 light nuclei produces an increase of binding energy per nucleon



General remarks (2)

- Advantages of fusion (compared to fission) as energy source:
 1. Light nuclei are abundant and easy to obtain
 2. End products are lights and usually stable
 3. Production of small quantity of short-live radioactive wastes
 4. No risk of core meltdown
 5. No proliferation
- Disadvantages of fusion:
 1. Before combination of light nuclei → overcoming of the Coulomb repulsion → enormous disadvantage
- To reach fusion → kinetic energy at least equal to energy repulsion has to be provided to nuclei
- Use of accelerators → current very low → power output very low (a few W)
- Use of thermal energy → very high temperature has to be reached → magnetic or inertial confinement

Basic processes (1)

- In controlled fusion reactors $\rightarrow$ to overcome the Coulomb barrier $\rightarrow$ choice of nuclei with small charge
- Most elementary fusion reaction $\rightarrow p + p \rightarrow {}^2\text{He} \rightarrow$ not possible (2 + charges) $\rightarrow$ attention: other possible pp reaction: positron emission $\rightarrow$ see solar fusion section
- Another elementary reaction $\rightarrow {}^2\text{H} + {}^2\text{H} \rightarrow {}^4\text{He} + \gamma$ (γ is necessary for energy balance because ${}^4\text{He}$ has no excited states) $\rightarrow$ problem because $Q = 23.8 \text{ MeV} >$ the separation energy of both proton and neutron of ${}^4\text{He}$
- More likely reactions $\rightarrow {}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + n$ ($Q = 3.3 \text{ MeV}$) or ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{H} + p$ ($Q = 4.0 \text{ MeV}$) $\rightarrow$ deuterium-deuterium (D-D) reactions
- But more stable is the fusion product greater is the energy released $\rightarrow$ a reaction that form ${}^4\text{He}$ has a particularly large energy release $\rightarrow {}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + n$ ($Q = 17.6 \text{ MeV}$) $\rightarrow$ deuterium-tritium (D-T) reaction

Basic processes (2)

- The D-T reaction is often used as neutron source
- The D-T reaction produces a large amount of energy (17.6 MeV)
→ separated into ^4He and n
- The D-T reaction has a large cross section (see below)
- The D-T reaction is characterized by the same Coulomb barrier as D-D reactions
- For these reasons the D-T reaction has been selected to be used in controlled fusion reactors
- Other basic processes will be explained in the solar fusion section

Characteristics of fusion: Energy release

- Calculation of the Q value $\rightarrow$ for controlled fusion reactors and solar processes the kinetic energy of the initial particles is small (in the 1-10 keV energy range) compared to $Q \rightarrow$ energy release = final total energy of the products particles $\rightarrow$

$$A + B \rightarrow C + D \quad \longrightarrow \quad \frac{1}{2}m_C v_C^2 + \frac{1}{2}m_D v_D^2 \simeq Q$$

- Again neglecting the initial motions $\rightarrow m_C v_C \simeq m_D v_D$
- We obtain thus $\rightarrow$

$$\frac{1}{2}m_C v_C^2 \simeq \frac{Q}{1 + m_C/m_D} \quad \frac{1}{2}m_D v_D^2 \simeq \frac{Q}{1 + m_D/m_C} \quad \frac{\frac{1}{2}m_C v_C^2}{\frac{1}{2}m_D v_D^2} \simeq \frac{m_D}{m_C}$$

- For D-T reaction $\rightarrow$ 80% of energy is taken by neutron $\leftrightarrow$ for D-D reaction $\rightarrow$ 75% of energy is taken by neutron/proton

Characteristics of fusion: Coulomb barrier

- With R_A and R_B the radii of reacting particles $\rightarrow$ the Coulomb barrier for just touching particles is

$$V_C = \frac{e^2}{4\pi\epsilon_0} \frac{Z_A Z_B}{R_A + R_B}$$

- The effect of the Coulomb barrier on the fusion reaction is similar to the effect of the Coulomb barrier on α decay $\rightarrow$ the product $Z_A Z_B$ will appear in an exponential barrier penetration probability $\rightarrow$ cross section very sensitive to its $\rightarrow$ the barrier is lowest for the hydrogen isotopes
- For D-T reaction $V_C = 0.4$ MeV $\rightarrow$ it is low but still far above the typical incident particle energy of 1-10 keV $\rightarrow$ tunnel effect

Characteristics of fusion: Cross section (1)

- Calculations from basic expression for nuclear reaction cross sections
→ use of the Breit-Wigner formula (eq. 10.102 of N. Cerf, Mécanique quantique, Première partie: Notions de base) →

$$\sigma_l = \frac{4\pi}{k^2} (2l + 1) \frac{\Gamma^2/4}{(E_r - E)^2 + \Gamma^2/4}$$

- Particles reacting at thermal energies → reaction occurs far from any resonance → energy dependence of the cross section comes mainly from two terms →
 1. The k^{-2} factor → v^{-2} dependence
 2. A barrier penetration factor on the form e^{-2G} as for α decay but substituting for Q the center of mass energy E of reacting particles


$$\sigma \propto \frac{1}{v^2} e^{-2G}$$

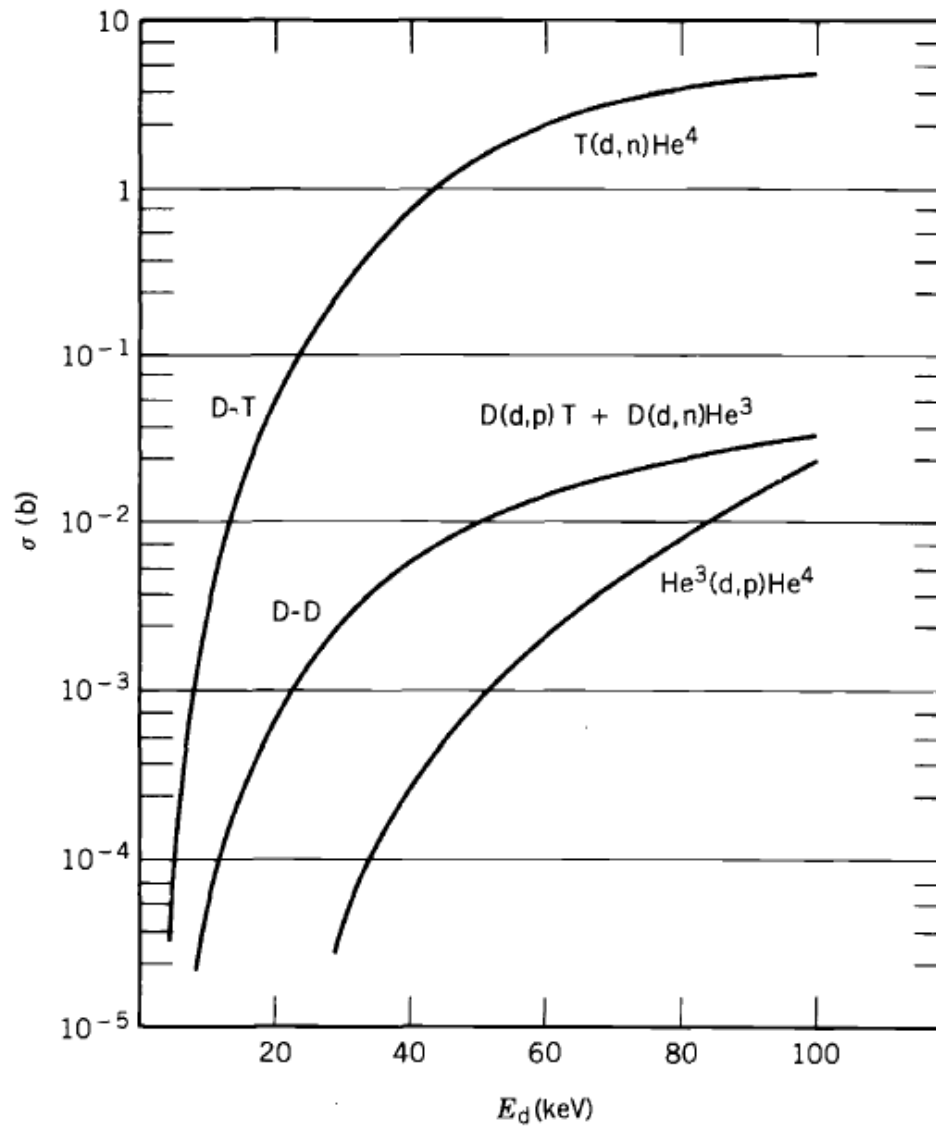
Characteristics of fusion: Cross section (2)

