

Chapter V: Alpha decay

Summary

1. General principles
2. Energy and momentum conservations
3. Theory of α emission
4. Angular momentum and parity

General principles (1)

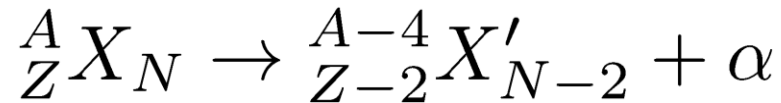
- The nucleus emit an α particle i.e. a nucleus of helium: ${}^4_2\text{He}_2$
- Alpha emission is a Coulomb repulsion effect $\rightarrow$ becomes important for heavy nuclei (Coulomb repulsion in heavy nuclei due to the larger number of protons present) $\rightarrow$ Bethe-Weizsäcker formula $\rightarrow$ Coulomb force $\nearrow$ with size at a faster rate (in $\sim Z^2$) than does the specific nuclear binding force (in $\sim A$)
- The α emission is particularly favored (compared to other particles) due to
 - Its very stable and tightly bound structure
 - Its small mass
 - Its small charge
- Theoretically some heavier particles as ${}^8\text{Be}$ or ${}^{12}\text{C}$ may be emitted or the fission into equal daughter nuclides may happen $\rightarrow$ but very penalized

General principles (2)

- For a nucleus to be an α emitter $\rightarrow$ not enough for the α decay to be energetically possible $\rightarrow$ the disintegration constant must also not be too small $\rightarrow$ the α emission would occur so rarely that it may never be detected ($T_{1/2} < 10^{16}$ years)
- Moreover if β decay is present $\rightarrow$ can mask the α decay
- Most nuclei with $A > 190$ and many with $150 < A < 190$ are energetically possible emitters but $\frac{1}{2}$ of them effectively meet the 2 other conditions

Energy conservation (1)

- The α emission process (between ground state levels) is:



- Rutherford shows in 1908 that the α particle is a nucleus of ${}^4\text{He}$
 $\rightarrow$ constituted of 2 protons and 2 neutrons
- Energy conservation with the initial decaying nucleus X at rest $\rightarrow$

$$m_X c^2 = m_{X'} c^2 + T_{X'} + m_\alpha c^2 + T_\alpha$$

- Due to the linear momentum conservation $\rightarrow$ X' and α are in motion $\rightarrow$ T is the kinetic energy
- Equivalently we write $\rightarrow$

$$T_{X'} + T_\alpha = (m_X - m_{X'} - m_\alpha) c^2$$

Energy conservation (2)

- $Q = (m_X - m_{X'} - m_\alpha)c^2$ = the net energy released in the decay $\rightarrow$ the decay occurs spontaneously only if $Q > 0$
- Q can be calculated from atomic masses (even we discuss about nuclear processes) because the electron masses cancel in the subtraction
- For a typical α emitter (^{232}U) $\rightarrow Q$ may be calculated from the known masses for various emitted particles:

Emitted Particle	Energy Release (MeV)	Emitted Particle	Energy Release (MeV)
n	-7.26	^4He	+5.41
^1H	-6.12	^3He	-2.59
^2H	-10.70	^6He	-6.19
^3H	-10.24	^6Li	-3.79
^3He	-9.92	^7Li	-1.94

Energy conservation (3)

- Only α emission is possible in this case $\rightarrow \alpha$ is very stable $\rightarrow \alpha$ has a relatively small mass compared with the mass of its separate constituents
- The Q value is also the total kinetic energy given to the decay fragments $Q = T_{X'} + T_{\alpha}$
- For $Q > 0 \rightarrow$ we find back the condition $m_X > m_{X'} + m_{\alpha}$ of instability in particles
- Remark: α disintegration towards excited levels of X' are possible $\rightarrow$ the excitation energy of the nucleus X' has to be subtracted from Q

Linear momentum conservation

- As the original nucleus is at rest $\rightarrow X'$ and α move with equal and opposite momenta $\rightarrow p_\alpha = p_{X'}$
- As the α decay released typically 5 MeV $\rightarrow$ we can use nonrelativistic kinematics $\rightarrow m_\alpha T_\alpha = m_{X'} T_{X'} \rightarrow$

$$T_\alpha = \frac{Q}{1 + m_\alpha/m_{X'}}$$

- As X' is a heavy nucleus $\rightarrow A \gg 4 \rightarrow$

$$T_\alpha = Q(1 - 4/A), \quad T_{X'} = 4Q/A$$

- Typically the α carries 98% of the Q energy and X' carries 2% (recoil energy) corresponding for $\alpha = 5$ MeV to $T_{X'} = 100$ keV

