Chapter IV:
Interactions of photons with matter

Contents of the chapter

Basic considerations

- Photons are classified according to their mode of origin:
  - γ rays accompany nuclear transitions $E_\gamma = h\nu = E_i - E_f$ (E_γ > 100 keV)
  - Bremsstrahlung (continuous X rays) result from a charged particle acceleration
  - Characteristic X rays are emitted in atomic transitions of bound electrons between the K, L, M, … shells in atoms (E_X < 100 keV)
- Momentum $\overrightarrow{p} = \hbar \overrightarrow{k}$ with $p = E_\gamma / c = \hbar k$ and k the wavenumber
- Photons interact with matter in a single event → between two events they have no interaction with matter (unlike the charged particles via the Coulomb force)
- Photons are indirectly ionizing radiations

Different interactions of γ with matter (1)

For 1 keV < E_γ < 1 GeV → Classification of Fano: 4 types of interactions and 3 consequences of the interaction → 12 theoretical processes are possible (even if a few of them are extremely rare or were never observed)

Type of interaction	**Effect of the interaction**
1: Atomic electron	A: Coherent diffusion
2: Atomic nucleus	B: Incoherent diffusion
3: Electric field of the nuclei and atomic electrons	C:Ttotal absorption
4: Mesonic field of atomic nuclei	

Different interactions of γ with matter (2)

- Only 3 effects are dominating →
  1. Compton Effect (1B): the photon is scattered by a free or weakly bound electron → the sum of the scattered photon energy and of the electron kinetic energy is equal to the energy of the incident photon
  2. Photoelectric effect (1C): The photon is absorbed by the electronic system (atom) → it gives all his energy → an atomic electron is emitted out of the atom with a kinetic energy equal to the energy of the photon minus the binding energy of the atomic electron
  3. Pair creation (3C): In the electric field of a nucleus or of an electron → the photon disappears and an electron-positron pair appears
- 2 other processes can also play a role →
  1. Rayleigh scattering (1A): the photon is scattered without energy loss by an electronic system (atom)
  2. Photodisintegration of the nucleus (2C): the photon is absorbed by the nucleus and a particle is emitted (γ, α, p, n, …)

Correspondence atomic shell ↔ electronic configuration

Couche atomique	Configuration e^-
K	$1s$ (j = 1/2)
L_I	$2s$ (j = 1/2)
L_{II}	$2p$ ($j = l - s$ = 1/2)
L_{III}	$2p$ ($j = l + s$ = 3/2)
M_I	$3s$ (j = 1/2)
M_{II}	$3p$ ($j = l - s$ = 1/2)
M_{III}	$3p$ ($j = l + s$ = 3/2)
M_{IV}	$3d$ ($j = l - s$ = 3/2)
M_V	$3d$ ($j = l + s$ = 5/2)
N_I	$4s$ (j = 1/2)
N_{II}	$4p$ ($j = l - s$ = 1/2)
N_{III}	$4p$ ($j = l + s$ = 3/2)
N_{IV}	$4d$ ($j = l - s$ = 3/2)
N_V	$4d$ ($j = l + s$ = 5/2)
N_{VI}	$4f$ ($j = l - s$ = 5/2)
N_{VII}	$4f$ ($j = l + s$ = 7/2)

Examples of binding energies

Z	Élément	K	LI	LII	LIII	MI	MII	MIII	MIV	MV
1	H	0.014								
2	He	0.025	0.001							
3	Li	0.055	0.003	0.001	0.001					
4	Be	0.111	0.006	0.002	0.002					
5	B	0.188	0.009	0.004	0.004					
6	C	0.284	0.013	0.005	0.005					
7	N	0.4	0.018	0.007	0.007					
8	O	0.533	0.024	0.009	0.009					
9	F	0.687	0.032	0.012	0.012					
10	Ne	0.867	0.045	0.018	0.018	0.001				
11	Na	1.0721	0.063	0.032	0.032	0.002				
12	Mg	1.305	0.088	0.05	0.05	0.003				
13	Al	1.5596	0.118	0.073	0.073	0.005				
14	Si	1.8389	0.151	0.099	0.1	0.007	0.0011	0.001		
15	P	2.1455	0.188	0.1301	0.13	0.01	0.0021	0.002		
16	S	2.472	0.227	0.1651	0.165	0.014	0.0041	0.004		
17	Cl	2.8224	0.27	0.203	0.202	0.018	0.0071	0.007		
18	Ar	3.2029	0.32	0.247	0.245	0.025	0.0121	0.012		
19	K	3.6074	0.377	0.296	0.294	0.034	0.0181	0.018		
20	Ca	4.0381	0.438	0.35	0.346	0.044	0.0251	0.025		
21	Sc	4.4928	0.5	0.406	0.401	0.053	0.0321	0.032		
22	Ti	4.9664	0.563	0.462	0.456	0.06	0.0351	0.035		
23	V	5.4651	0.628	0.521	0.513	0.066	0.0381	0.038		
24	Cr	5.9892	0.696	0.584	0.575	0.074	0.0421	0.042	0.0011	0.001
25	Mn	6.539	0.769	0.651	0.64	0.084	0.0471	0.047	0.0021	0.002
26	Fe	7.112	0.846	0.721	0.708	0.093	0.0531	0.053	0.0031	0.003
27	Co	7.7089	0.926	0.794	0.779	0.101	0.0601	0.06	0.0041	0.004
28	Ni	8.3328	1.0081	0.871	0.845	0.111	0.0671	0.067	0.0051	0.005
29	Cu	8.9789	1.0961	0.953	0.933	0.122	0.0741	0.074	0.0071	0.007
30	Zn	9.6586	1.1936	1.0428	1.0197	0.138	0.0881	0.087	0.0101	0.01
31	Ga	10.367	1.2977	1.1423	1.1154	0.158	0.106	0.103	0.0171	0.017
32	Ge	11.103	1.4143	1.2478	1.2167	0.18	0.126	0.121	0.0281	0.028
33	As	11.867	1.5265	1.3586	1.3231	0.204	0.146	0.14	0.0411	0.041

Energies are in keV

Remark

- Photon **cannot** be absorbed by free electron (and then gives it all its energy)
- Conservation of the energy and of the momentum with $h\nu_0$ the energy of the photon and m, E et p, the mass, the total energy and the momentum of the electron→

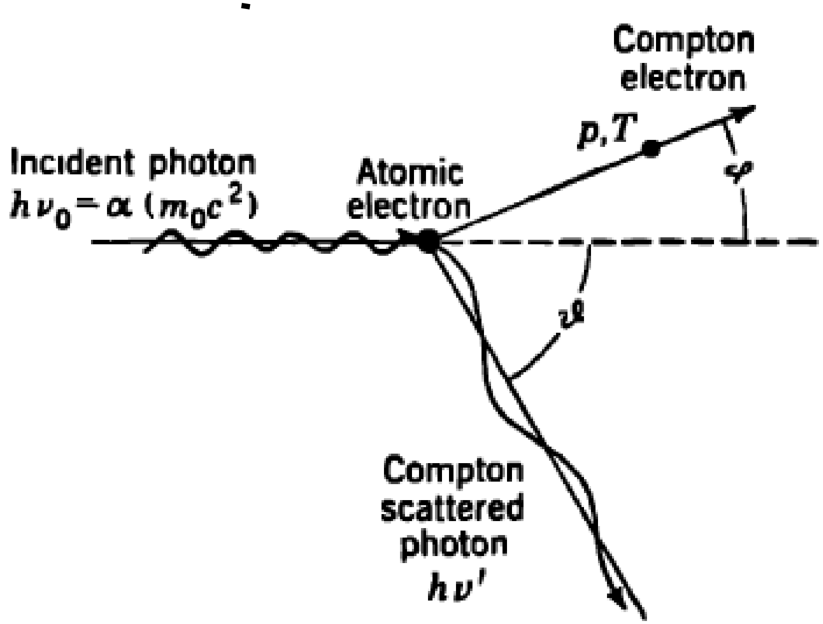
$$h\nu_0 + mc^2 = E$$

$$\frac{h\nu_0}{c} = p$$

- This implies $E = pc+mc^2$ and by definition → $E^2 = p^2c^2+m^2c^4$ → only possible for $p = h\nu_0 = 0$ → must be rejetcted

Compton effect

- Scattering by a free electron (in the range of energy where Compton effect occurs → electrons are considered as free → when this approximation is no more true → photoelectric effect is dominating)
- The photon gives a part of its energy to an electron



History of Compton effect: Thomson

- Thomson classically calculates (1906) the scattering cross section of an electromagnetic wave by a free electron
- Hypothese: Due to the force caused by the electric field of the wave → oscillation of the electron → this oscillation has same frequency as the wave and has the same direction as the electric field → electric dipole → the electron irradiate → diffusion of the incident wave <u>in a continuous way</u>
- For a non-polarized incident wave → Thomson calculates the diffusion cross section $d\sigma_0$ in solid angle $d\Omega$ (with θ the angle formed with the direction of the incident wave) →

$$\frac{d\sigma_0}{d\Omega} = \frac{r_e^2}{2}(1 + \cos^2\theta)$$

with $r_e = e^2/4\pi\epsilon_0 mc^2 = 2.8\ 10^{-15}$ m, the classique radius of the electron

Demonstration of Thomson (1)

- Let's consider an electromagnetic wave with frequency ν that interacts with a free electron (mass *m*, charge -*e*) → the electron undergoes a force *F* due to the incident electric field:

$$\overrightarrow{E} = E_0 \exp\left(i\overrightarrow{k}\,\overrightarrow{r} - i\nu t\right)\overrightarrow{1_E}$$

with $\overrightarrow{1_E}$ the direction of polarization

- The motion equation of the electron is →

$$m\frac{d^2\overrightarrow{r}}{dt^2} = -e\overrightarrow{E}$$

- In the dipolar approximation the emitted power per unit of solid angle is (differential equation of Larmor - see electromagnetism teaching) →

$$\frac{dP}{d\Omega} = \frac{e^2}{16\pi^2\epsilon_0 c^3}\langle a^2\rangle \sin^2\Theta$$

with Θ the angle between the polarization direction and the observer

Demonstration of Thomson (2)

- From equation of motion we directly obtain $\langle a^2 \rangle$, the mean square acceleration →

$$\langle a^2 \rangle = \frac{e^2}{2m^2} |E_0|^2$$

- The differential power becomes →

$$\frac{dP}{d\Omega} = r_e^2 \frac{\epsilon_0 c |E_0|^2}{2} \sin^2 \Theta$$

- We consider now the modulus of the Poynting vector that is for the energy flux by second (see you know what…) →

$$I = \epsilon_0 c \frac{|E_0|^2}{2}$$

Demonstration of Thomson (3)

- The differential cross section is obtained with →

$$\frac{d\sigma_0}{d\Omega} = \frac{dP/d\Omega}{I} = r_e^2 \sin^2 \Theta$$

- If we consider an non-polarized incident wave → we have to work out the average of Θ →

$$\overline{\sin^2 \Theta} = \frac{1}{2}(1 + \cos^2 \theta)$$

with θ, the scattering angle

- We thus obtain the equation of Thomson →

$$\frac{d\sigma_0}{d\Omega} = \frac{r_e^2}{2}(1 + \cos^2 \theta)$$

Total cross section of Thomson

- By integration over angles → total scattering cross section of Thomson →

$$\sigma_0 = \frac{8\pi}{3} r_e^2 = 0.665 \times 10^{-28}\ \mathrm{m}^2 \approx \frac{2}{3}\ \ \mathrm{barn/electron}$$

- In his demonstration → the incident wave is scattered in a continuous way

History of Compton effect: Compton

- Measurement of Compton (1922) → the scattered wave has not a continuum spectrum but follows the relation →

$$\lambda_1 - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$$

  with λ_0 and λ_1 the wavelengths of incident and scattered photons and $h/mc = \lambda_C$, the wavelength of Compton
- This expression shows that the wavelength displacement does not depend on Z and on the incident wavelength and that the energy and momentum lost by the photon goes to only one electron

Demonstration of the expression of Compton (1)

- In the laboratory frame → conservation law of the four-vector energy-momentum before and after the scattering →

$$\text{photon before} \rightarrow \left(\frac{h\nu_0}{c}, \frac{h\nu_0}{c}\overrightarrow{n_0}\right)$$

$$\text{photon after} \rightarrow \left(\frac{h\nu_1}{c}, \frac{h\nu_1}{c}\overrightarrow{n_1}\right)$$

$$\text{electron before} \rightarrow \left(\frac{mc^2}{c}, 0\right)$$

$$\text{electron after} \rightarrow \left(\frac{E}{c}, \overrightarrow{p}\right)$$

Demonstration of the expression of Compton (2)

- Conservation laws →

$$h\nu_0 + mc^2 = h\nu_1 + E$$

$$h\nu_0 \overrightarrow{n_0} = h\nu_1 \overrightarrow{n_1} + \overrightarrow{p}c$$

- With $E^2 = p^2c^2 + m^2c^4$ →

$$(h(\nu_0 - \nu_1) + mc^2)^2 = h^2(\nu_0 \overrightarrow{n_0} - \nu_1 \overrightarrow{n_1})^2 + m^2c^4$$

- And with: $\overrightarrow{n_0}\overrightarrow{n_1} = \cos\theta$

$$h\nu_0\nu_1(1 - \cos\theta) = mc^2(\nu_0 - \nu_1) \Rightarrow \frac{hc^2}{\lambda_0\lambda_1}(1 - \cos\theta) = mc^3\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_1}\right)$$

$$\lambda_1 - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$$

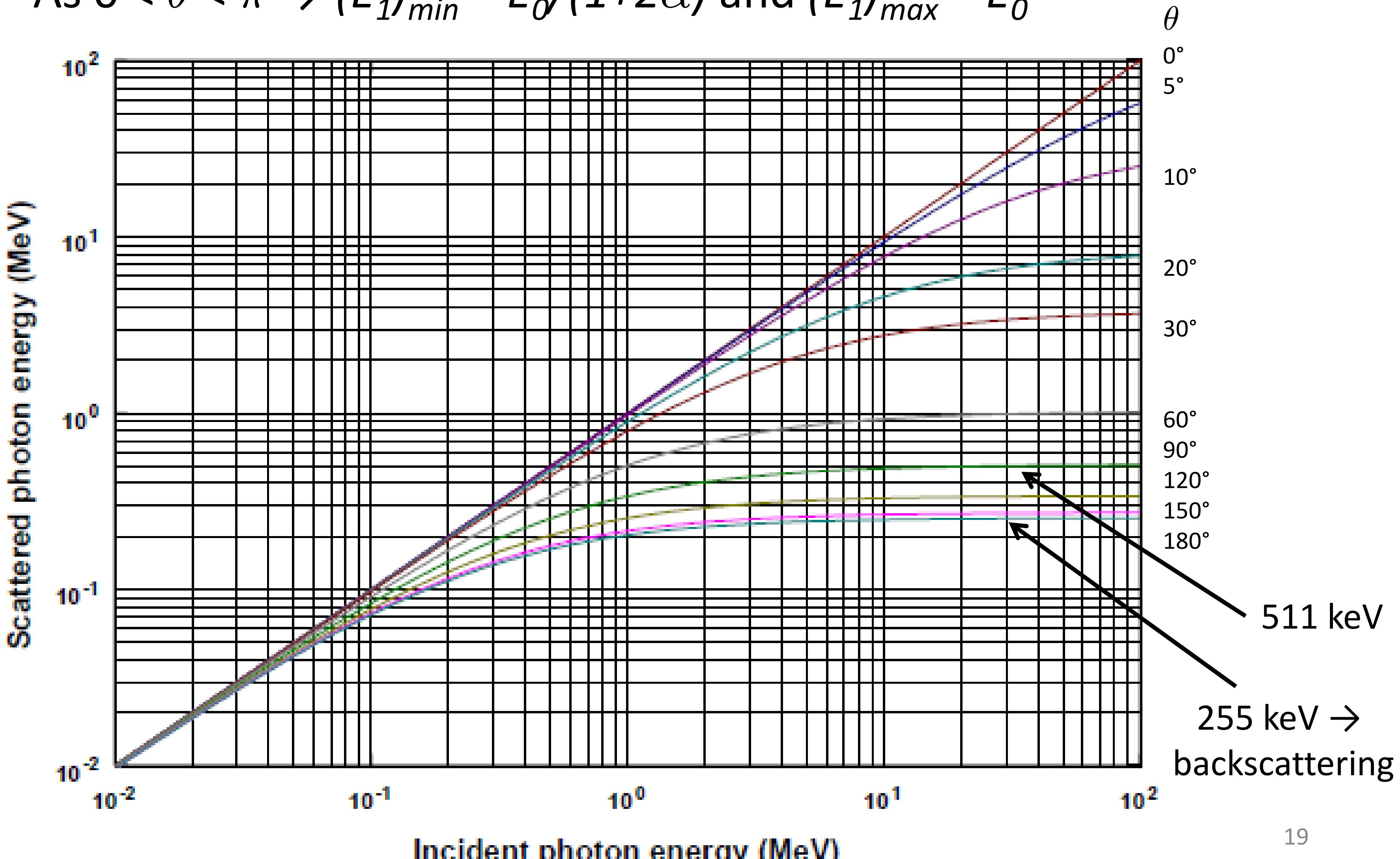
Relations between energies and angles (1)