- G is the Gamow factor of α decay but substituting for Q the center of mass energy E of reacting particles $\rightarrow$ as $E \ll B \rightarrow$ with v the relative velocity of interacting particles $\rightarrow$

$$G \simeq \frac{e^2}{4\pi\epsilon_0} \frac{\pi Z_A Z_B}{\hbar v}$$

- In σ the proportionality factor involve nuclear matrix elements and statistical factors depending on the spins of the particles but energy dependence is correct

Characteristics of fusion: Cross section (3)



Characteristics of fusion: Reaction rate (1)

- The reaction rate is $\propto$ to $\sigma \times v$ (see Nuclear Metrology Techniques)
- In thermonuclear fusion the distribution of particle speeds is described by the usual Maxwell-Boltzmann velocity distribution for particles in thermal equilibrium (with k the Boltzmann constant and T the temperature) $\rightarrow$

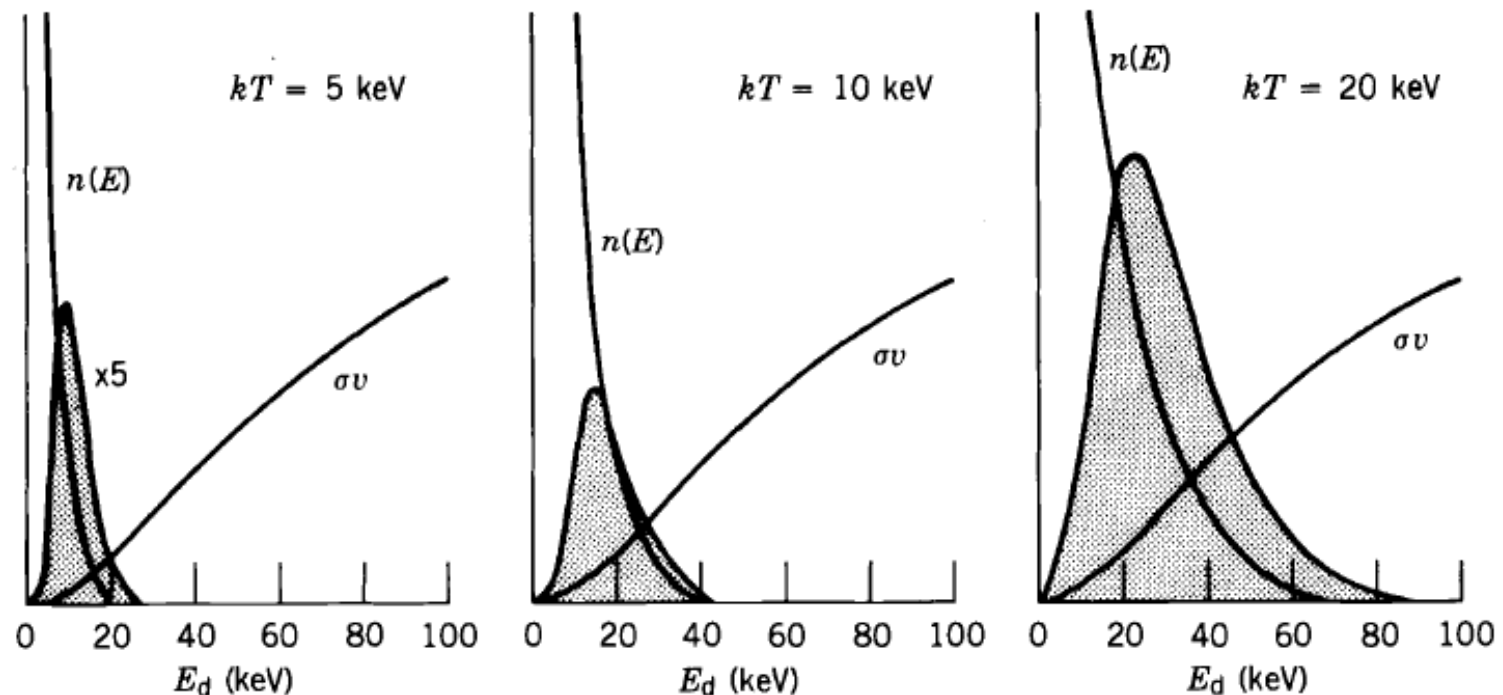
$$n(v) \propto e^{-mv^2/2kT}$$

- $n(v)v^2dv$ gives the relative probability to find a particle with speed between v and $v + dv$ in a collection of particles in thermal equilibrium at T
- For a gas of identical particles $\rightarrow$

$$\langle \sigma v \rangle \propto \int_0^\infty \frac{1}{v} e^{-2G} e^{-mv^2/2kT} v^2 dv \quad \longleftrightarrow \quad \langle \sigma v \rangle \propto \int_0^\infty e^{-2G} e^{-E/kT} dE$$

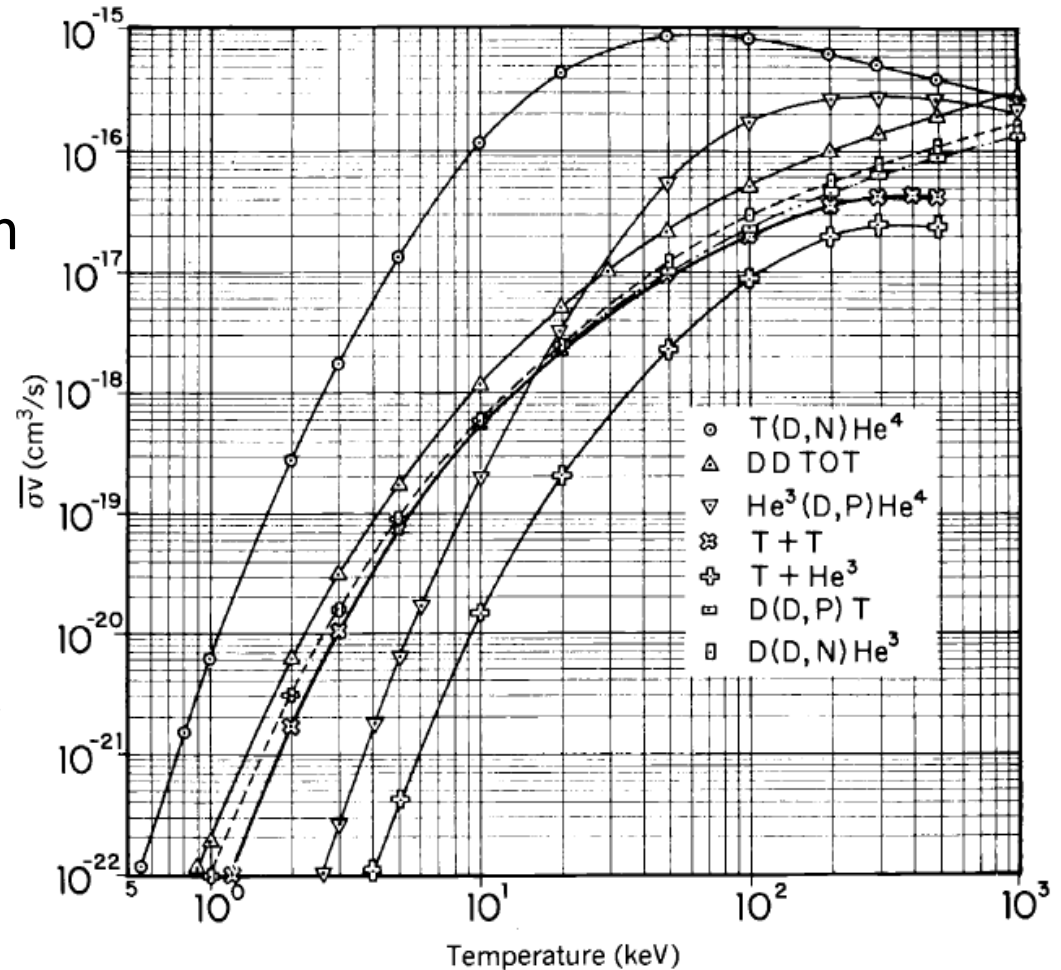
Characteristics of fusion: Reaction rate (2)