Released energy (1)

- Introducing the binding energies the energy released during the α decay may be written $\rightarrow$

$$Q = B(4, 2) + B(A - 4, Z - 2) - B(A, Z)$$

- Thus $Q > 0$ becomes $\rightarrow$

$$\begin{aligned} B(4, 2) &> B(A, Z) - B(A - 4, Z - 2) \\ &= A \frac{B(A, Z)}{A} - (A - 4) \frac{B(A - 4, Z - 2)}{A - 4} \\ &= 4 \left(\frac{B(A, Z)}{A} \right)_m + (A - 2) \Delta \end{aligned}$$

- $(B/A)_m$ is the mean binding energy by nucleon of parent and daughter nuclei and Δ is their difference (≈ 30 keV for heavy nuclei)

Released energy (2)

- As $B(4,2) \approx 28 \text{ MeV} \rightarrow$ we have thus approximately $\rightarrow$

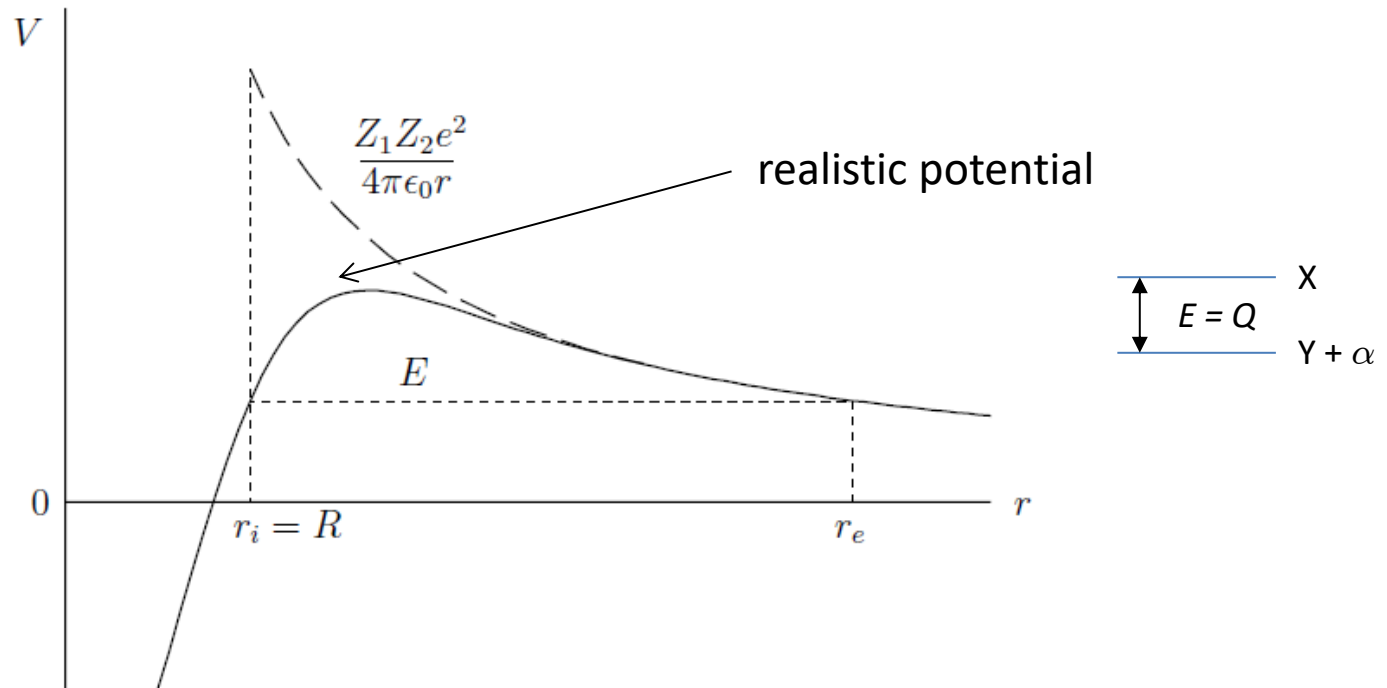
$$\left(\frac{B(A, Z)}{A} \right)_m < \frac{B(4, 2)}{4} \approx 7 \text{ MeV}$$

- Du to the term $\Delta \rightarrow \alpha$ decay becomes frequent for nuclei with $A > 200$ for which B/A is $< 7.8 \text{ MeV}$
- This also explains why the α emission is favored compared to other nuclei as deuteron ($B(2,1)/2 \approx 1.11 \text{ MeV}$) or tritium ($B(3,1)/3 \approx 2.83 \text{ MeV}$) $\rightarrow$ indeed the α particle has tightly bound structure $\rightarrow$ a pair of neutrons and a pair of protons inside a nucleus is favored to form an α -cluster