- We consider E_0 = the incident photon energy, E_1 = the scattered photon energy, $T = E_0 - E_1$ = the kinetic energy given to the electron, θ = the photon diffusion angle, ϕ = the angle between the trajectory of the electron and the photon initial direction and $\alpha = E_0/mc^2$ = the ratio between incident photon energy et the electron mass energy →

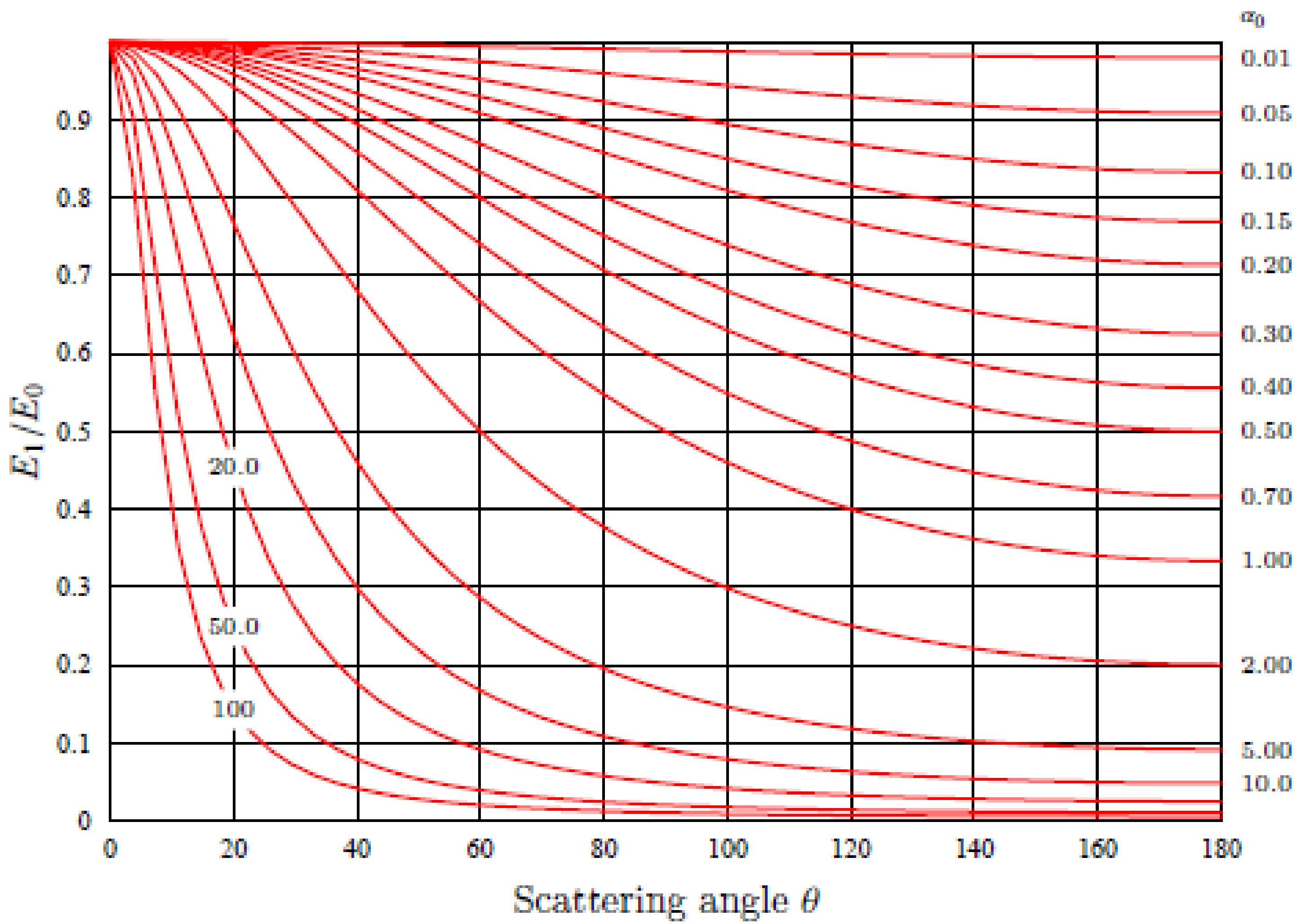
$$
\begin{aligned}
E_1 &= E_0 \frac{1}{1+\alpha(1-\cos\theta)} \\
T &= E_0 \frac{2\alpha\cos^2\phi}{(1+\alpha)^2-\alpha^2\cos^2\phi} = E_0 \frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)} \\
\cot\phi &= (1+\alpha)\tan\frac{\theta}{2}
\end{aligned}
$$

Relations between energies and angles (2)

- As $0 < \theta < \pi \rightarrow (E_1)_{min} = E_0/(1+2\alpha)$ and $(E_1)_{max} = E_0$



Relations between energies and angles (3)

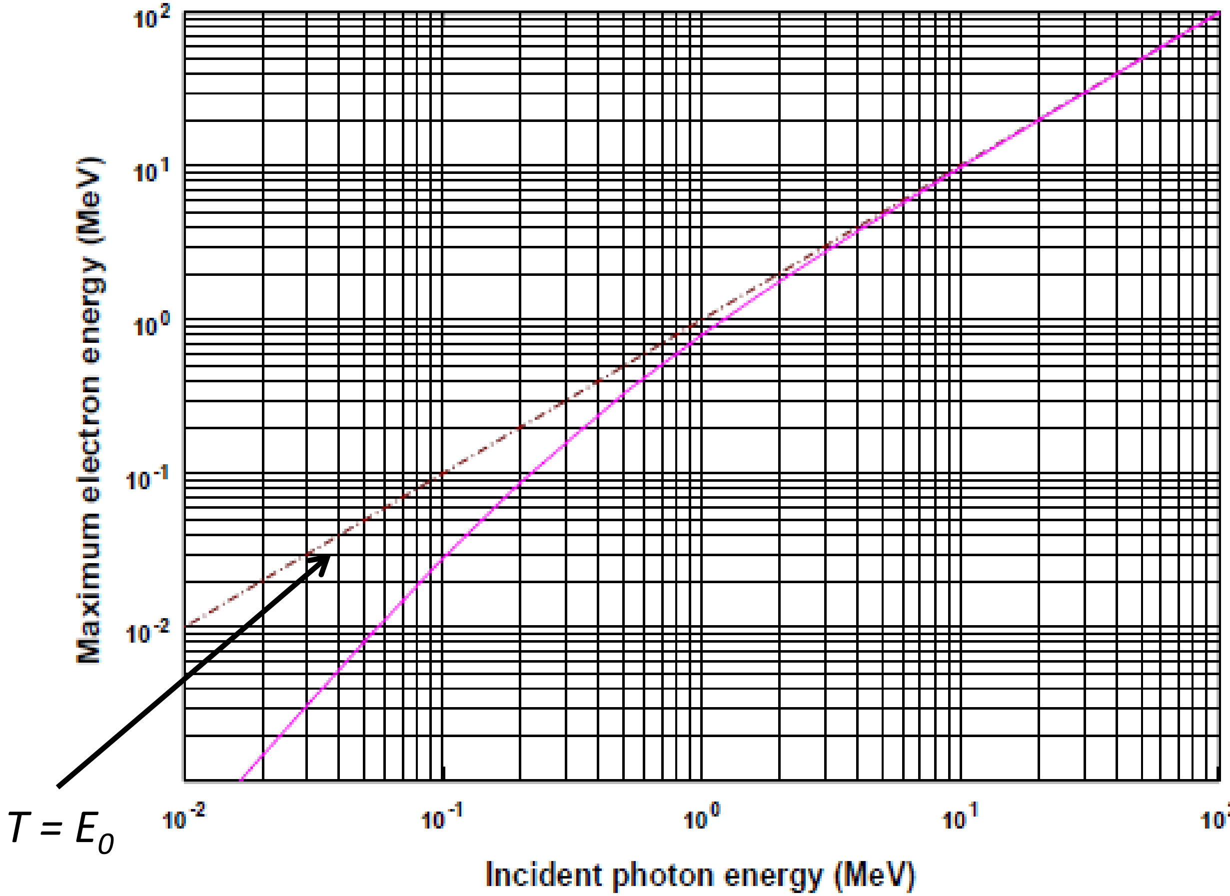


Remarks on the energy of the scattered photon

- The energy modification for the photon depends on the incident photon energy
- For small E_0 → small energy loss for the photon (for any θ)
- For E_0 ↗ → the variation of the energy of the scattered photon as a function of the angle becomes important
- For 90° → E_1 always < 511 keV ($= mc^2$)
- For 180° → E_1 always < 255 keV ($= mc^2/2$) → backscattered photon → backscattering peak in γ spectra

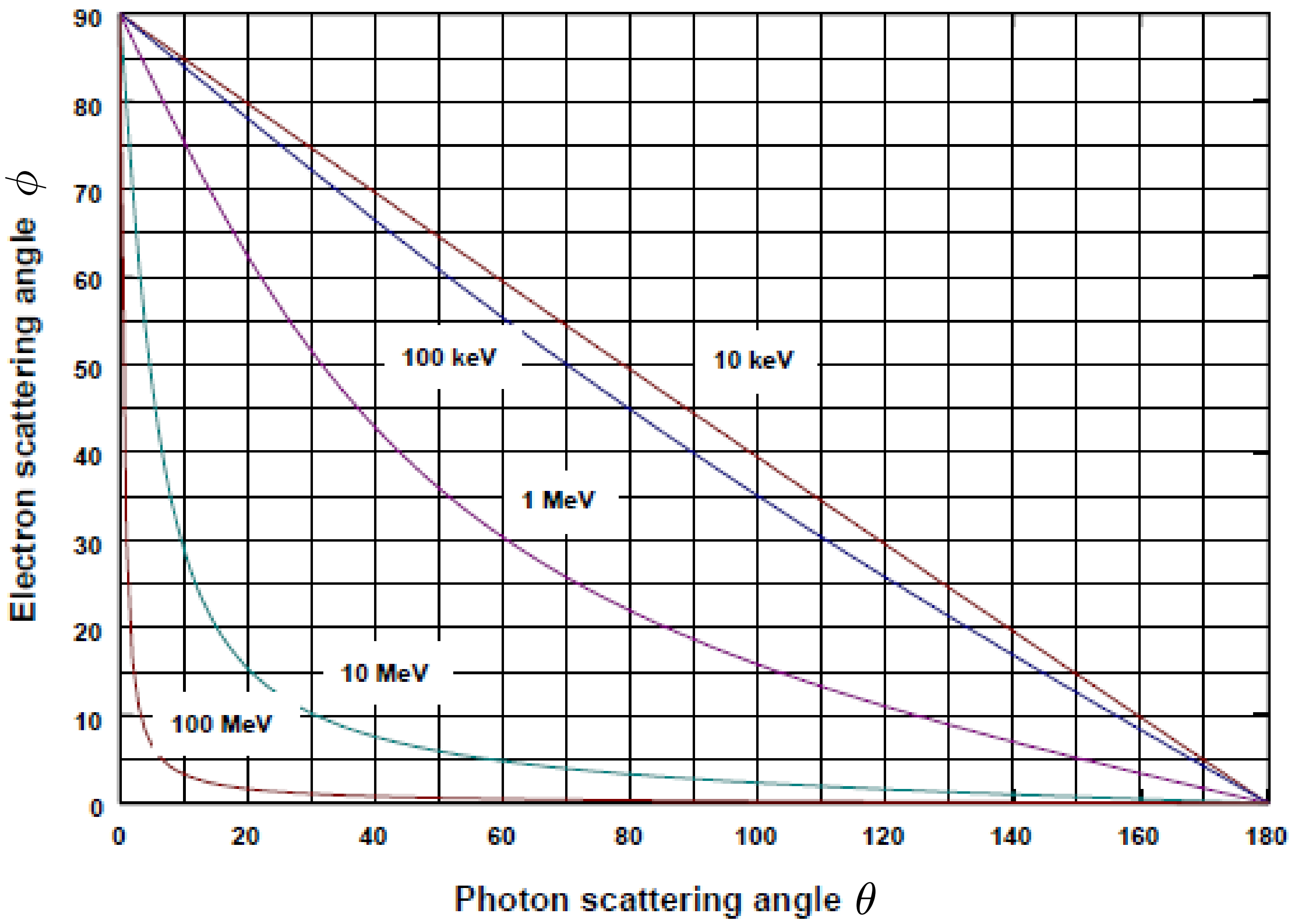
Relations between energies and angles (4)

- We have $0 < \phi < \pi/2 \rightarrow T_{min} = 0$ and $T_{max} = E_0/[1+(1/2\alpha)]$



- For large $E_0 \rightarrow T_{max} \approx E_0$ - 255 keV (backscattered photon)

Relations between energies and angles (5)



Angular-differential cross section for Compton effect (1)