- At low $T \rightarrow$ little overlap between $n(E)$ and $\sigma v \rightarrow$ the average is small
- At very high $T \rightarrow$ the area of the Maxwell-Boltzmann distribution becomes small $\rightarrow$ the average value of σv is small
- At intermediate $T \rightarrow \langle \sigma v \rangle$ reaches a maximum



Characteristics of fusion: Reaction rate (3)

- At extremely high $T \approx 10^{10}$ K (corresponding to MeV energies) $\rightarrow$ the D-T reaction may become less favorable than others
- In the temperature region achievable in a thermonuclear fusion reactor (1-10 keV or $T \approx 10^7$ - 10^8 K) $\rightarrow$ the D-T reaction is clearly favored



Characteristics of fusion: Reaction rate (4)

- The simple theory used here is appropriate only for the D-D reaction (only 1 type of nuclei)
- For reactions involving two different nuclei (D-T, ...) → the velocities of the different species have to be considered
- The cross section and reaction rate involve a relative velocity → $\sigma(v_{rel})$ and $\langle \sigma(v_{rel}) v_{rel} \rangle$ have to be determined → average is done over the Maxwell-Boltzmann distribution of both species
- More complicated calculations but previous general conclusions about fusion reaction rates remain valid

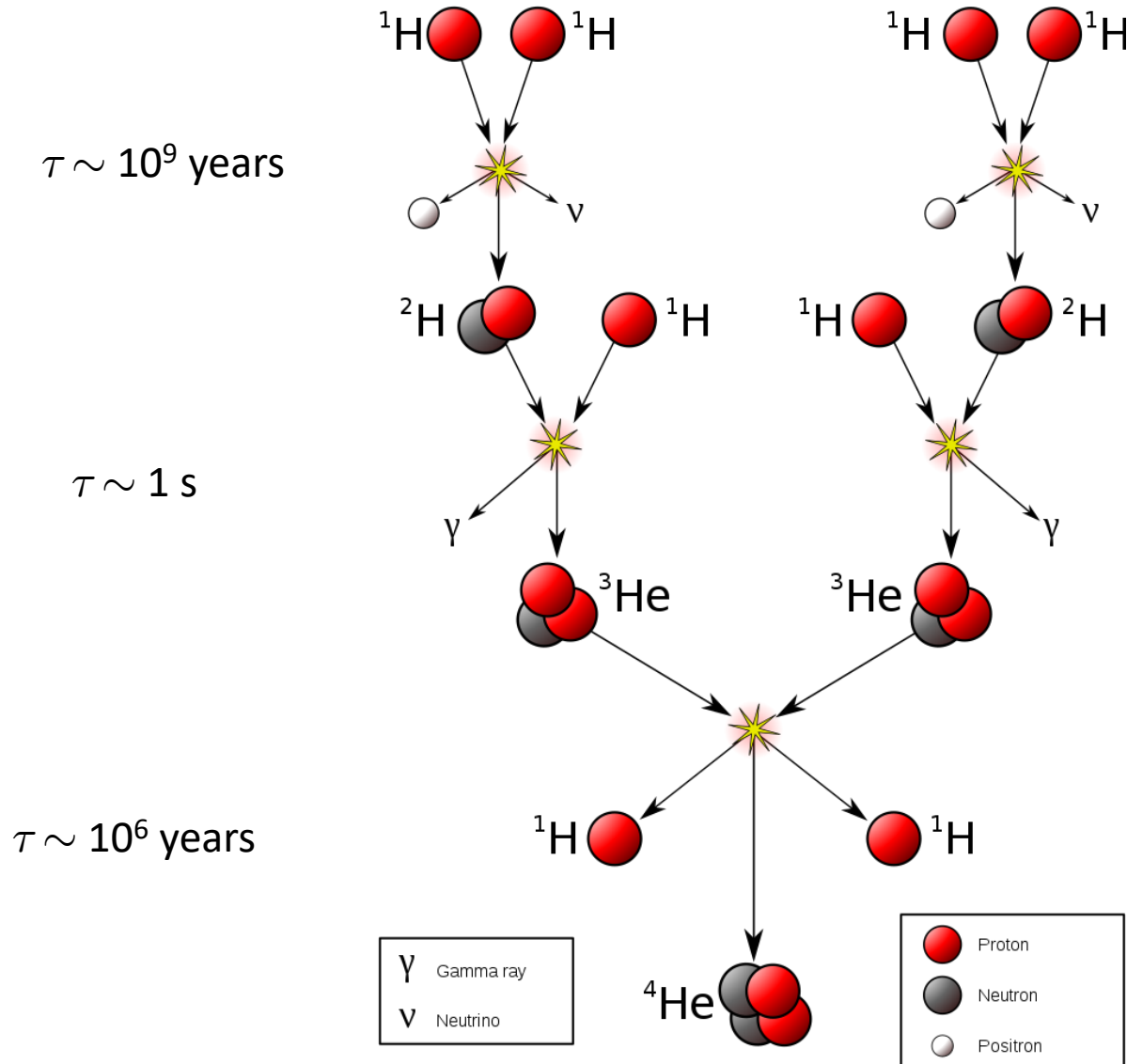
Solar fusion: General remarks

- Sun = extremely successful prototype of a self-sustaining thermonuclear reactor at (in the core) $T \approx 15 \times 10^6$ K (output constant during 10^9 years)
- Basic process in the Sun (and in most other stars) $\rightarrow$ fusion of H into He
- Abundance of atoms in universe $\rightarrow$ about 92% of the atoms in the universe are hydrogen $\leftrightarrow$ 7% are He (formed during the early stages of universe) $\leftrightarrow$ less than 1% are other ones
- All reactions in any fusion cycle must be two-body reactions because the simultaneous collision of three particles is too improbable

Solar fusion: pp chain (1)