Theory of α emission (1)

- General features of α emission theory have been developed by Gamow, Gurney and Condon in 1928
- The α particle is assumed to move in a spherical region determined by the daughter nucleus $\rightarrow$ one-body model
- The α particle is preformed inside the parent nucleus $\rightarrow$ there is no proof that it is well the case but it works quite well
- Deeply inside the heavy nucleus $\rightarrow$ attractive nuclear force dominates the Coulomb repulsion force
- Outside the nucleus $\rightarrow$ only remains the Coulomb force
- Between the 2 (close to the nucleus surface) $\rightarrow$ number of neighbours $\searrow \rightarrow$ nuclear attractive force $\searrow \rightarrow$ equilibrium with repulsion force $\rightarrow$ potential barrier
- To be emitted the α particle has to cross this potential barrier by tunnel effect

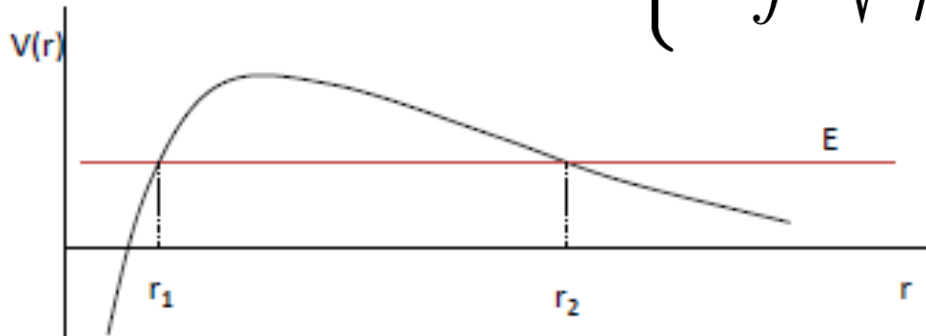
Theory of α emission (2)



- The probability of disintegration per time unit W is supposed to be $\propto$ to the probability of crossing the barrier $\rightarrow W \propto |T|^2$ with T the transmission coefficient
- The transmission probability is given by the WKB approximation

WKB approximation

- For decay between ground state levels (e.g. 0^+) $\rightarrow$ we have to resolve $u''(r) - 2\mu/\hbar^2 (V(r) - E) u(r) = 0$ (with $\mu \approx m_\alpha m_X/m_X$ the reduced mass of the α nucleus and the daughter nucleus)
- By choosing $u(r) = \exp(iS(r))$ with $S(r)$ slowly varying $\rightarrow$
 $S(r)'^2 = 2\mu/\hbar^2 (E - V(r))$
- Si $E < V(r) \rightarrow u(r) = \exp \left\{ - \int \sqrt{\frac{2\mu}{\hbar^2} (V(r) - E)} dr \right\}$



G = Gamow factor

$$|T|^2 = \left| \frac{u(r_2)}{u(r_1)} \right|^2 = \exp \left\{ -2 \int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2} (V(r) - E)} dr \right\} = \exp(-2G)$$

Theory of α emission (3)

- WKB approximation applied to α emission $\rightarrow r_1 = R$ (the radius of the nucleus), $r_2 = 2(Z-2)e^2/4\pi\epsilon_0 E$, with

$$V(r) = \begin{cases} 0 & r < R \\ 2(Z-2)e^2/4\pi\epsilon_0 r & r \geq R \end{cases}$$



$$|T|^2 = \exp \left\{ -2 \left(\frac{2\mu E}{\hbar^2} \right)^{1/2} \int_R^{r_2} \left(\frac{r_2}{r} - 1 \right)^{1/2} dr \right\}$$

- Transforming $r = r_2 \cos^2 u \rightarrow$

$$\int_R^{r_2} \left(\frac{r_2}{r} - 1 \right)^{1/2} dr = r_2 \left[\arccos \sqrt{\frac{R}{r_2}} - \sqrt{\frac{R}{r_2}} \sqrt{1 - \frac{R}{r_2}} \right]$$

Theory of α emission (4)

- For a heavy nucleus emitting 5 MeV $\alpha \rightarrow r_2 \approx 0.6 Z \text{ fm} > R \approx 1.25 A^{1/3} \text{ fm}$
- Assuming $R/r_2 \approx 0 \rightarrow$ we obtain the Gamow approximation $\rightarrow$
 $|T|^2 \approx \exp(-2\pi\eta)$

where η the Sommerfeld factor is ($\alpha = 1/137$)