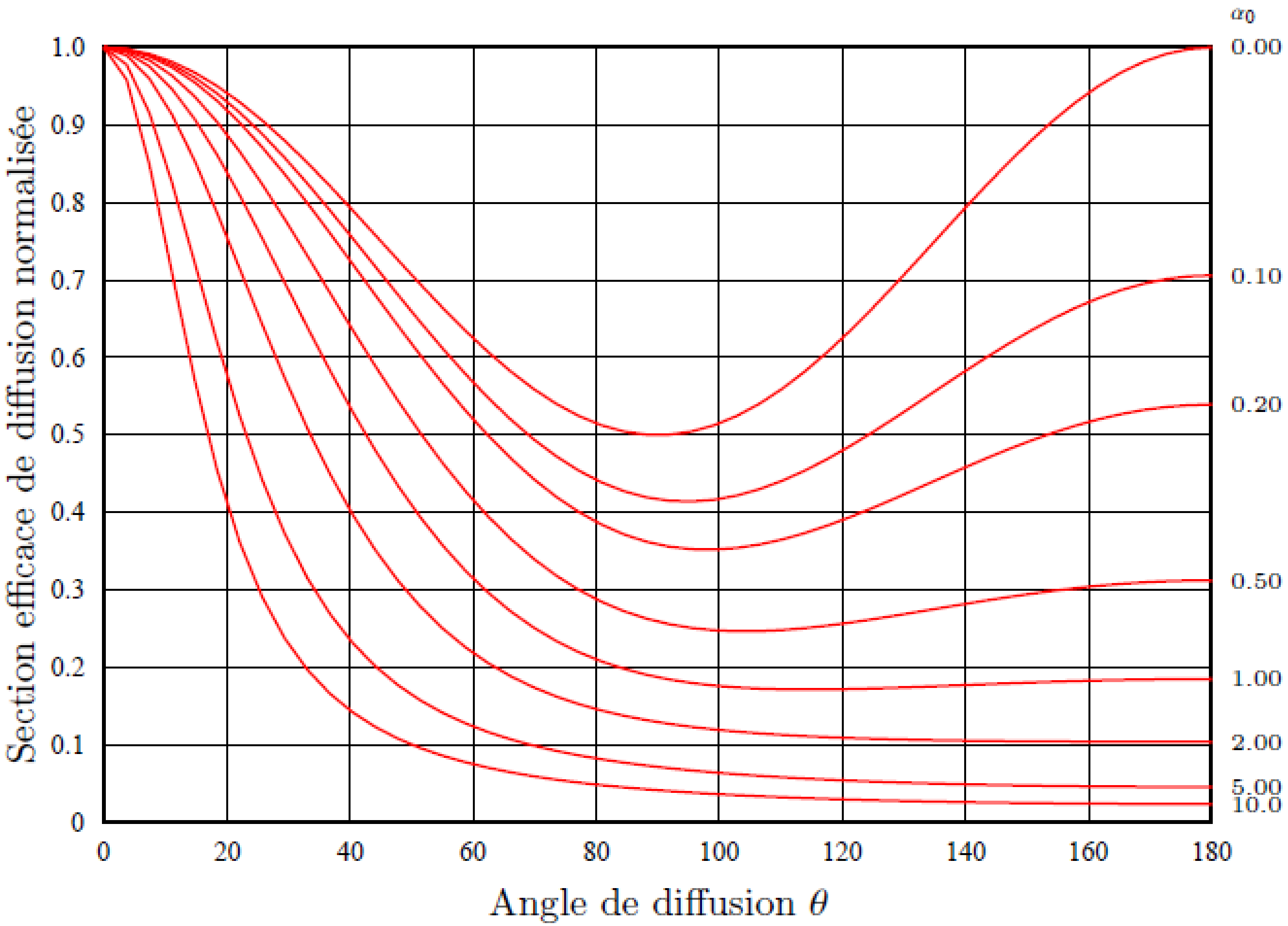
- Equation of Klein-Nishina (quantum electrodynamics): valid for free electrons at rest
- The angular-differential scattering cross section of a non-polarized photon in the solid angle $d\Omega$ around the direction making an angle θ with the initial direction of the photon is given by →

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}\left(\frac{\nu'}{\nu_0}\right)^2\left(\frac{\nu_0}{\nu'} + \frac{\nu'}{\nu_0} - \sin^2\theta\right)$$

with r_e, the classical radius of the electron

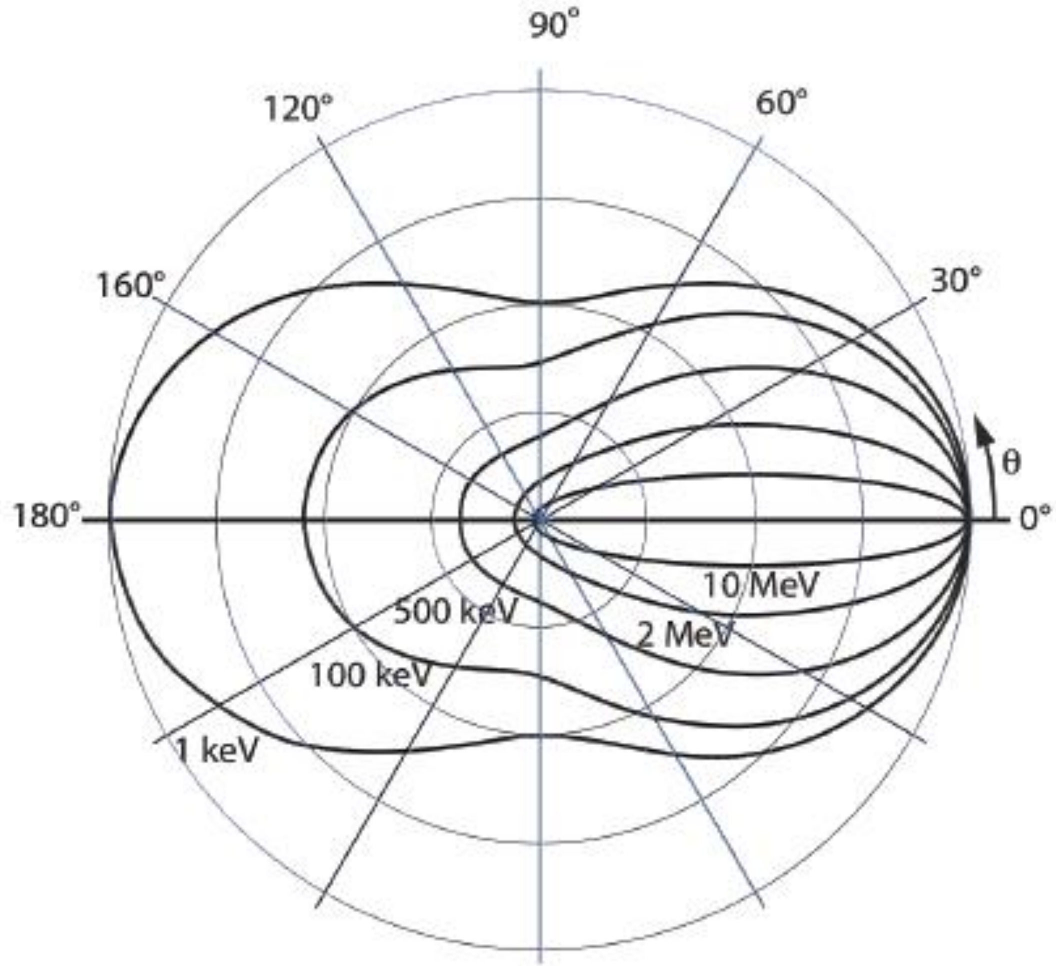
- Remark → no dependence on Z

Angular-differential cross section (2)



For $\alpha \ll$ → $E_1 \approx E_0$ → $d\sigma \approx d\sigma_0$ → we obtain the Thomson cross section

Angular-differential cross section (3)



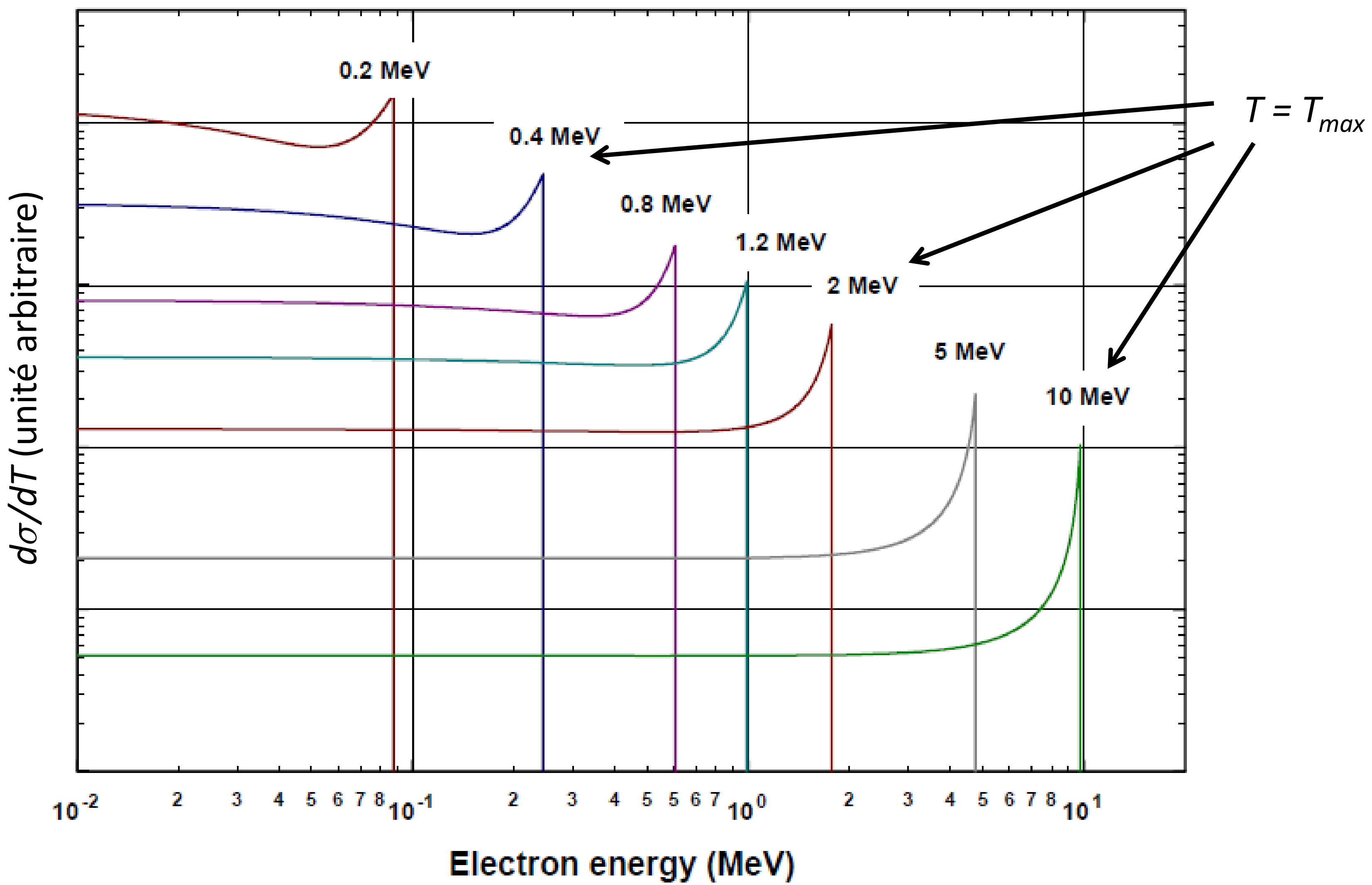
Energy-differential cross section (1)

- From the angular-differential cross section → Energy-differential cross sections →

$$\frac{d\sigma}{dE_1} = \frac{\pi r_e^2}{\alpha^2 m_e c^2} \left\{ 2 + \left(\frac{E_0 - E_1}{E_1} \right)^2 \left[\frac{1}{\alpha^2} + \frac{E_1}{E_0} - \frac{2}{\alpha} \left(\frac{E_1}{E_0 - E_1} \right) \right] \right\}$$

$$\frac{d\sigma}{dT} = \frac{\pi r_e^2}{\alpha^2 m_e c^2} \left\{ 2 + \left(\frac{T}{E_0 - T} \right)^2 \left[\frac{1}{\alpha^2} + \frac{E_0 - T}{E_0} - \frac{2}{\alpha} \left(\frac{E_0 - T}{T} \right) \right] \right\}$$

Energy-differential cross section (2)

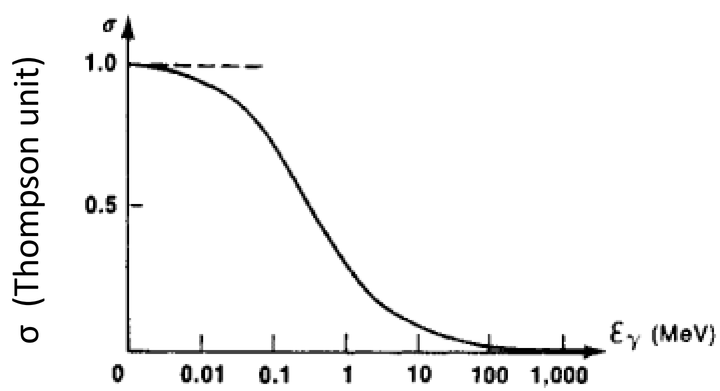


Total cross section for the Compton effect (1)

- After integration of the Klein-Nishina cross section over angles →

$$\sigma = 2\pi r_e^2 \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}$$

- For $\alpha \ll$ → $\sigma \approx \sigma_0 = 8\pi r_e^2/3$ (Thomson cross section)



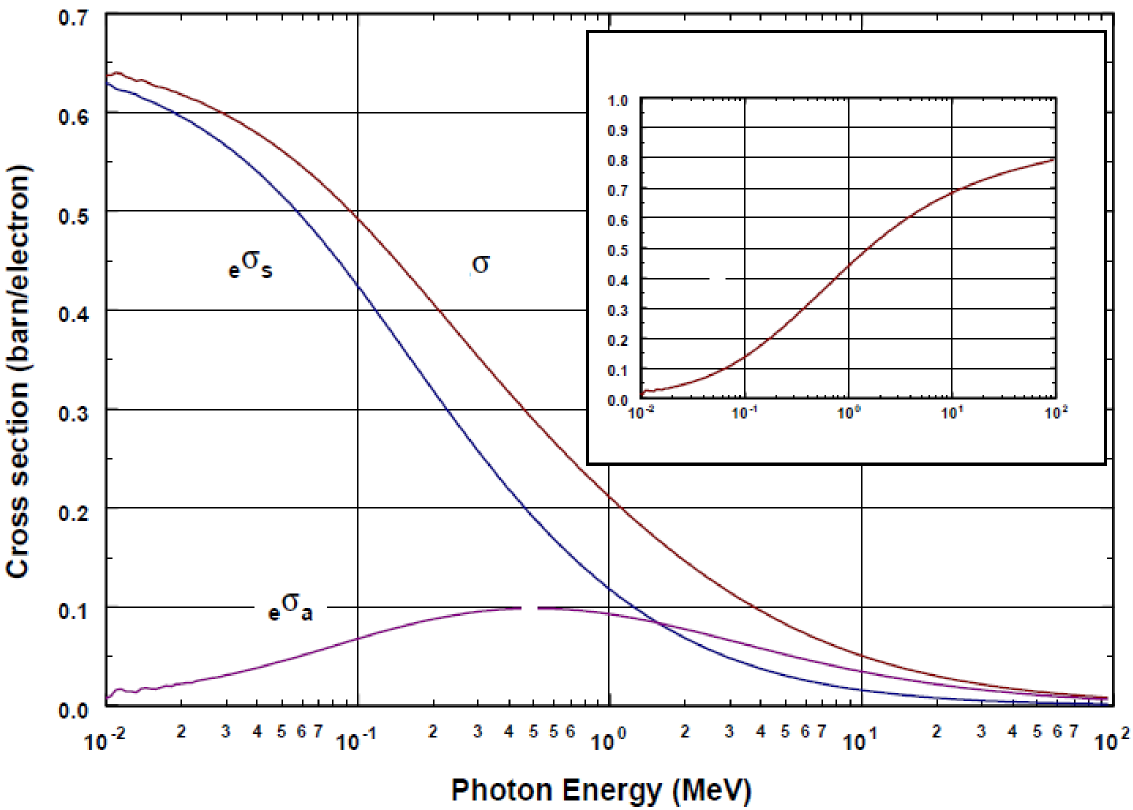
Total cross section for the Compton effect (2)