- First step: $p + p \rightarrow {}^2\text{H} + e^+ + \nu$ ($Q = 0.42 \text{ MeV}$) $\rightarrow$ very small reaction rate ($5 \times 10^{-18} \text{ s}^{-1}$ per proton) $\rightarrow$ bottleneck process $\rightarrow$ but number of protons in the Sun $\approx 10^{56}$
- Second step: $p + {}^2\text{H} \rightarrow {}^3\text{He} + \gamma$ ($Q = 5.49 \text{ MeV}$) $\rightarrow$ D-D reaction is very unlikely because the number of deuterons is small (1 ${}^2\text{H}$ for $\approx 10^{18}$ ${}^1\text{H}$)
- Third step: ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2{}^1\text{H} + \gamma$ ($Q = 12.86 \text{ MeV}$) $\rightarrow$ ${}^3\text{He}$ -p reaction is not possible (${}^4\text{Li}$ does not exist as a bound system) and ${}^3\text{He}$ -D is unlikely because density of D is very low and because D is rapidly converted to T
- The net reaction called the proton-proton (pp) chain is thus the conversion of 4 protons to helium : $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu$ with a total $Q = 2 \times (0.42 + 5.49) + 12.86 + 4 \times 0.51 = 26.72 \text{ MeV}$

Solar fusion: pp chain (2)

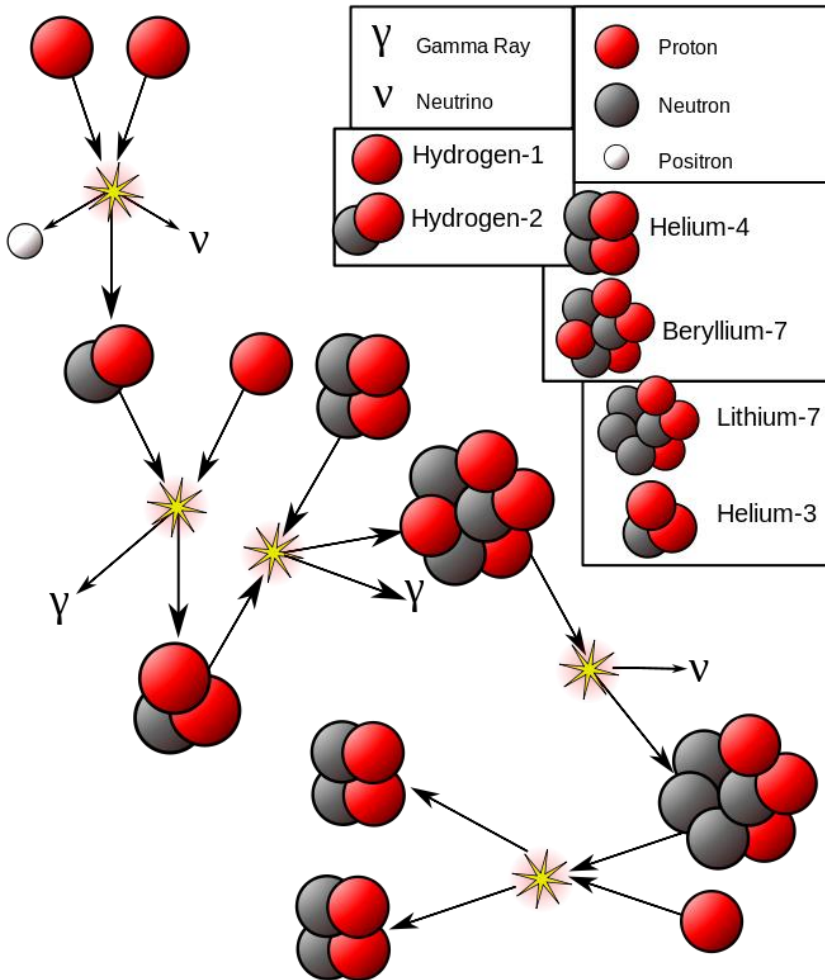


Solar fusion: pp chain (3)

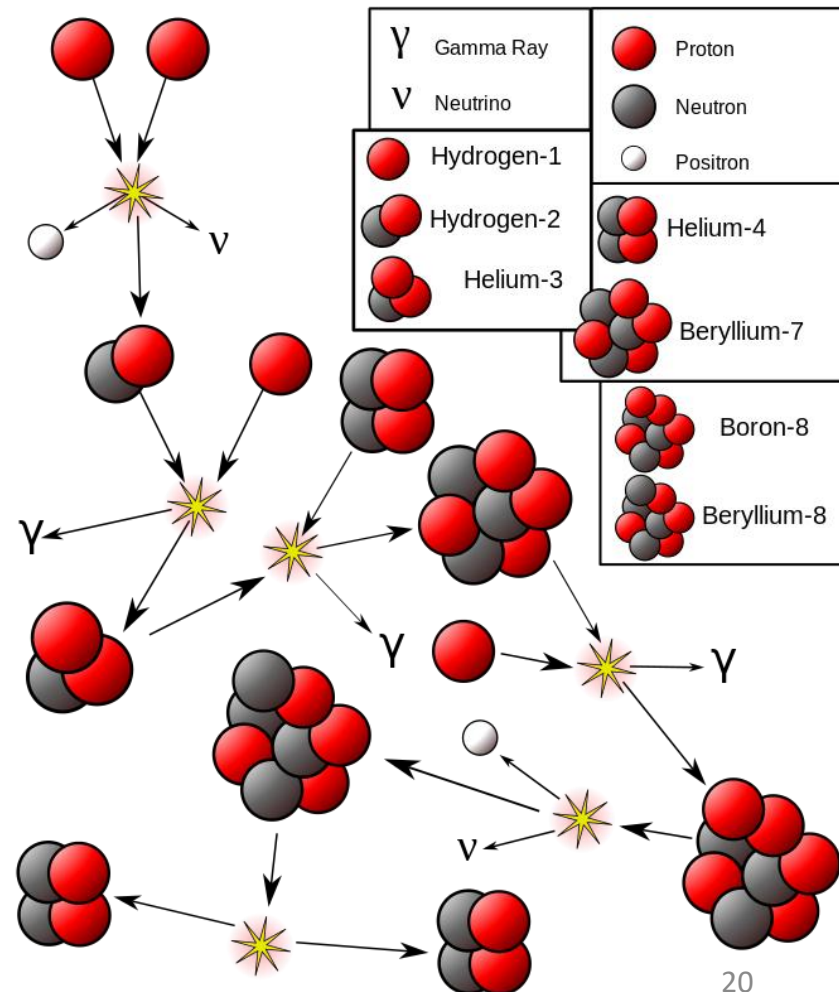
- More precisely this chain is called pp1 and is dominating for $T \approx 10-14 \cdot 10^6 \text{ K}$
- An alternative chain (pp2) is dominating for $T \approx 14-23 \cdot 10^6 \text{ K} \rightarrow {}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma \rightarrow {}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu \rightarrow {}^7\text{Li} + p \rightarrow 2{}^4\text{He}$
- pp3 is dominating for $T > 23 \cdot 10^6 \text{ K} \rightarrow {}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma \rightarrow {}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma \rightarrow {}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu \rightarrow {}^8\text{Be} \rightarrow 2{}^4\text{He}$
- The net reaction and the net Q value are the same for these three possible paths