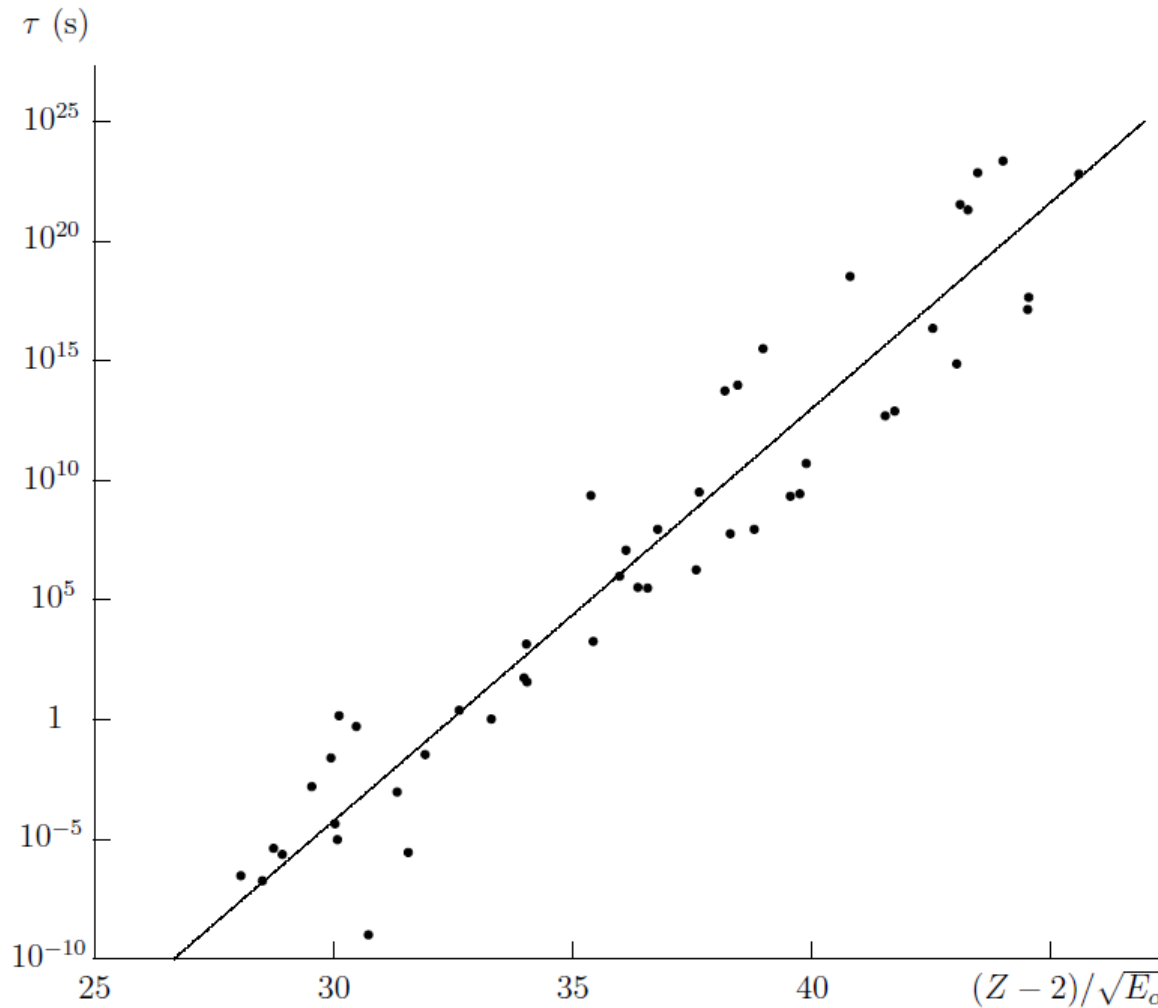
$$\eta = (Z - 2)\alpha \sqrt{\frac{2\mu c^2}{E}}$$

- The mean lifetime $\tau = W^{-1}$ approximately follows $\rightarrow$

$$\log_{10} \tau \approx C_1 + C_2 \frac{Z - 2}{\sqrt{E}}$$

$$\text{with } C_2 = 2\pi(\log_{10} e)\alpha \sqrt{2\mu c^2}$$

Theory of α emission (5)