- For $\alpha \gg$ → σ → $(ln\ \alpha)/\alpha$ → Compton cross section ↘ when photon energy ↗
- Indeed we observe for $\alpha \gg$ → angular distribution predominant in θ = 0 → no diffusion → no energy transfer → no effect

- The <u>atomic</u> cross section $_{a}\sigma = Z\sigma$ is thus $\propto Z$

Collision = Scattering + Absorption

- The cross section σ represents the scattering probability → a part of the energy is scattered and the other one is given to the e^- (absorbed)
- To characterize this aspect → we define a scattered cross section σ_s and an absorption cross section σ_a with $\sigma = \sigma_s + \sigma_a$



$$\sigma_s = \frac{\langle E_1 \rangle}{E_0}\sigma$$

$$\sigma_a = \frac{\langle T \rangle}{E_0}\sigma$$

Coherent and incoherent scattering (1)

- For $E_0 \ll$ → we cannot consider that e^- are free and at rest → scattering by the whole electronic system (atom)
- If the atom stays in its initial state → the energy of the photon does not change but it changes its direction (the atom takes the momentum difference) → Rayleigh scattering (coherent scattering)
- If the atom changes its state → the photon loss energy → incoherent scattering
- For large energies → incoherent scattering = Compton scattering

Coherent and incoherent scattering (2)

- Approximation for coherent scattering → the electronic system corresponds to a system with charge *Ze* and mass *Zm* → the Rayleigh scattering cross section is →

$$_a\sigma_{coh} = Z^2\sigma_0$$

- In reality → the structure of the electronic cloud implies a decrease of the scattering → introduction of the factor of atomic structure *F* →

$$_a\sigma_{coh} = F^2\sigma_0$$

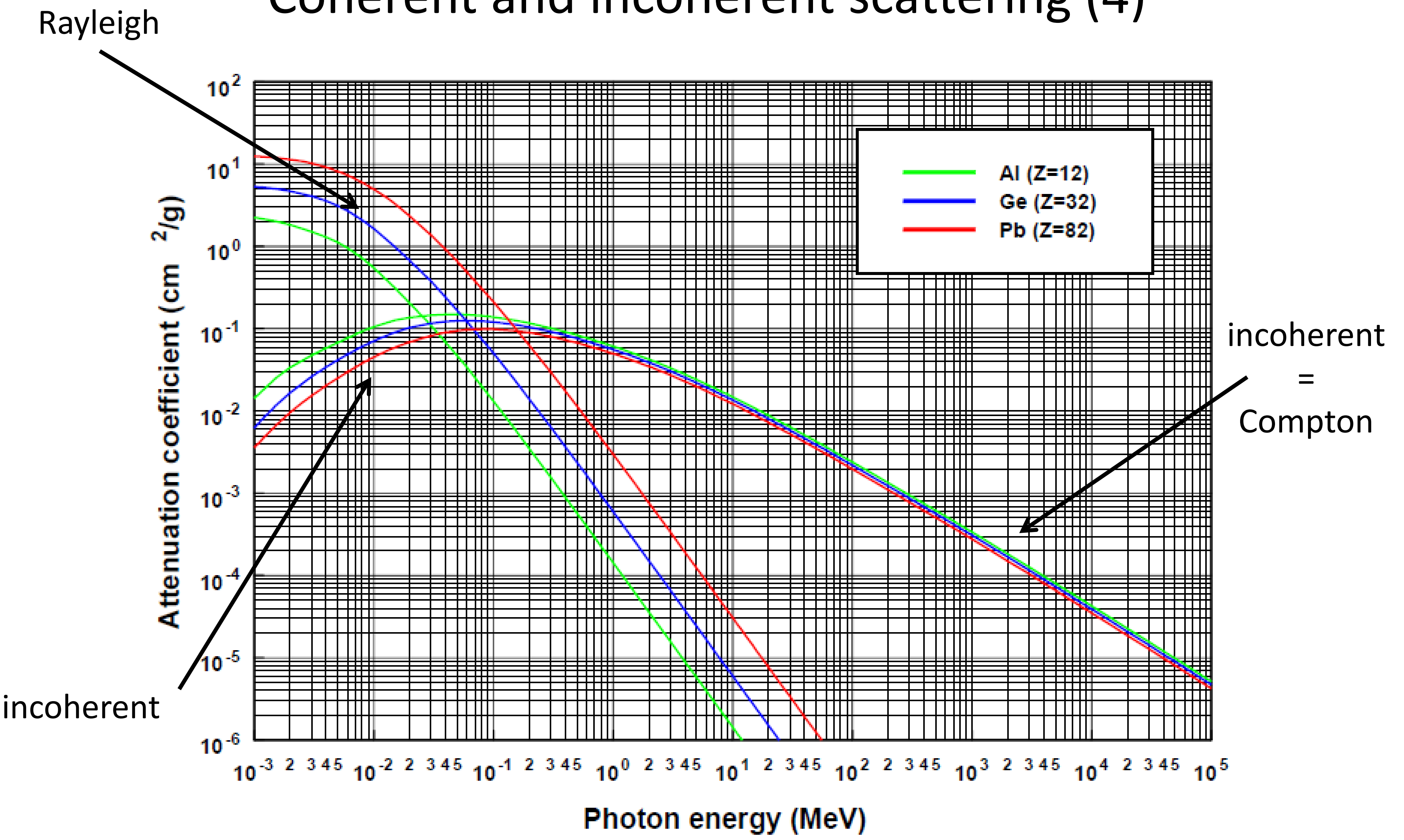
Coherent and incoherent scattering (3)

- For incoherent scattering → modification of the Klein-Nishina cross section by the function of incoherent scattering *S* → *S* considers the fact that the electrons of the atom are bonded → the photon can be incapable of ejecting an electron from the atom →

$$_{a}\sigma_{incoh} = Z\sigma S$$

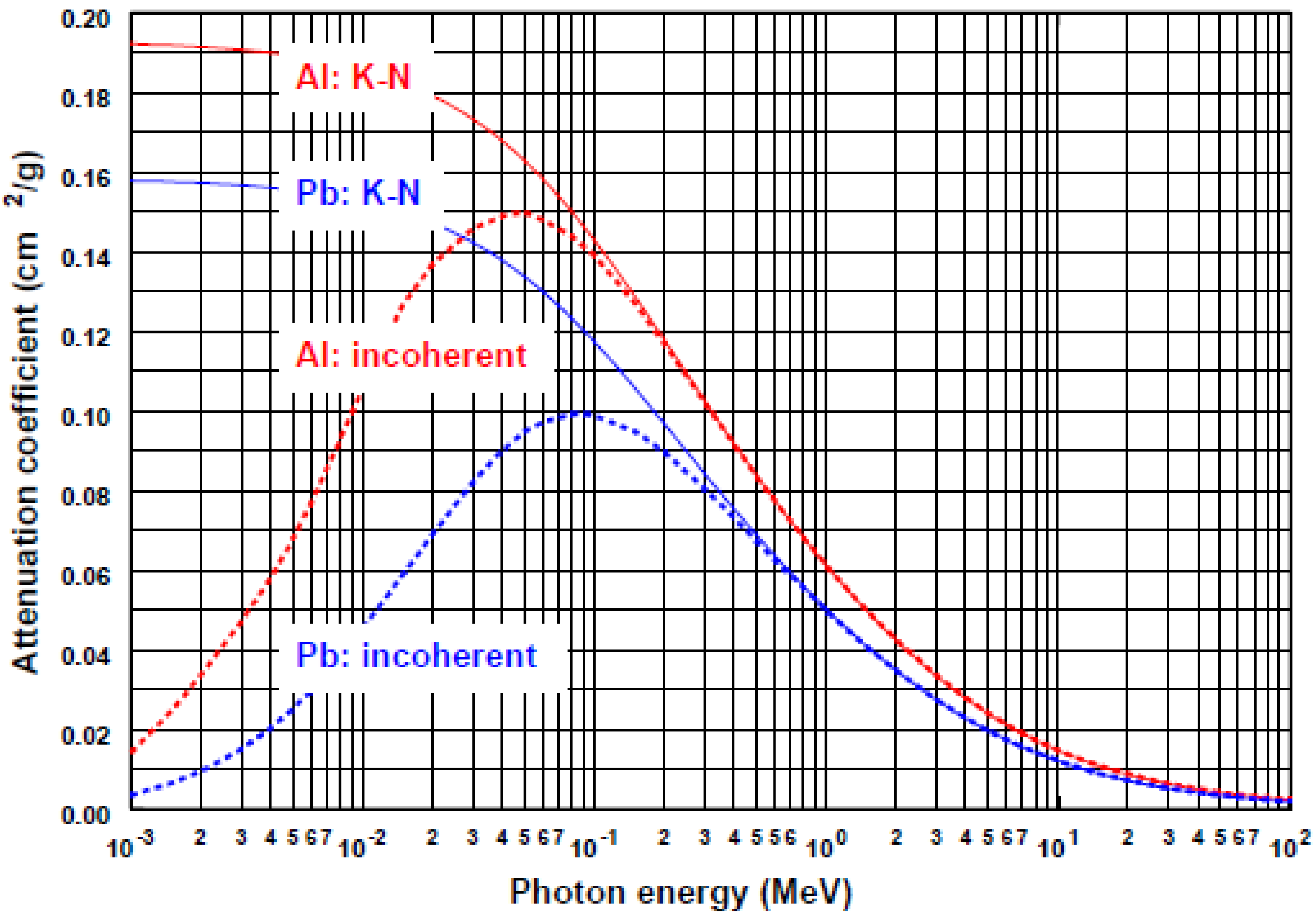
- For large energies → *S* → 1 → Compton cross section

Coherent and incoherent scattering (4)



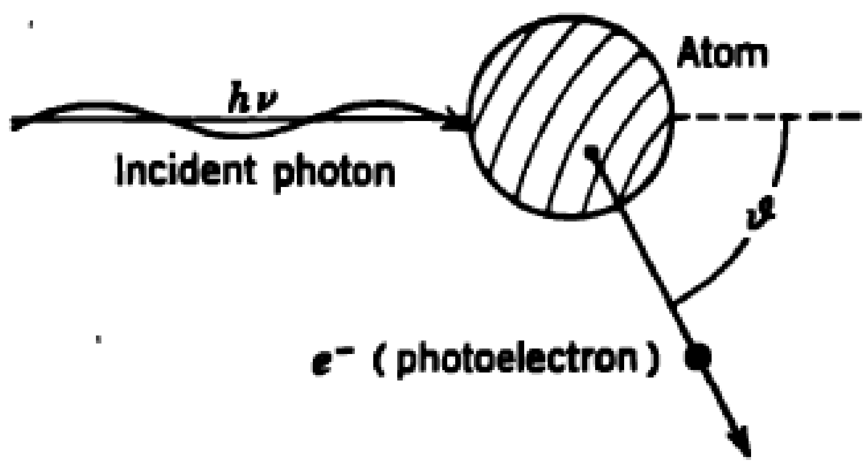
Remark → coherent scattering ↗ for Z ↗

Comparison incoherent - Compton



Photoelectric effect

- The photoelectric effect is a process in which an incident photon interacts with an atom and an electron is emitted (process correctly explained by Einstein in 1905)
- This process of photon capture by an atom with an electron excited in a continuous state is the inverse process of spontaneous emission of a photon by an excited atom



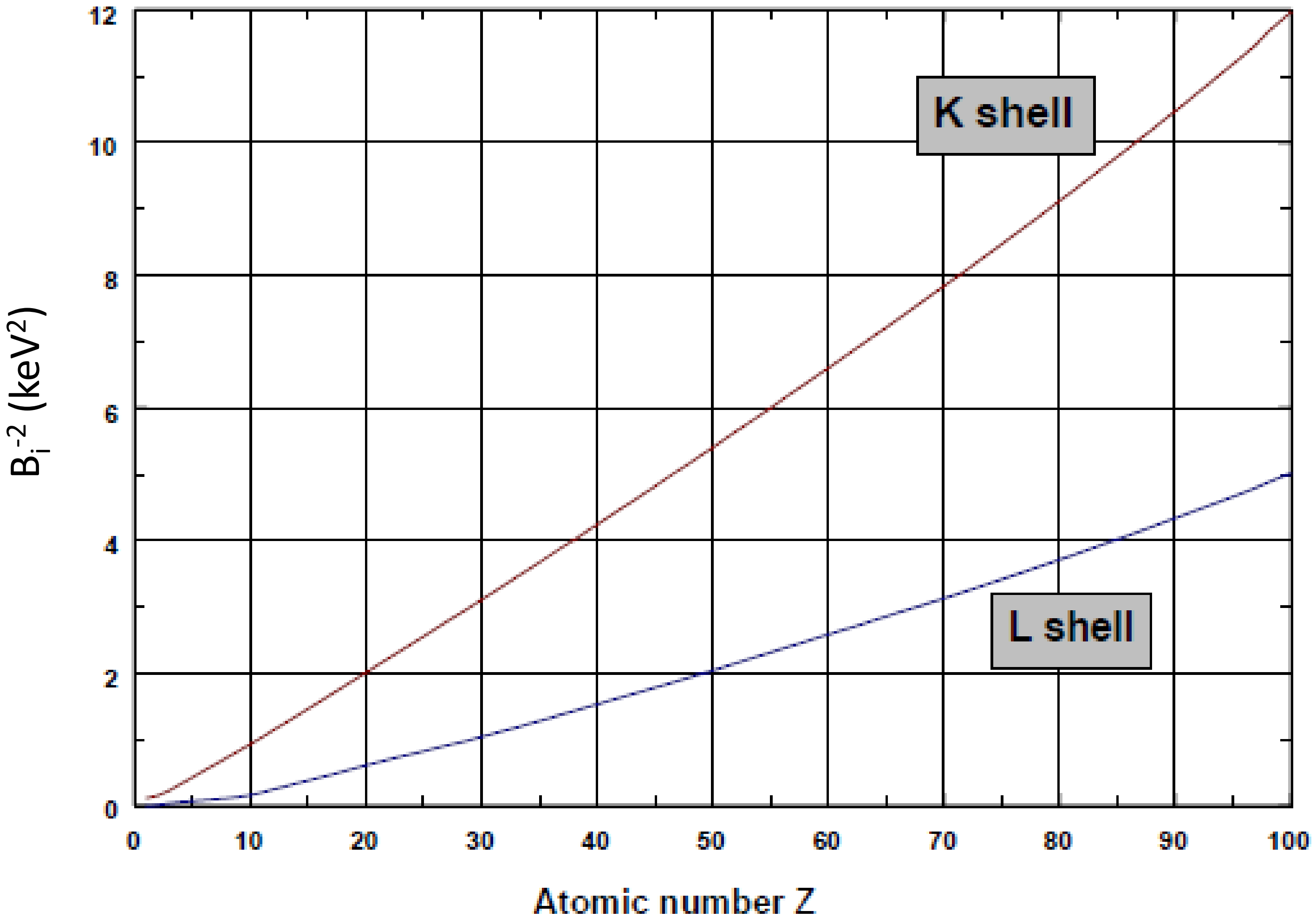
Energy conservation

- There are absorption of a photon with energy $h\nu_0$ that is completely absorbed by an atom and consequent emission of an electron (called photoelectron) with a kinetic energy T out of an atomic shell characterized by a binding energy B_i (i = K, L_I, L_{II}, L_{III}, …) → by neglecting the recoil energy of the nucleus (due to ≠ of mass) →

$$h\nu_0 = T + B_i$$

- The energy conservation implies $h\nu_0 > B_i$
- When $h\nu_0$ ↗ the probability of photoelectric effect ↘ since the behaviour of the electron more and more approaches the one of a free electron (and the absorption by a free e^- is impossible)
- The more bound electrons (shell K) have the largest probability to absorb the photon (with always the condition $h\nu_0 > B_K$)