Solar fusion: pp chain (4)

pp2

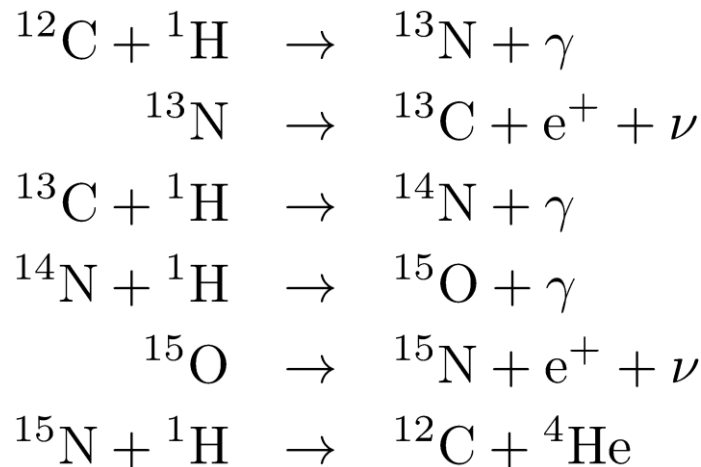


pp3



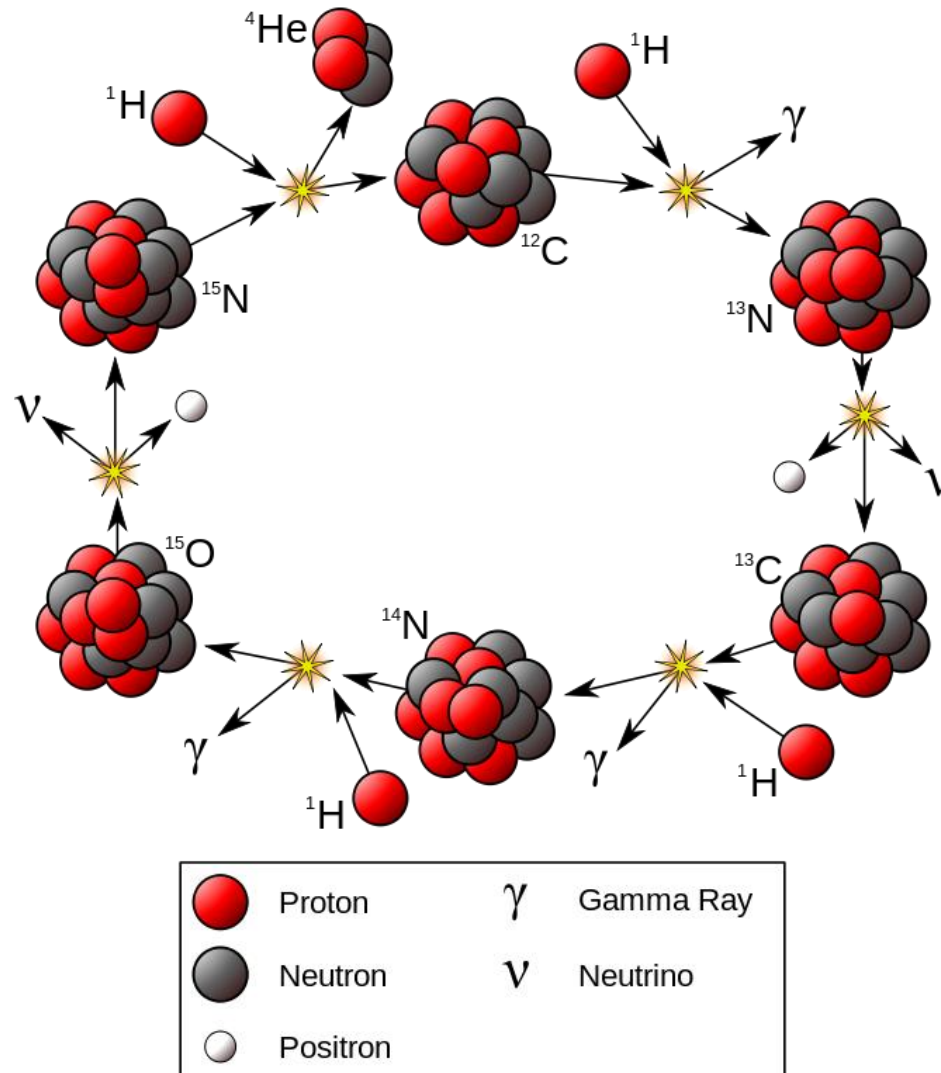
Solar fusion: CNO cycle (1)

- Heavier elements are present in a star $\rightarrow$ different series of fusion reactions can occur
- One important process is the carbon or CNO cycle $\rightarrow$



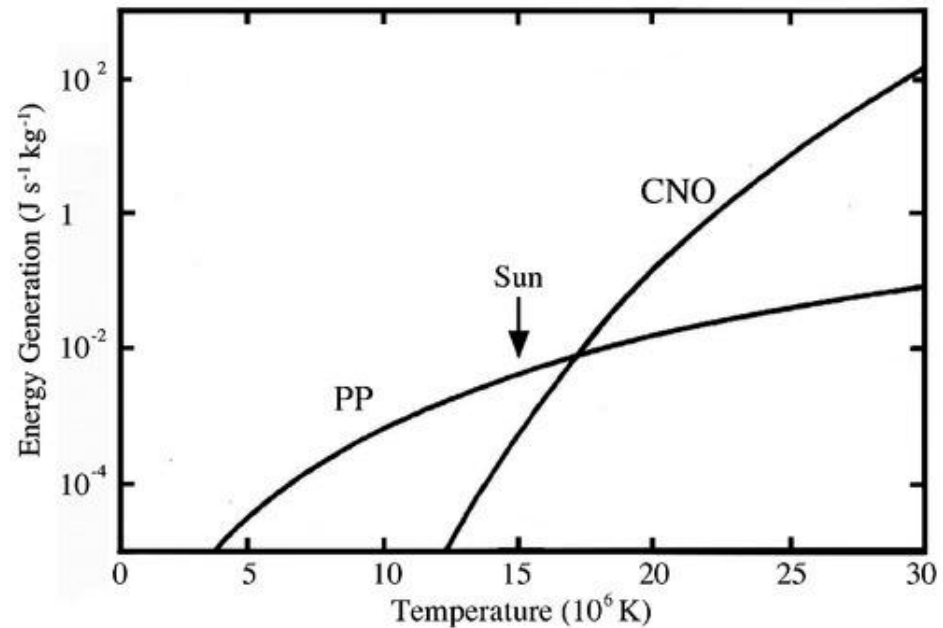
- ${}^{12}\text{C}$ is neither created nor destroyed $\rightarrow$ acts as a catalyst to aid in the fusion process
- The net process is $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu$ as in pp chain and Q is the same

Solar fusion: CNO cycle (2)



Solar fusion: CNO cycle (3)

- The CNO cycle proceeds more rapidly than the pp cycle (no deuterium bottleneck)
- However the Coulomb barrier is 6 or 7 times higher for proton reactions with carbon and nitrogen than for proton-proton reactions
- The CNO cycle is thus dominant at higher T (additional thermal energy is needed to increase the probability to penetrate the Coulomb barrier)



Solar fusion: After...