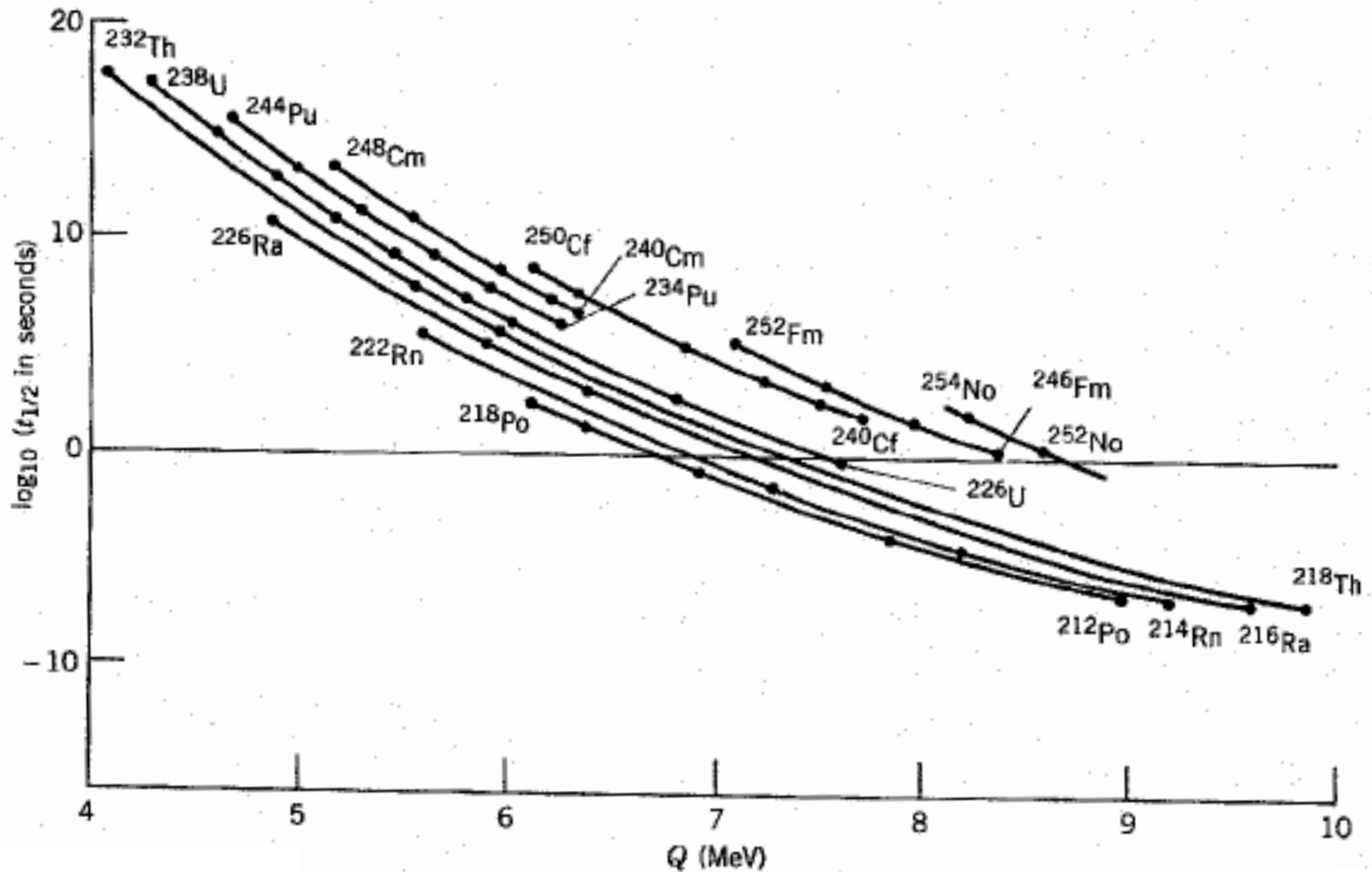
E_α is the kinetic energy of the α particle in MeV

Theory of α emission (6)