Binding energy



For Z > 30 → binding energies approximatively follow $B_i = a_i(Z-c_i)^2$ (with a_i and c_i constant for each shell)

Cross section (1)

- The cross section per atom ${}_a\tau$ can be decomposed into a sum of partial cross sections (${}_a\tau_i$) corresponding to the emission of an electron from a given shell $i \rightarrow$

$$ {}_a\tau = \sum_i {}_a\tau_i $$

- The calculation of ${}_a\tau_K$ has been done for a hydrogen-like atom in the Born approximation using a plane wave as wave function for the emitted electron
- We suppose $h\nu_0 \ll mc^2$ (non-relativistic approximation) and $h\nu_0 \gg B_K$ (interaction between the nucleus and the electron neglected)

Cross section (2)

- We find (with α, the fine structure constant and σ_0, the Thomson cross section) →

$$
\begin{aligned}
{}_a\tau_K &= \frac{8\pi r_e^2}{3} Z^5 \alpha^4 2^{5/2} \left(\frac{mc^2}{h\nu_0}\right)^{7/2} \\
&= \sigma_0 Z^5 \alpha^4 2^{5/2} \left(\frac{mc^2}{h\nu_0}\right)^{7/2}
\end{aligned}
$$

- As $r_e / a_0 = \alpha^2$ (with a_0, the Bohr radius) →

$$
{}_a\tau_K = \frac{8\pi}{3} \left(\frac{a_0}{Z}\right)^2 Z^7 \alpha^8 2^{5/2} \left(\frac{mc^2}{h\nu_0}\right)^{7/2}
$$

- And as, for a hydrogen-like atom, $B_K = \alpha^2 Z^2 mec^2/2$ →

$$
{}_a\tau_K = \frac{8\pi}{3} \left(\frac{a_0}{Z}\right)^2 32\alpha \left(\frac{B_K}{h\nu_0}\right)^{7/2}
$$

Cross section (3)

- Factor a_0/Z → approximatively = to the atom size
- Variation with $h\nu_0^{-7/2}$ for the energy
- When $h\nu_0 \approx B_K$ → Born approximation no more valid → introduction of a correction term $f(\xi)$ →

$$f(\xi) = 2\pi \left(\frac{B_K}{h\nu_0}\right)^{1/2} \frac{e^{-4\xi \operatorname{arccot} \xi}}{1 - e^{-2\pi\xi}} \quad \text{with} \quad \xi = \left(\frac{B_K}{h\nu_0 - B_K}\right)^{1/2}$$

- For $\xi \to 0$ (i.e. $h\nu_0 \gg B_K$) → $f(\xi)$ → 1 → previous situation
- For $\xi \to \infty$ (i.e. $h\nu_0 \approx B_K$) → $f(\xi)$ →

$$f(\xi) \approx 2\pi e^4 \left(\frac{B_K}{h\nu_0}\right)^{-5/6} \quad \text{and} \quad {}_a\tau_K \approx \frac{6.28 \times 10^6}{Z^2} \left(\frac{B_K}{h\nu_0}\right)^{8/3}$$

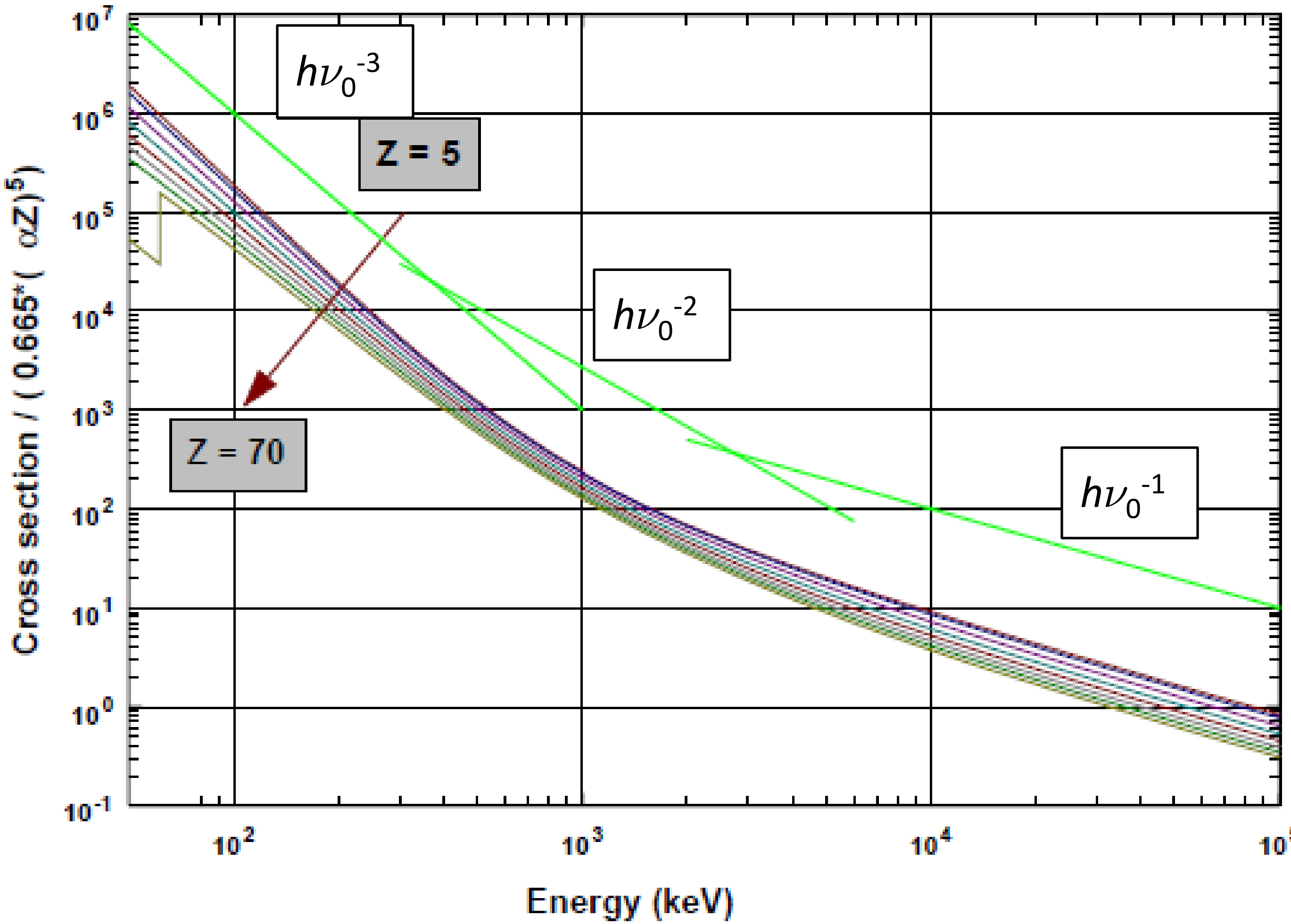
Cross section (4)

- The other partial cross sections have the same behaviour as ${}_a\tau_K$ → we generally write the total cross section as→

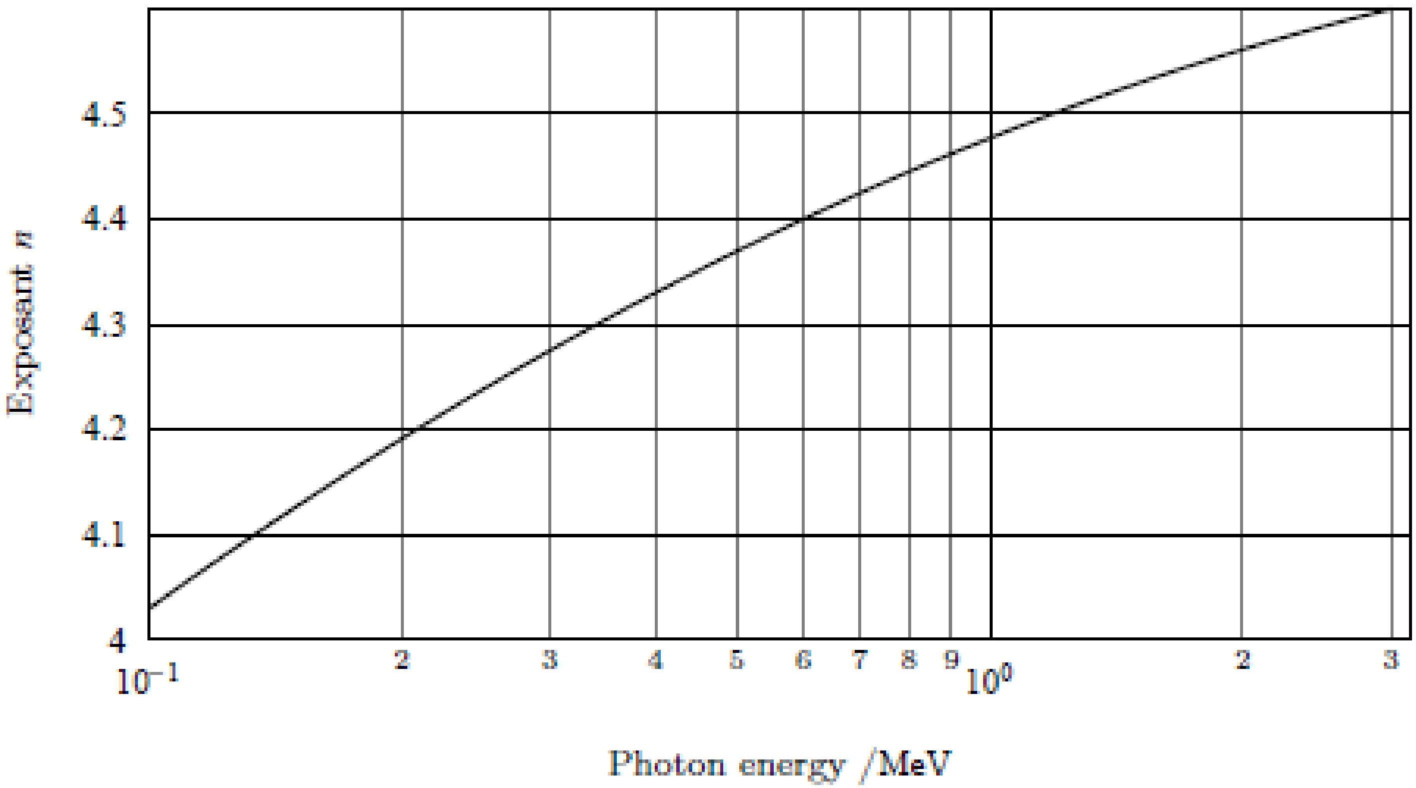
$$ {}_a\tau = C\frac{Z^n}{(h\nu_0)^k} $$

  with *n* varying between 4 and 4.6 and *k* varying between 1 and 3 (*C* is a constant)
- For $h\nu_0 \lessapprox$ 0.1 MeV (most important energy range for the photoelectric effect) → n ≈ 4 and k ≈ 3
- For $h\nu_0 \gtrapprox$ 1 MeV → n ≈ 4.5 and k ≈ 1

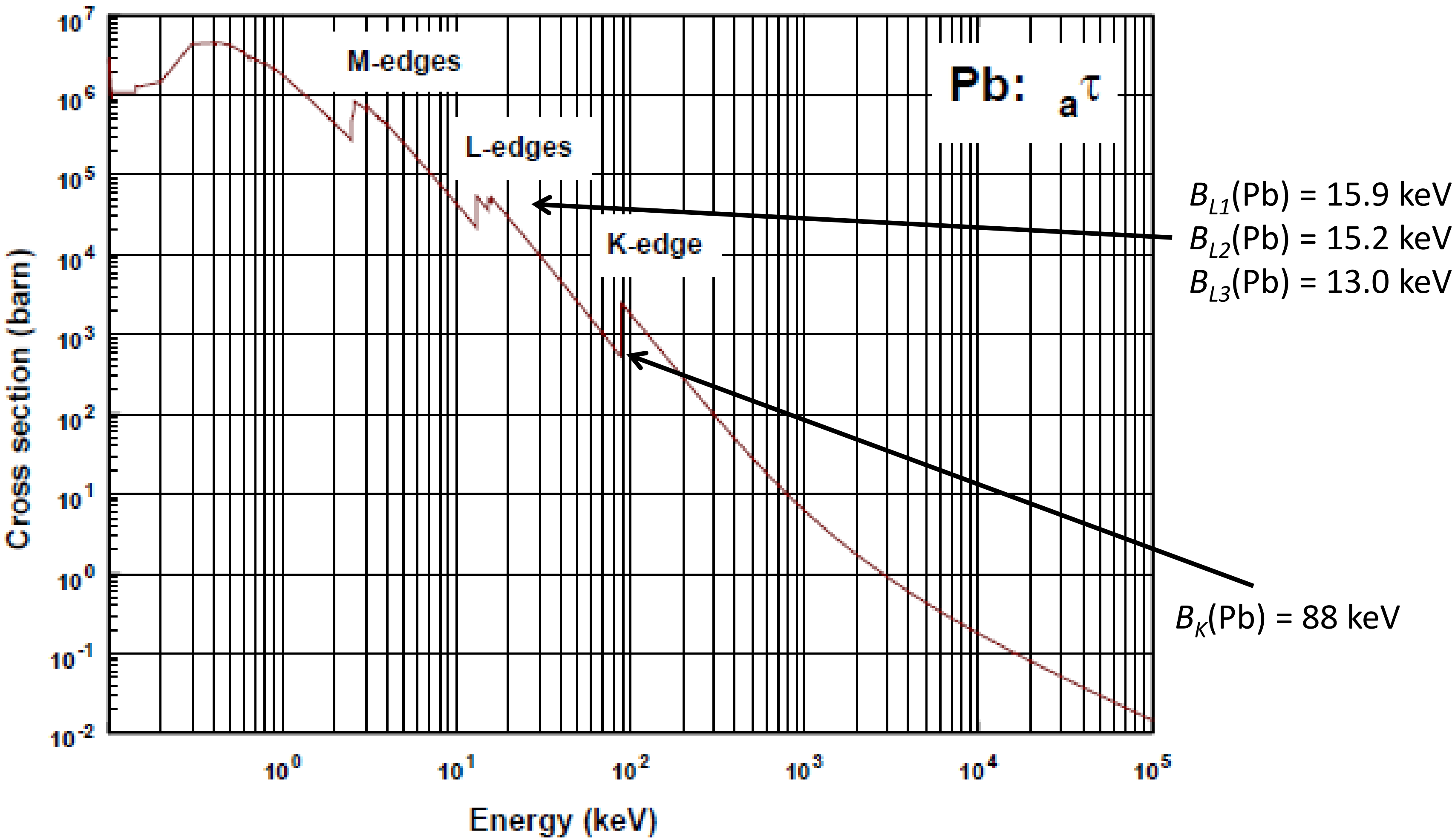
Variation of ${}_a\tau$ with $h\nu_0$



Variation of $_a\tau$ with Z

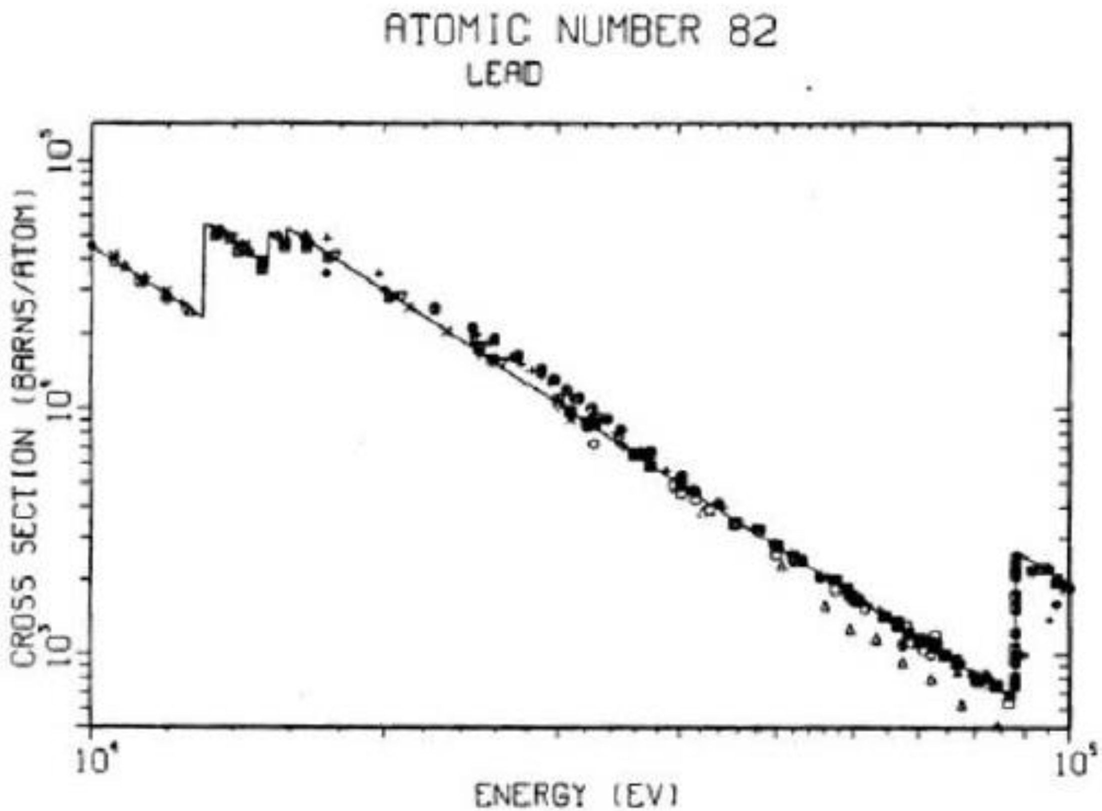


Cross section: Example (1)