- Once a star has consumed its hydrogen $\rightarrow$ helium fusion reactions takes place with $3^4\text{He} \rightarrow ^{12}\text{C}$ at the higher T needed to penetrate the Coulomb barrier
- Other reactions involving fusion of light nuclei and α -particle capture continues to release energy
- The process ends near $^{56}\text{Fe} \rightarrow$ beyond it there is no energy gain in combining nuclei

Controlled fusion: General remarks

- High T ($\sim 10^8$ K $\rightarrow$ mean particle kinetic energies of 10 keV) $\rightarrow$ the atoms are ionized $\rightarrow$ fuel is a hot mixture of clouds of positive ions and negative electrons (overall electrically neutral) $\rightarrow$ plasma (see Y. Louis, Statistical physics and plasma physics)
- In a star $\rightarrow$ gravitational force confine the plasma $\rightarrow$ allows high temperature and pressure $\rightarrow$ fusion
- In Earth $\rightarrow$ gravitational confinement is impossible $\rightarrow$ magnetic or inertial confinement (electrostatic confinement is also possible but not mature)

Controlled fusion: Lawson criterion (1)

- In a plasma $\rightarrow$ high agitation of ions and electrons $\rightarrow$ many collisions between $\rightarrow$ to obtain fusion 3 quantities have to be considered $\rightarrow$
 1. Temperature T
 2. Density N
 3. Confinement time τ
- Considering a D-T plasma $\rightarrow$ we assume that $\rightarrow$
 - densities of D and T are each equal to $n/2$
 - Q is the energy released per reaction (17.6 MeV for D-T)
 - τ is the confinement time during which reactions occur
- The energy released per unit volume from fusion reactions in the plasma is $\rightarrow$

$$E_f = \frac{1}{4}n^2\langle\sigma v\rangle Q\tau$$

Controlled fusion: Lawson criterion (2)

- The thermal energy per unit volume needed to raise both ions and electrons to temperature T is $3/2nkT$ (with $n = n_e$) $\rightarrow$ the total thermal energy is $\rightarrow$

$$E_{th} = 3nkT$$

- Energy E_{th} is supplied to heat the plasma $\rightarrow$ during time τ plasma is confined and energy E_f can be extracted
- The reactor shows a net energy gain if $\rightarrow$

$$E_f > E_{th}$$

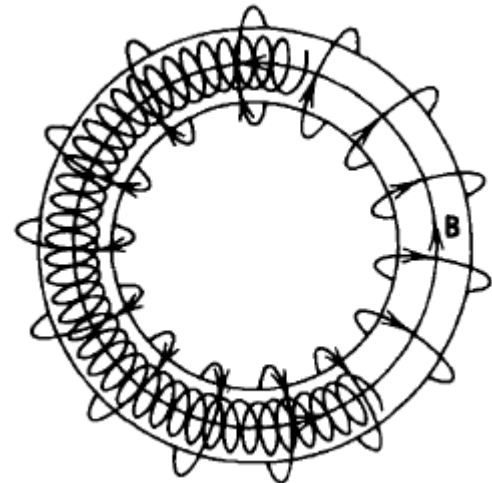
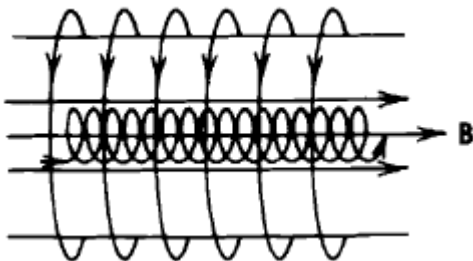
$$\frac{1}{4}n^2\langle\sigma v\rangle Q\tau > 3nkT$$

$$n\tau > \frac{12kT}{\langle\sigma v\rangle Q}$$

- This is called the Lawson criterion $\rightarrow$ represents the goal of reactor designers $\rightarrow$ for D-T reaction $\rightarrow n\tau > 10^{20} \text{ s/m}^3$

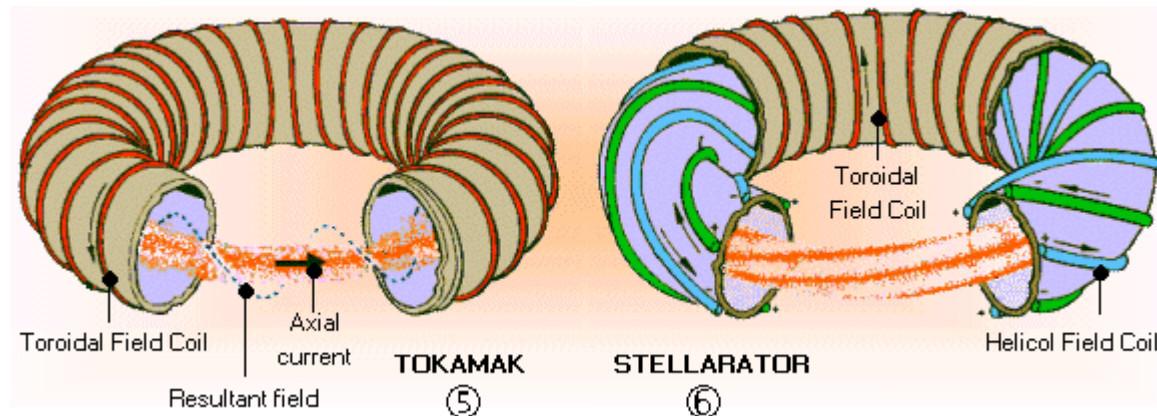
Controlled fusion: Magnetic confinement (1)

- In a magnetic confinement $\rightarrow$ the plasma is trapped and maintained at high temperature
- It must be confined in a limited immaterial box far from any material wall $\rightarrow$ the simplest magnetic confinement is a uniform magnetic field $\rightarrow$ charged particles spiral about the field direction
- To prevent the loss of particles along the axis $\rightarrow$ the line forms a torus $\rightarrow$ the spiral is kept in a ring



Controlled fusion: Magnetic confinement (2)

- In a toroidal geometry $\rightarrow$ the field is weaker at larger radii $\rightarrow$ as a particle spirals it sees a region of lower field which lets the spiral radius become larger $\rightarrow$ the particle approach the outer wall
- To reduce this effect a magnetic field component along the surface of the toroid is introduced $\rightarrow$ the poloidal field $\rightarrow$ helical path
- It can be achieved using
 - a set of external coils $\rightarrow$ stellarator
 - a current along the axis of the toroid through the plasma itself $\rightarrow$ tokamak



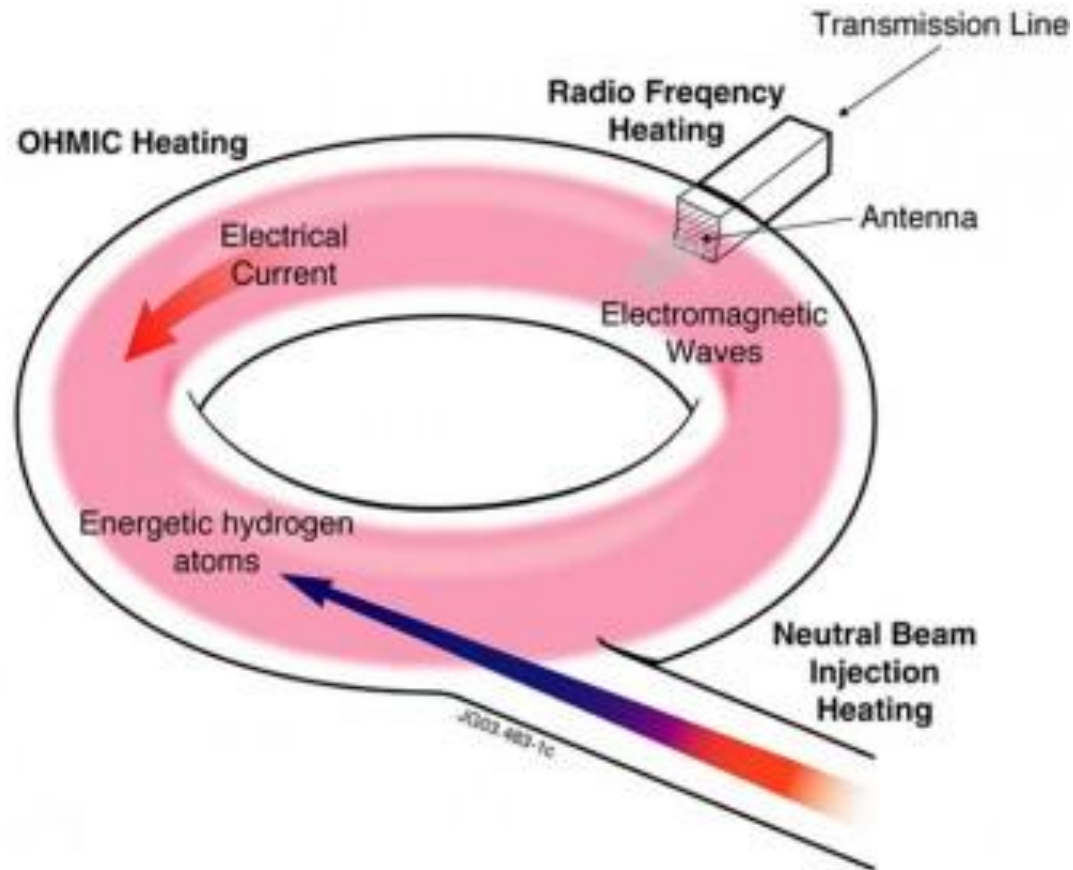
Controlled fusion: Plasma heating (1)