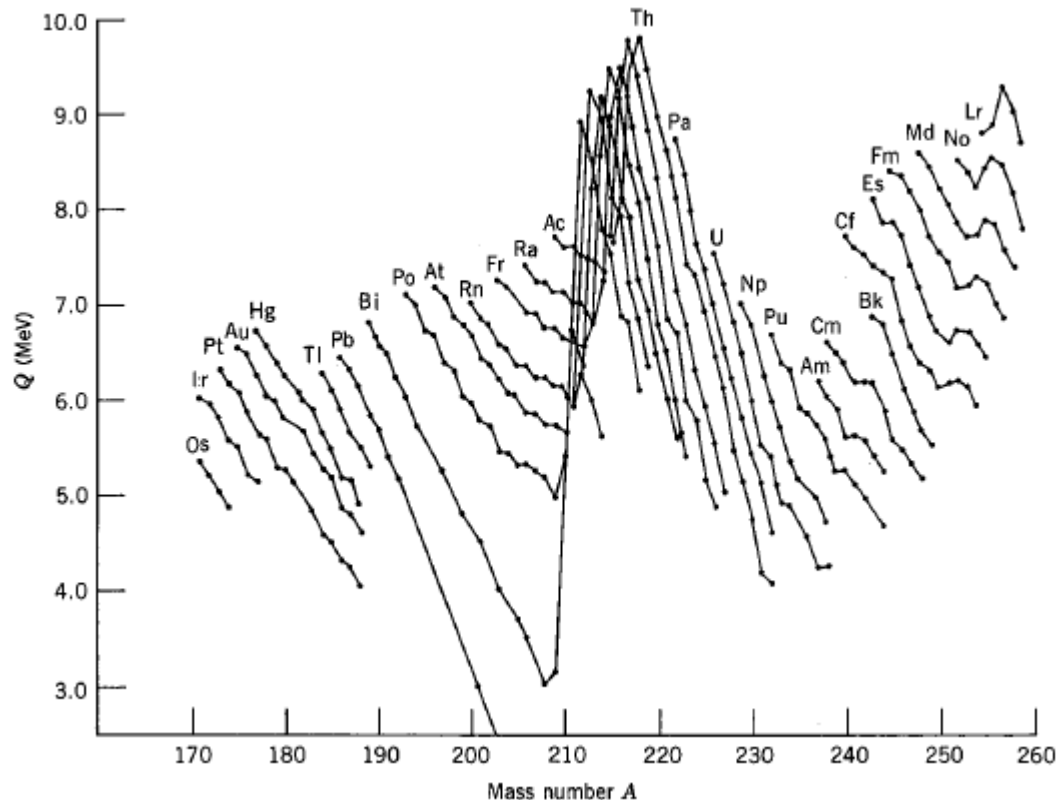
- The α emitters with short mean lifetime have large disintegration energy (and conversely) $\leftrightarrow$ observed in 1911: Geiger-Nutall rule
- Examples: ^{232}Th : $T_{1/2} = 1.4 \times 10^{10}$ years, $Q = 4.08$ MeV and ^{218}Th : $T_{1/2} = 1.0 \times 10^{-7}$ s, $Q = 9.85$ MeV
- A factor 2 in energy implies a factor 10^{24} in half-life
- Correct tendency for all isotopes but very good only for Z and N both even nuclei

A	Q (MeV)	$t_{1/2}$ (s)	
		Measured	Calculated
220	8.95	10^{-8}	3.3×10^{-9}
222	8.13	2.8×10^{-3}	6.3×10^{-5}
224	7.31	1.04	3.3×10^{-2}
226	6.45	1854	6.0×10^1
228	5.52	6.0×10^7	2.4×10^8
230	4.77	2.5×10^{12}	1.0×10^{13}
232	4.08	4.4×10^{17}	2.6×10^{16}

Theory of α emission (7)



Theory of α emission (8)



- When $A > 212 \rightarrow$ adding neutrons to a nucleus reduces the disintegration energy $\rightarrow$ due to the Geiger-Nuttall rule $\rightarrow$ increase of the half-life $\rightarrow$ the nucleus becomes more stable
- The discontinuity near $A = 212$ occurs where $N = 126 \rightarrow$ another example of nuclear shell structure.

Theory of α emission (9)

- Geiger-Nutall rule enables us to understand why other decays into light particles are not commonly seen (even though they are allowed by the Q value)

Theory of α emission (10)

- Geiger-Nuttall rule is able to reproduce $T_{1/2}$ within 1-2 orders of magnitude over a range of more than 20 orders
- Approximations in previous calculations:
 1. Initial and final wave functions ($\leftrightarrow$ Fermi Golden Rule) are not considered
 2. The nucleus is assumed to be spherical with $R \approx 1.25 A^{1/3}$ fm
 $\rightarrow$ heavy nuclei (specially with $A \geq 230$) have strongly deformed shape
 3. The angular momentum carried by the α particle is neglected

Angular momentum and parity (1)

- In previous calculations $\rightarrow$ transition between ground state levels (e.g. 0^+) $\rightarrow$ but an initial state can populate different final states in the daughter nucleus $\rightarrow$ « fine structure » of α decay
- In that case we have to consider the angular momentum J_i and J_f of the initial and final nuclear state $\rightarrow$ and consequently the angular momentum of the α particle ℓ_α
- Consequently $\rightarrow$ α decay must follow the laws of the conservation of angular momentum and of parity

Angular momentum and parity (2)

- Definition of the total angular momentum $\mathbf{J}$ for i nucleons:

$$\mathbf{J} = \sum_{i=1}^A (\mathbf{L}_i + \mathbf{S}_i)$$

with $\mathbf{L}_i$ and the orbital angular momentum operator and $\mathbf{S}_i$ the spin operator of the i^{th} nucleon