For E > 88 keV → the 2 e^- of the K shell contribute for 3/4 of the cross section (by comparison to the 80 other e^-) ↔ large importance of B_i in $_a\tau$

Cross section: Example (2)



Comparison theory ↔ experiment

Angular distribution of the photoelectrons (1)

- In non-relativistic case → the differential cross section $d_a\tau/d\Omega$ is $\propto$ f(θ) →

$$f(\theta) = \frac{\sin^2\theta}{(1-\beta\cos\theta)^4}$$

  with $\beta = v/c$, the relative velocity of the photoelectron and θ, the emission angle of the photoelectron relatively to the initial direction of the photon
- Cross section = 0 in the direction of the incident photon (θ = 0) → the electron aims to be emitted in the direction of the electric field of the electromagnetic wave
- For E photons ↗ → more and more electrons are ejected in the forward direction

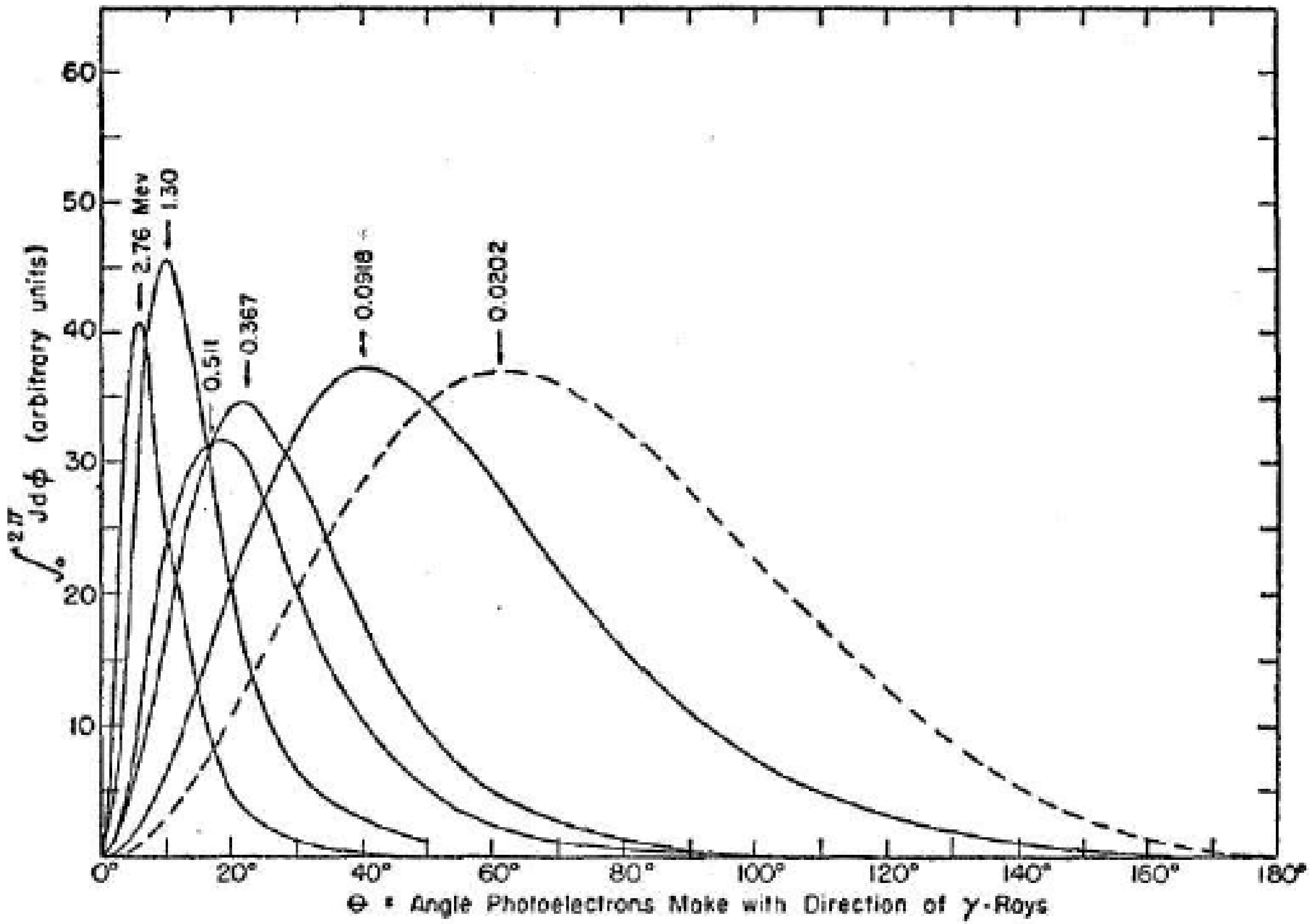
Angular distribution of the photoelectrons (2)

- In the relativistic case →

$$f(\theta) = \frac{\sin^2\theta}{(1-\beta\cos\theta)^4} + \frac{3(1-\sqrt{1-\beta^2})-2\beta^2}{2(1-\beta^2)^{3/2}} \times \frac{\sin^2\theta}{(1-\beta\cos\theta)^3}$$

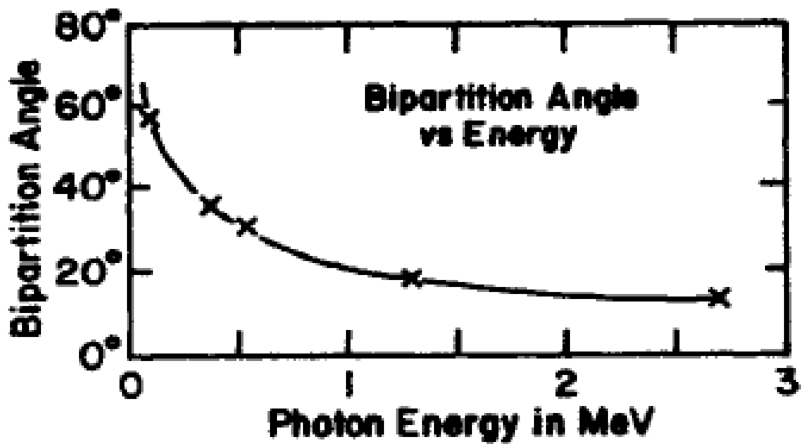
- In both cases → For photons energy ↗ → more and more electrons are ejected in the forward direction

Angular distribution of the photoelectrons (3)



Bipartition angle

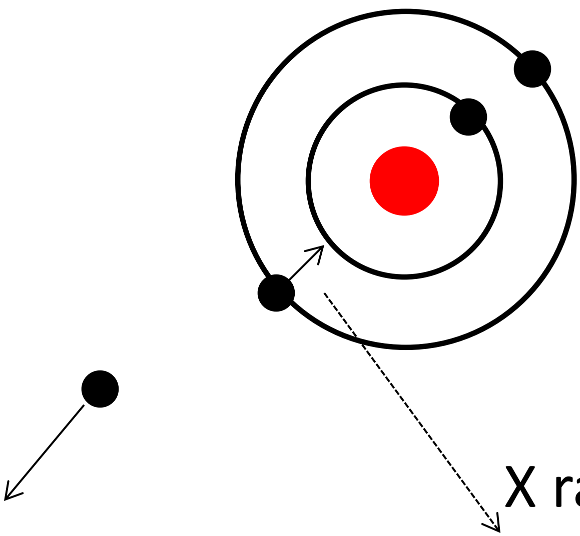
- Bipartition angle θ_b: angle for which 1/2 of the photoelectrons are emitted in the forward direction inside a cone with half-angle smaller than θ_b

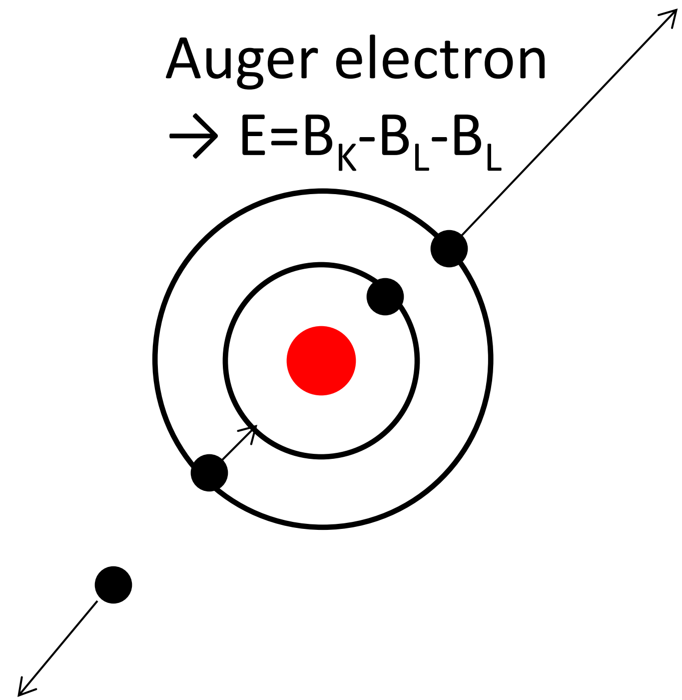


- For instance → for $h\nu_0$ = 0.5 MeV → 1/2 of the photoelectrons are emitted inside a cone with half-angle $\simeq$ 30°

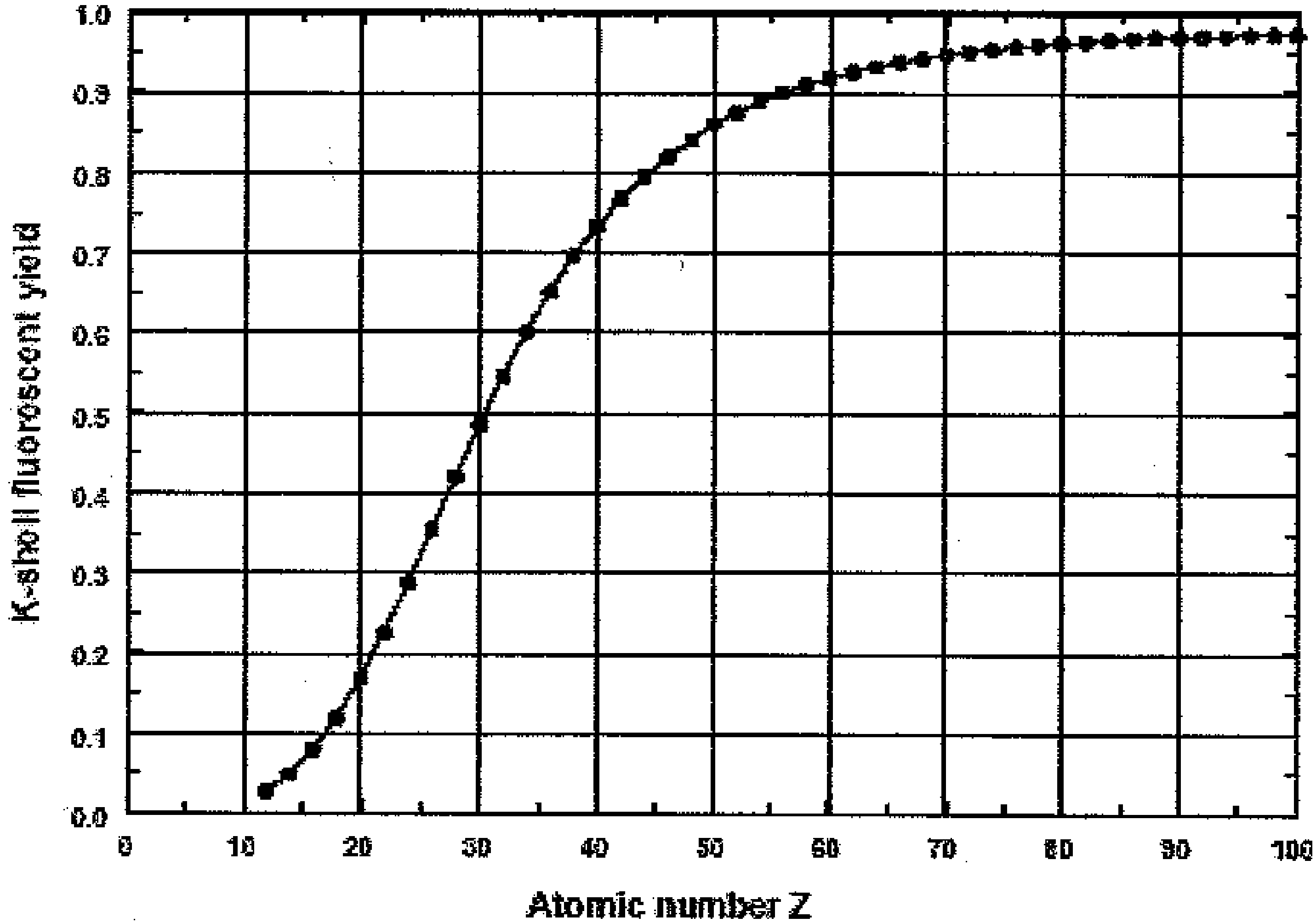
Consecutive phenomena

After a photoelectric effect → hole in an inner-shell → electronic rearrangement → emission of a X-ray (fluorescence) or an Auger electron → definition of the fluorescence yield ωi (photon emission probability after a transition to the shell i)



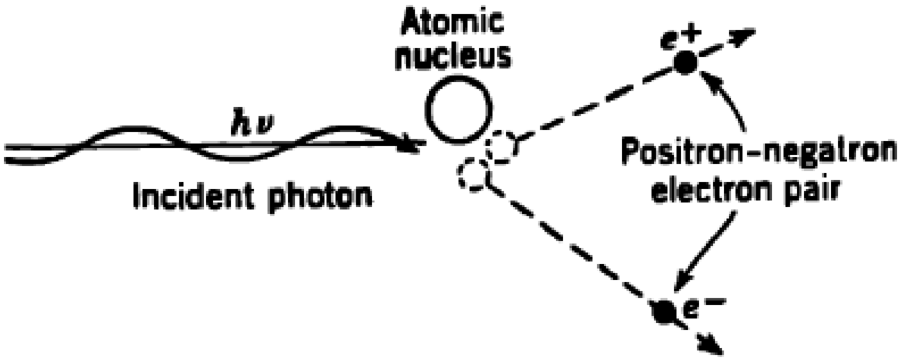


K-shell fluorescence yield



Pair production

- The photon is completely absorbed and its place appears a electron-positron pair. This process only takes place in the field of a nucleus or of an electron (more rarely)



- For the pair production in the electric field of an atomic electron → triplet production (a part of the energy is transferred to the initial electron)
- What is the minimal energy of the incident photon to have pair production?