- The plasma is created inside the confinement structure at temperature too low to reach fusion → 3 methods are possible to heat the plasma up
 1. The current in the plasma itself is used to heat the plasma by Joule effect → effective to $T \approx 10^7$ K → beyond plasma resistivity becomes too weak → effectiveness $\searrow$ (impossible in stellarator $\leftrightarrow$ no central current)
 2. Heating by injection of neutrals → a beam of ions (outside the machine) is created → neutralized before entering the plasma → collisions transfer energy → $T \nearrow$
 3. Absorption of energy from electromagnetic waves → heating is transmitted to the plasma by antennas covering part of the confinement area

Controlled fusion: Plasma heating (2)

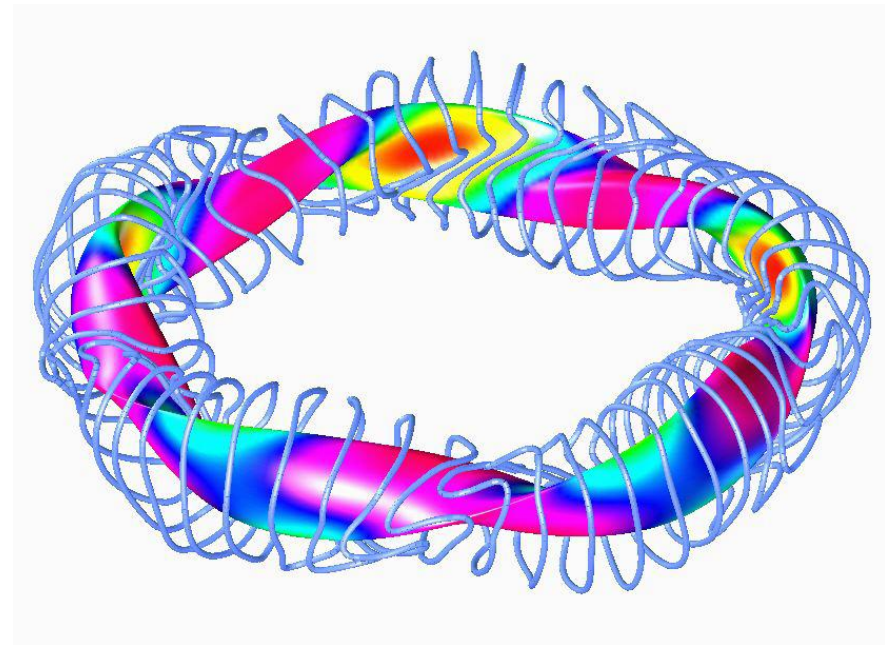
- Generally in fusion reactor with magnetic confinement → temperature is raised by a combination of the 3 methods
- When the number of fusion reactions $\nearrow$ → the energy carried by the helium nuclei is confined in the plasma → heating
- If this contribution becomes equal to the energy lost by the plasma → previous heating methods are no more necessary → plasma is self-maintained → ignition process
- We define the amplification factor Q as the ratio between the power generated by the plasma and the heating power injected into the plasma → If $Q = \infty$ → plasma is self-maintained → If $Q = 1$ → plasma supplies as much energy as injected → « break even »
- European tokamak JET (Joint European Torus) is close to break even ($Q \approx 0.65$)

Controlled fusion: JET heating system



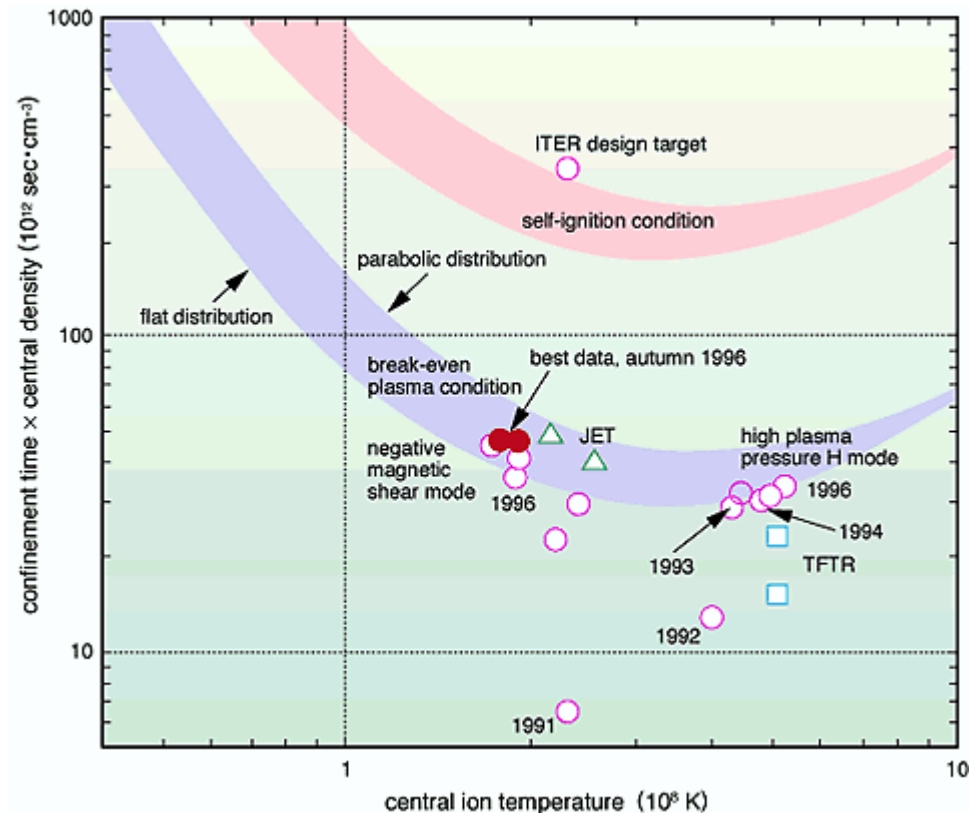
Controlled fusion: Stellarator

- The stellarator attempts to create a natural twist plasma path using external magnets
- In first tests → stellarator suffers from instabilities and plasma leakage
- Now → development of computers → new calculations possibilities → building of the largest stellarator (Wendelstein 7-X) at the Max Planck Institute of Plasma Physics in 2015