- In the particular case of α decay $\rightarrow$ this expression may be written (with $\mathbf{I}_\alpha$ and $\boldsymbol{\ell}_\alpha$ the spin and angular momentum of the α particle and $\mathbf{J}_i$ and $\mathbf{J}_f$ written for initial and final nuclear states) $\rightarrow$

$$\mathbf{J}_i = \mathbf{J}_f + \mathbf{I}_\alpha + \boldsymbol{\ell}_\alpha$$

- The α particle wave function is then represented by a $Y_{\ell m}$ with $\ell = \ell_\alpha$

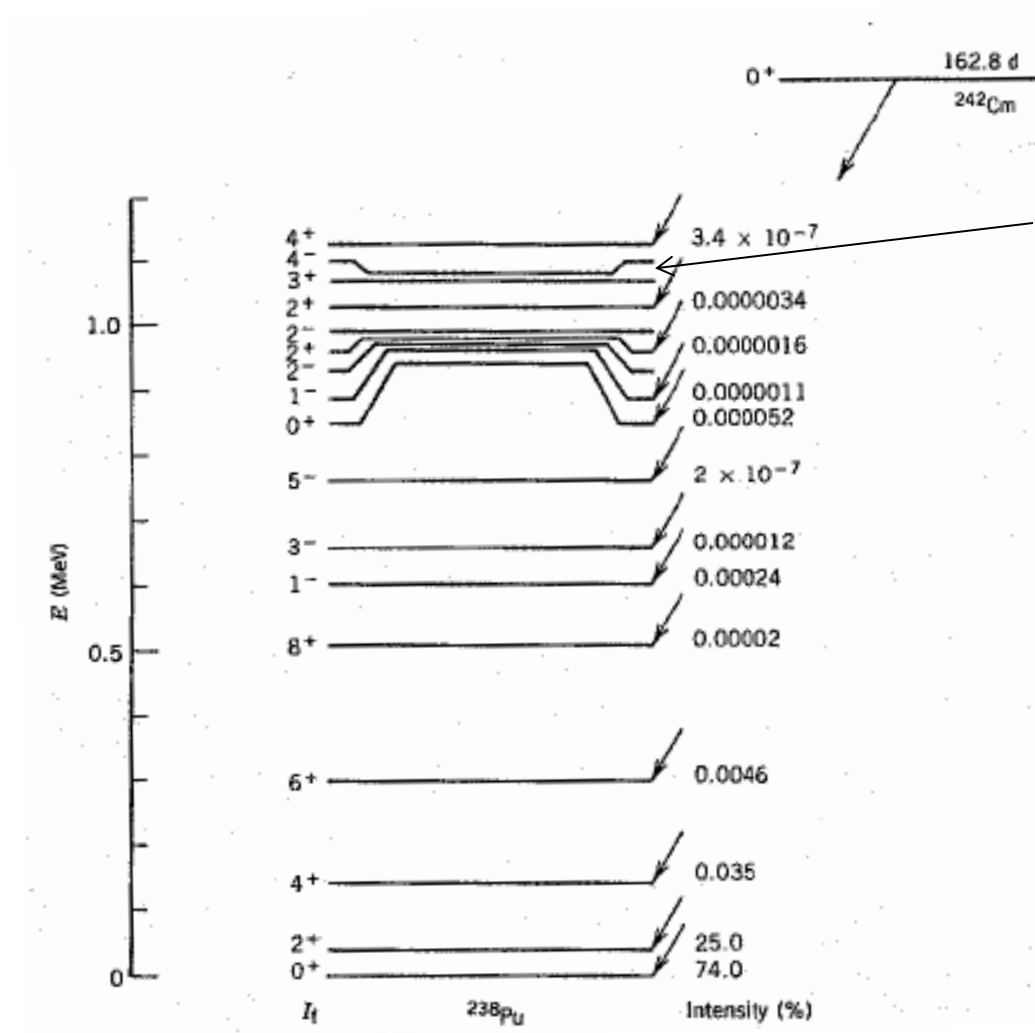
Angular momentum and parity (3)