Conservation laws (1)

- We work initially in the frame of the « target » particle with mass *M* → this particle is at rest →

$$\text{photon before} \quad \rightarrow \quad \left(\frac{h\nu_0}{c}, \frac{h\nu_0}{c}\right)$$

$$\text{target particle before} \quad \rightarrow \quad \left(\frac{Mc^2}{c}, 0\right)$$

- After interaction → we work in the center of mass frame →

$$\text{electron after} \quad \rightarrow \quad \left(\frac{mc^2 + T_e}{c}, \overrightarrow{p_e}\right)$$

$$\text{positron after} \quad \rightarrow \quad \left(\frac{mc^2 + T_p}{c}, \overrightarrow{p_p}\right)$$

$$\text{target particle after} \quad \rightarrow \quad \left(\frac{Mc^2 + T_C}{c}, \overrightarrow{p_C}\right)$$

Conservation laws (2)

- After the interaction → center of mass frame →

$$\overrightarrow{p_e} + \overrightarrow{p_p} + \overrightarrow{p_C} = 0$$

- We note $T_{tot} = T_e + T_p + T_C$
- By conservation of the invariant $P^2 = (E/c)^2 + p^2$ →

$$\text{before} \quad \rightarrow \quad P^2 = \left(\frac{h\nu_0}{c} + \frac{Mc^2}{c}\right)^2 - \left(\frac{h\nu_0}{c}\right)^2$$

$$\text{after} \quad \rightarrow \quad P^2 = \left(\frac{2mc^2 + Mc^2 + T_{tot}}{c}\right)^2$$

Conservation laws (3)

- The minimum energy $h\nu_{0,min}$ is obtained by equalizing both expressions and by considering the kinetic energy $T_{tot} = 0$ →

$$h\nu_{0,min} = 2mc^2 \left(1 + \frac{m}{M}\right)$$

- In the nucleus field → $M \gg m$ → $h\nu_{0,min} = 2mc^2$
- In the electron field → $M = m$ → $h\nu_{0,min} = 4mc^2$

- Remark → It is possible to have pair production in the electron field for photon energy between $2mc^2$ and $4mc^2$ because the atom can take a part of initial momentum → however the probability of this process is extremely weak

Pair production cross section in a nucleus field (1)

- Pair creation occurs inside the electronic cloud → the screening effect due to atomic electrons is important
- Cross sections calculations made by Bethe and Heitler (1934)
- In the nucleus field → attractive force for the electron and repulsive force the positron → their energy distributions are different
- However → weak effect (< 0.0075 $\times$ Z MeV) → can be neglected → the differential cross section for the creation of an electron with kinetic energy T_- is equal to the one of creation of a positron with kinetic energy $T_+ = h\nu_0 - 2mc - T_-$ and is symmetric with respect to the mean energy →

$$\langle T \rangle = \frac{h\nu_0 - 2mc^2}{2}$$

Pair production cross section in a nucleus field (2)

- The energy-differential cross section is →

$$\frac{d_a\kappa}{dT_+} = \frac{\sigma_p Z^2 P(T_+, h\nu_0, Z)}{h\nu_0 - 2mc^2} \qquad \text{for} \qquad h\nu_0 > 2m_e c^2$$

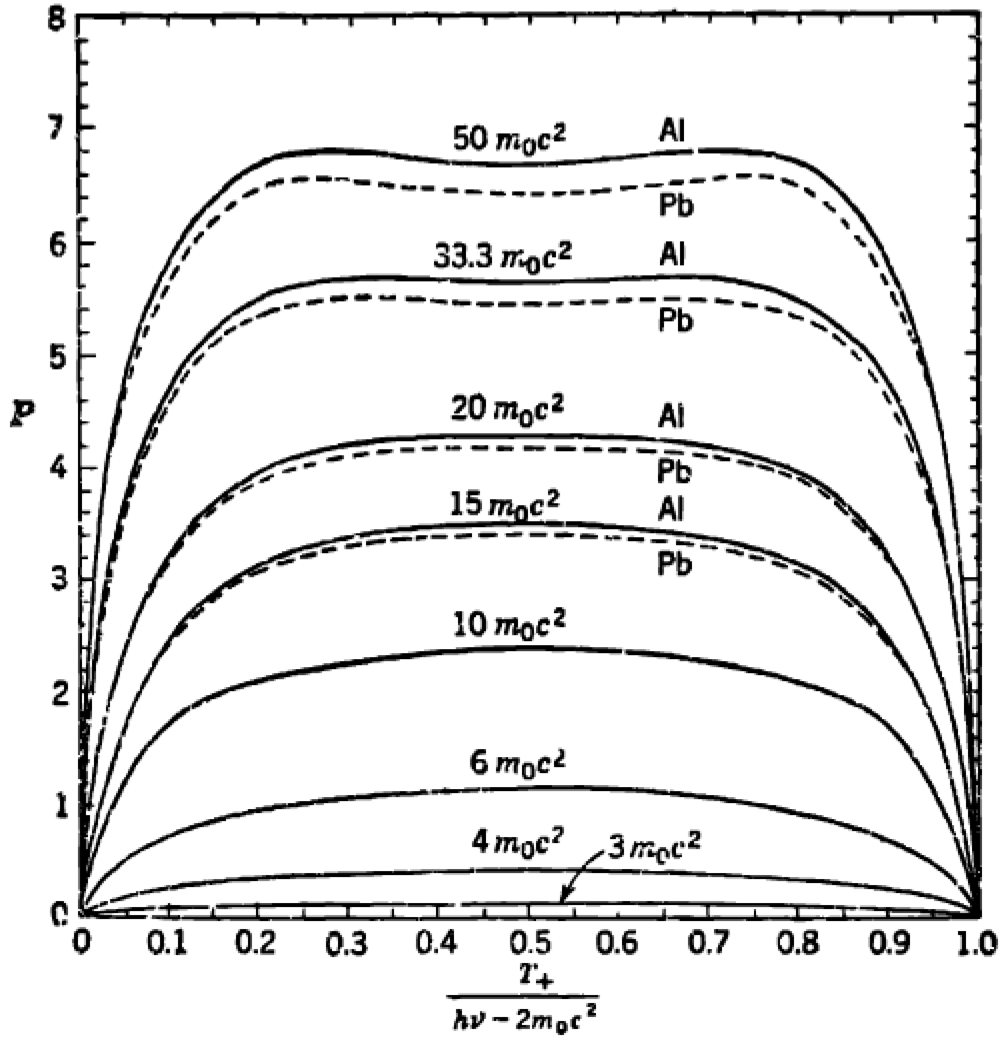
with $\sigma_p = \alpha r_e^2 = 5.80 \times 10^{-32}$ m^2

- It can be written →

$$\frac{d_a\kappa}{dx} = \sigma_p Z^2 P(x, h\nu_0, Z)$$

with $x = T_+/(h\nu_0 - 2m_e c^2)$

Results for $P(x,h\nu_0,Z)$



Comments on the function $P(x,h\nu_0,Z)$

- The function is symmetric

- The function P does not depend a lot on the atomic number Z
→ the cross section is thus proportional to Z^2

- The function P varies slowly with the energy $h\nu_0$ of the photon

- The shapes of various curves are similar

- For $0.2 < x < 0.8$ → P is approximately constant

Total cross section

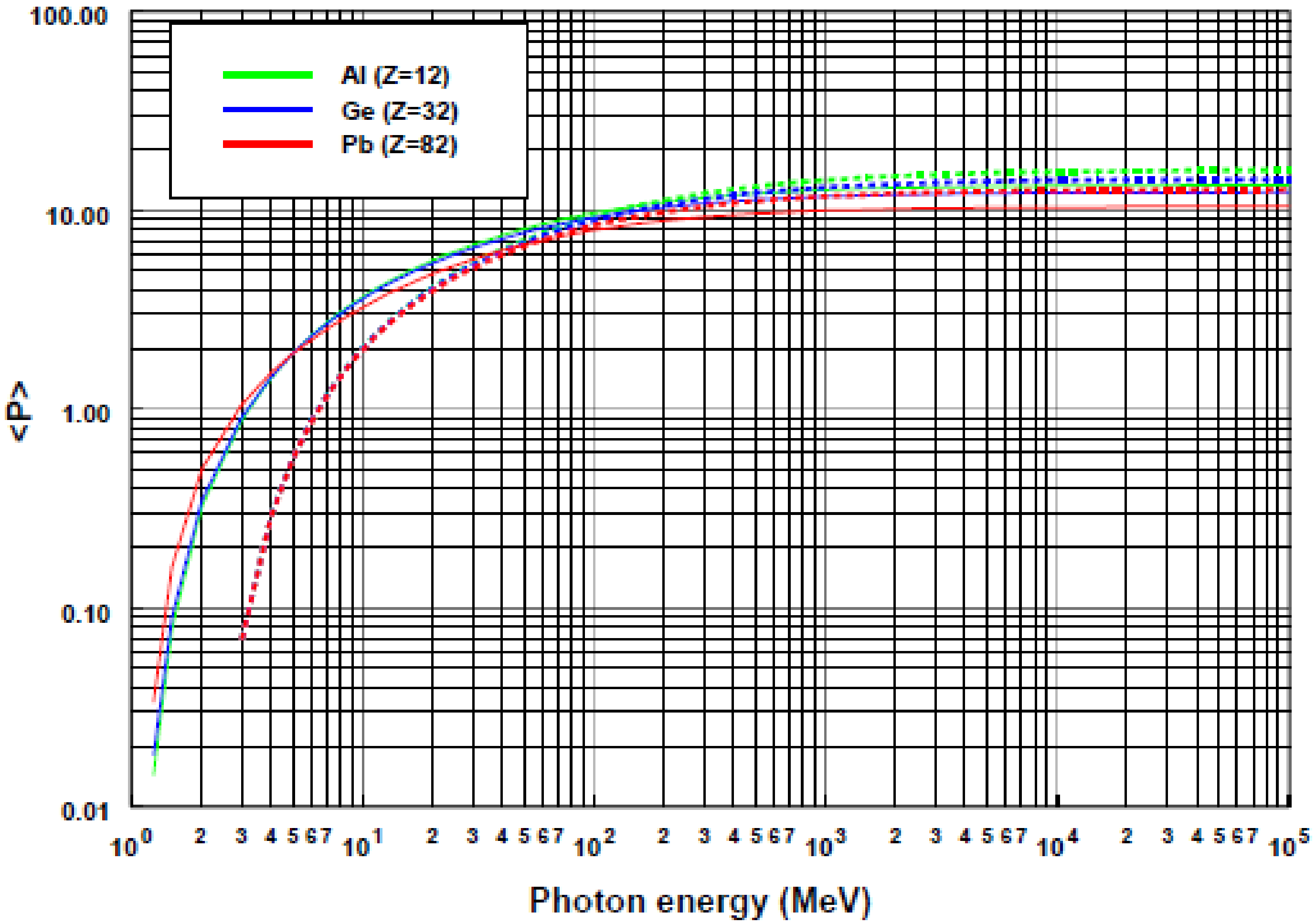
- The total cross section is given by integration on T_+ →

$$
\begin{aligned}
{}_a\kappa &= \int_{T_+} d_a\kappa = \sigma_p Z^2 \int_0^{h\nu_0 - 2mc^2} \frac{P dT_+}{h\nu_0 - 2mc^2} \\
&= \sigma_p Z^2 \int_0^1 P d\left(\frac{T_+}{h\nu_0 - 2mc^2}\right) \\
&= \sigma_p Z^2 \langle P \rangle
\end{aligned}
$$

with $\langle P \rangle$, the mean value of *P*

- $\langle P \rangle$ does not depend a lot on Z and is slowly increasing with $h\nu_0$ → becomes constant for large energies (> 100 MeV) due to the screening of the nuclear field by the atomic electrons

Fuction ⟨ P ⟩



- Line: nucleus field
- Dash: electron field

Cross section for triplet production

- Very complex calculations
- It is possible to show →

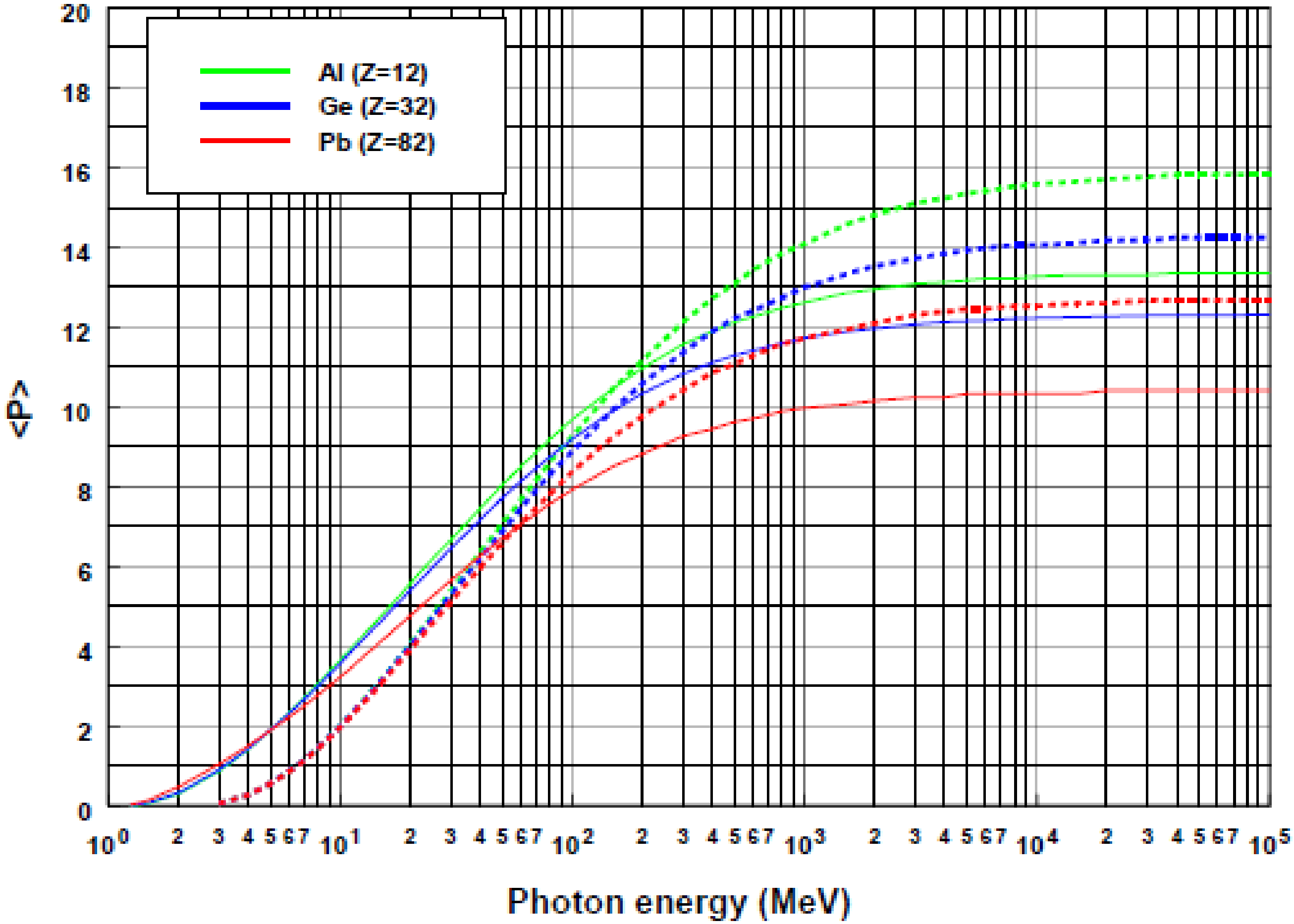
$$_a\kappa_{triplet} = \sigma_p Z \langle P \rangle_{triplet} \quad \text{for} \quad h\nu_0 > 4 m_e c^2$$