Controlled fusion: Tokamak

- Imagined by Tamm and Sakharov in 1950 → tokamak = **toroïdalnaïa kamera s magnitnymi katushkami** = toroidal chamber with magnetic coils
- Actually → the Lawson criterion is not reached (JET) → most interesting way to achieve controlled fusion for energy production → ITER project = International Thermonuclear Experimental Reactor

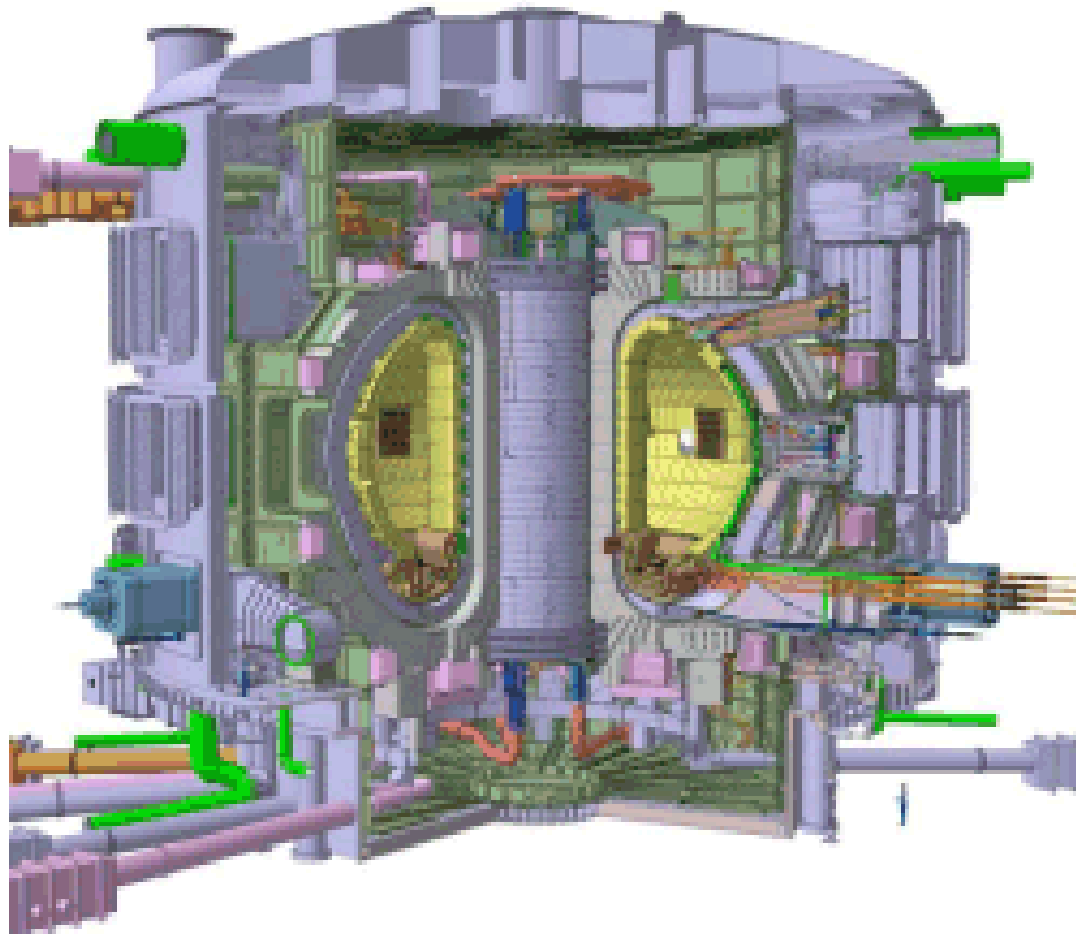


Controlled fusion: JET

- First plasma production: 1983
- Pulse duration: 20 s
- Maximum magnetic field (toroidal): 3.45 T
- Maximum plasma current: 5 MA
- Maximum injected power: 25 MW
- Mass (nucleus) : 2 800 tons

Controlled fusion: ITER (1)

- First expected test → 2035
- Electricity production → 2050



Controlled fusion: ITER (2)

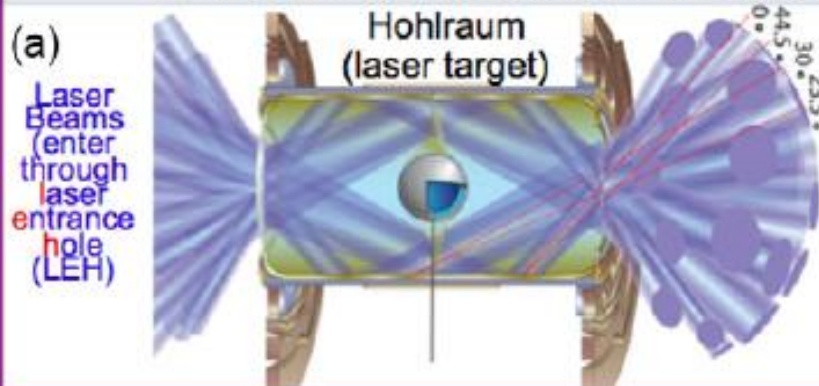
- Plasma volume: 840 m^3
- Plasma mass: $\sim \text{g}$
- Maximum plasma current: 15 MA
- Maximum magnetic field (toroidal): 5.3 T
- Pulse duration: 6 min to 1 h
- Expected Q: ~ 10

Controlled fusion: Inertial confinement (1)

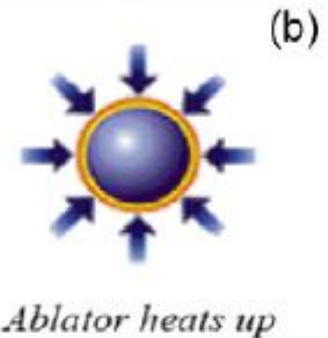
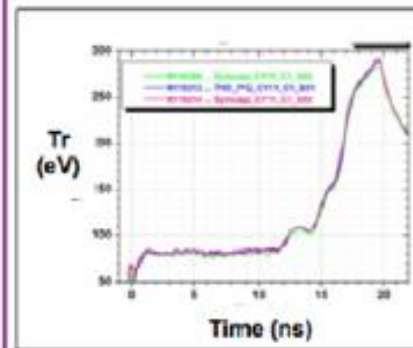
- We consider a microball generally in gold (« hohlraum ») enclosing a pellet of gaseous deuterium and tritium encased in ablator material
- Laser energy enters the hohlraum target → strikes the hohlraum walls → is converted to X-radiation
- X-radiation fills the hohlraum → creating a radiation oven that bathes the capsule → the ablator heats up
- The ablated shell expands outward → the remainder of the capsule is compressed inward
- Fusion initiates in a central hot spot where the ion temperature is high → a burn front propagates outward

Controlled fusion: Inertial confinement (2)

Coupling: laser energy couples to hohlraum and converts to x-rays



Drive: x-rays bathe capsule, heating it up -- it expands



Symmetry: radiation compresses capsule and it implodes



Rocket effect

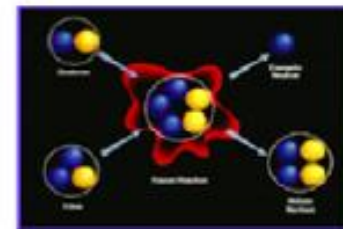
- conservation of momentum: ablated shell expands outward, rest of shell (frozen DT) is forced inward

(c)

Fusion initiates in a central hot spot and a burn front propagates outward



Ignition



Burn wave

(d)