- As the ^4He nucleus consists of 2 protons and 2 neutrons all in 1s state $\rightarrow$ their spins coupled pairwise $\rightarrow I_\alpha = 0$
- The composition of the 3 remaining angular momenta leads to $\rightarrow$
$$|J_i - J_f| \leq \ell_\alpha \leq J_i + J_f$$
- The conservation of parity implies $\rightarrow$
$$\pi_i = \pi_f \pi_\alpha (-1)^{\ell_\alpha}$$
- Moreover as the parity π_α of α particle is + (even-even nucleus) $\rightarrow$ the parity conservation rule becomes $\rightarrow$
$$(-1)^{\ell_\alpha} = \pi_i \pi_f$$
- If the initial and final parities are the same $\rightarrow \ell_\alpha$ must be even $\leftrightarrow$
If the parities are different $\rightarrow \ell_\alpha$ must be odd
- In particular for an initial state 0^+ (frequent case) $\rightarrow \ell_\alpha = J_f$

Angular momentum and parity (4)

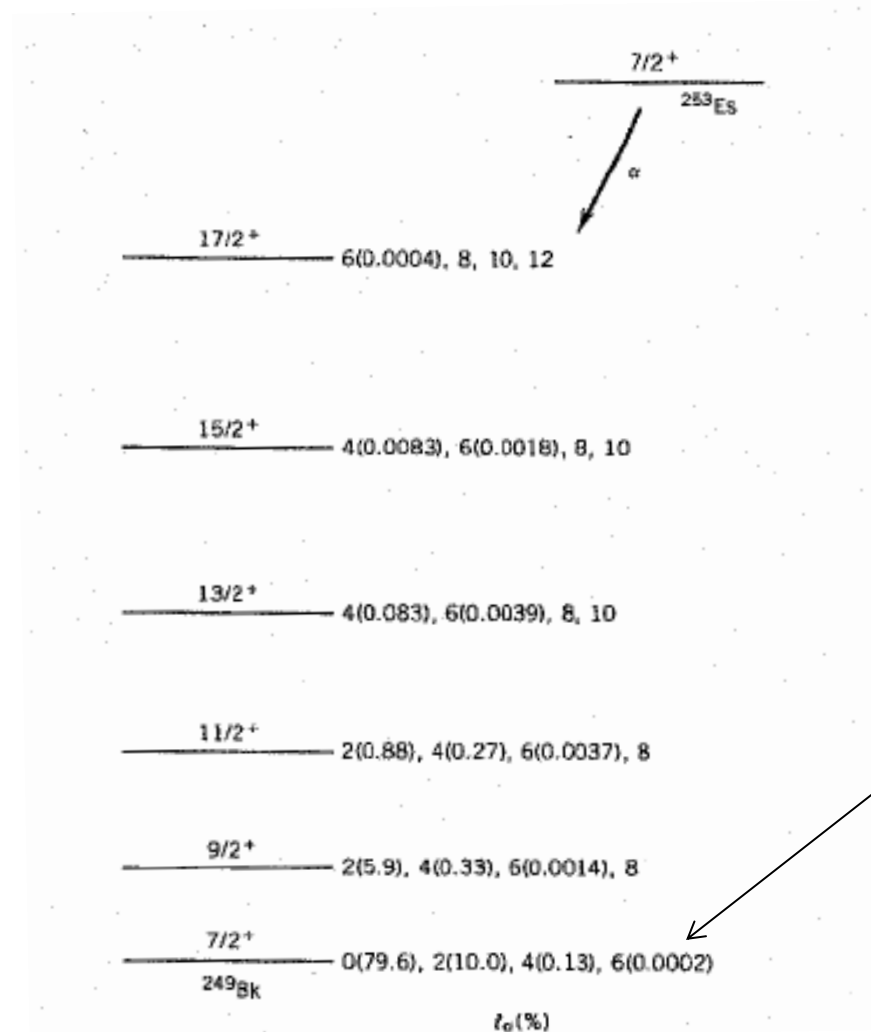
- Another consequence of the introduction of $\ell_\alpha \rightarrow$ the barrier of potential is raised and becomes (particle in a well):
$$V(r) + \hbar^2 \ell_\alpha (\ell_\alpha + 1) / 2mr^2$$
- The additional term is always $> 0 \rightarrow \nearrow$ of the barrier thickness $\rightarrow$ the probability transition $\searrow$
- Moreover the Q value $\searrow$ when the final state is not the ground state: $Q \rightarrow Q - E_x$ with E_x the energy of the excited state $\rightarrow$ application of the Geiger-Nuttall rule $\rightarrow$ a smaller Q value implies a large mean lifetime $\rightarrow$ a small transition probability $\rightarrow$ a small intensity in the decay branch
- These 2 reasons implies a $\searrow$ of the probability transition when the final state is not the ground state

Angular momentum and parity (5)



The 3^+ state is forbidden by the parity selection rule $\rightarrow 0 \rightarrow 3$ decay must have $\ell_\alpha = 3 \rightarrow$ the parity has to change

Angular momentum and parity (6)



intensity