- And thus →

$$\frac{_a\kappa_{triplet}}{_a\kappa} \simeq \frac{1}{CZ}$$

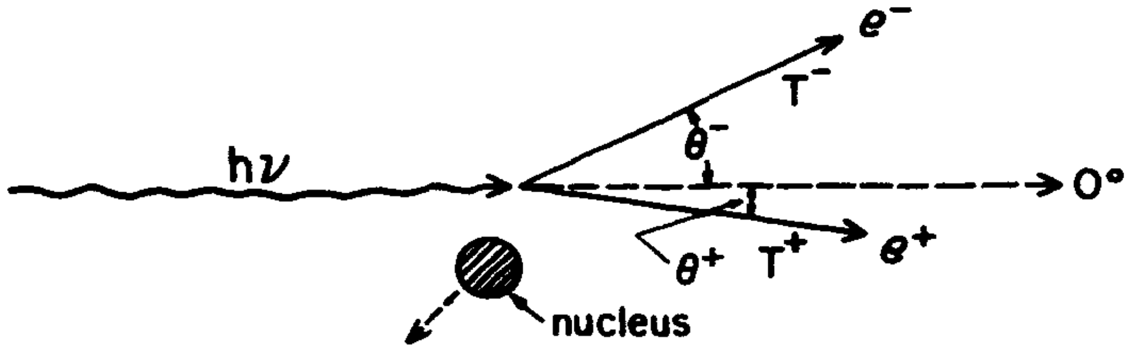
  with *C*, parameter dependent only on $h\nu_0$ such as C → 1 for $h\nu_0 \rightarrow \infty$ and ↗ slowly for $h\nu_0$ ↘ (C ≈ 2 for $h\nu_0$ = 5 MeV)
- The triplet production contributes little to the total cross section except for media with small Z (1% for Pb and 5-10% for Z $\sim$ 10)

Function $\langle P \rangle_{triplet}$



- Line: nucleus field
- Dash: electron field

e^- - e^+ direction of emission



- For $h\nu_0$ quite larger than the energy threshold, electrons and positrons are emitted in forward direction
- Mean emission angle (relatively to the direction of the photon is roughly (radians) →

$$\langle\theta\rangle \simeq \frac{mc^2}{\langle T\rangle}$$

- Example: For $h\nu_0$ = 5 MeV → $\langle T \rangle$ = 1.989 MeV and $\langle \theta \rangle$ = 0.26 radians $\simeq$ 15°

Consecutive phenomenon to the pair production

- First, positron is slowing down in the medium (large cross section for Coulombian interactions)

- Second, annihilation of the positron when it is (quasi) at rest with an electron at rest in the medium

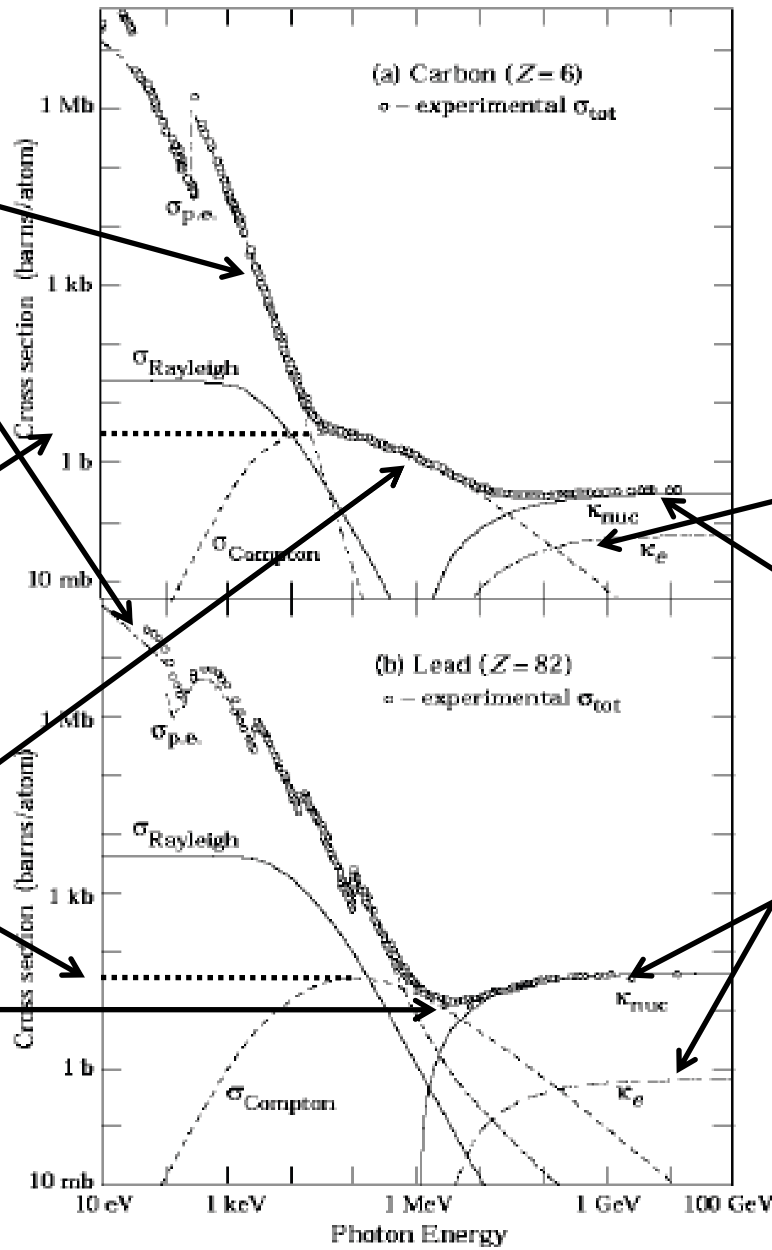
- After the annihilation → two photons of 511 keV energy are emitted with an angle of 180° between them (conservation laws)

Photodisintegration of nuclei

- The photon is absorbed by an atom and a particle is emitted.
- That particle can be a photon or a light particle: p, n, α,…
- This interaction is possible when the photon energy is larger to the threshold energy of the process (between 8 and 20 MeV)

Comparison of various effects

Photoelectrique: complex



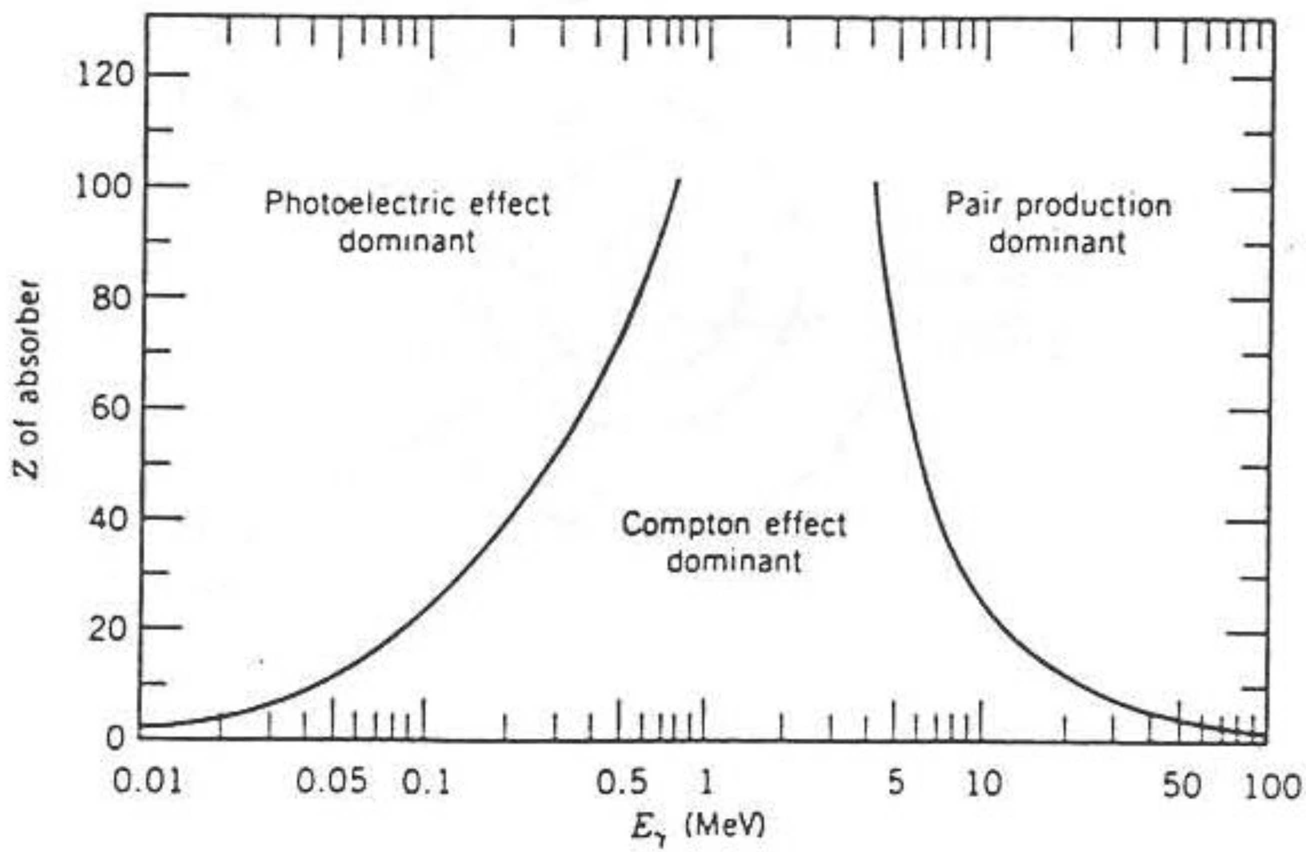
Z effect + C factor

Thomson

Z^2 effect

Z effect

Comparison of the three dominant effects



Attenuation coefficients (1)

- The 3 main photon interaction processes in matter have been characterized by their atomic scattering cross section :
  - Photoelectric effect: ${}_a\tau$
  - Compton effect: ${}_a\sigma = Z\sigma$
  - pair creation: ${}_a\kappa$
- As other processes play a negligible role in our energy range → the total atomic cross section ${}_a\mu$ is →

$$ {}_a\mu = {}_a\tau + {}_a\sigma + {}_a\kappa $$

Attenuation coefficients (2)

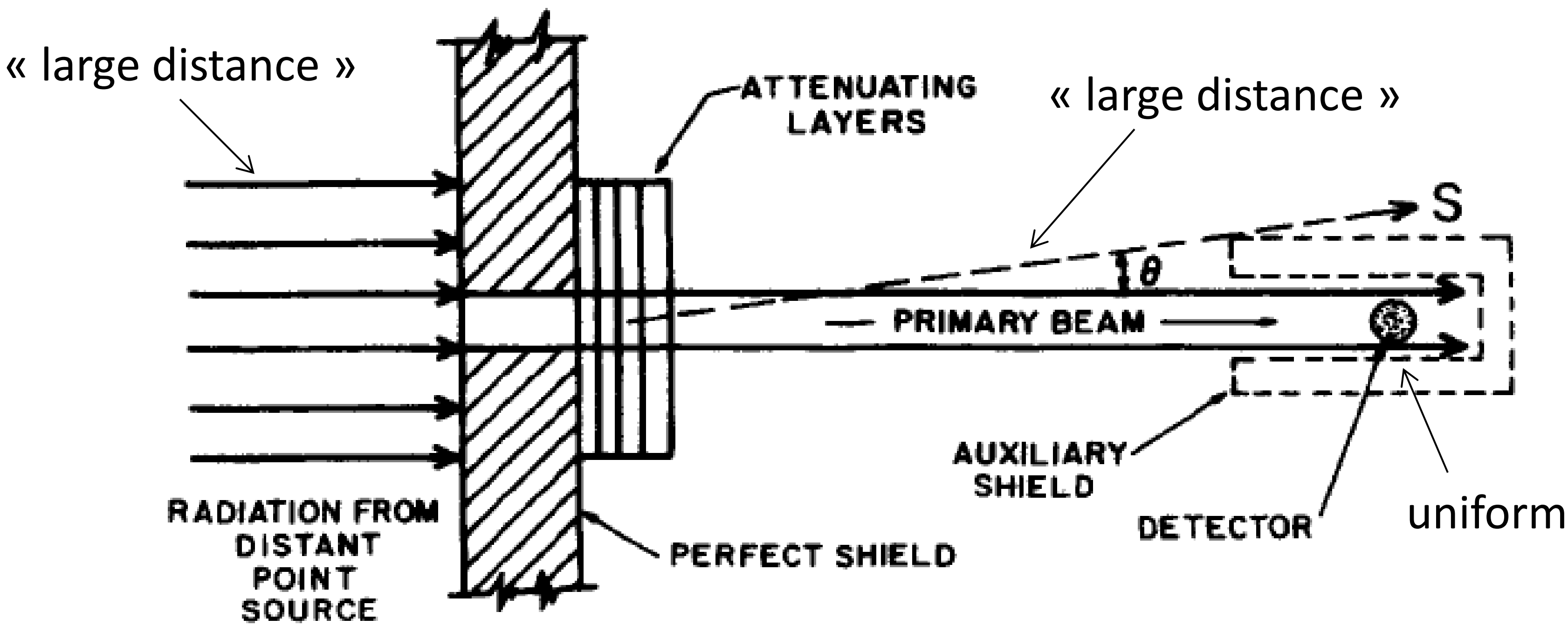
- As previously seen → in a thin target (with atomic density *N*) of width *dx*, the scattering probability for 1 photon is ${}_a\mu N dx$
- For a monoenergetic beam of I photons (||) per time unit → the collision rate is $I\,{}_a\mu N dx$
- The variation dI of the intensity after crossing the target is (by assuming that each collision implies a loss in the beam → all scattering are absorbing) → $dI = -I\,{}_a\mu N dx$
- For a thick target (width *l*) and an initial beam $\perp$ to the target with I_0 particles → the intensity after the target is →

$$I = I_0 \exp\left(-{}_a\mu N l\right)$$

- $\mu = {}_a\mu N$: **Linear attenuation coefficient** (unit: m^{-1}) → allows to the evaluate the scattering rate

Remark on experimental conditions

To check this exponential equation → a particular geometry is needed → *narrow beam geometry* that prevents deflected primaries and secondaries to reach the detector



Narrow beam geometry

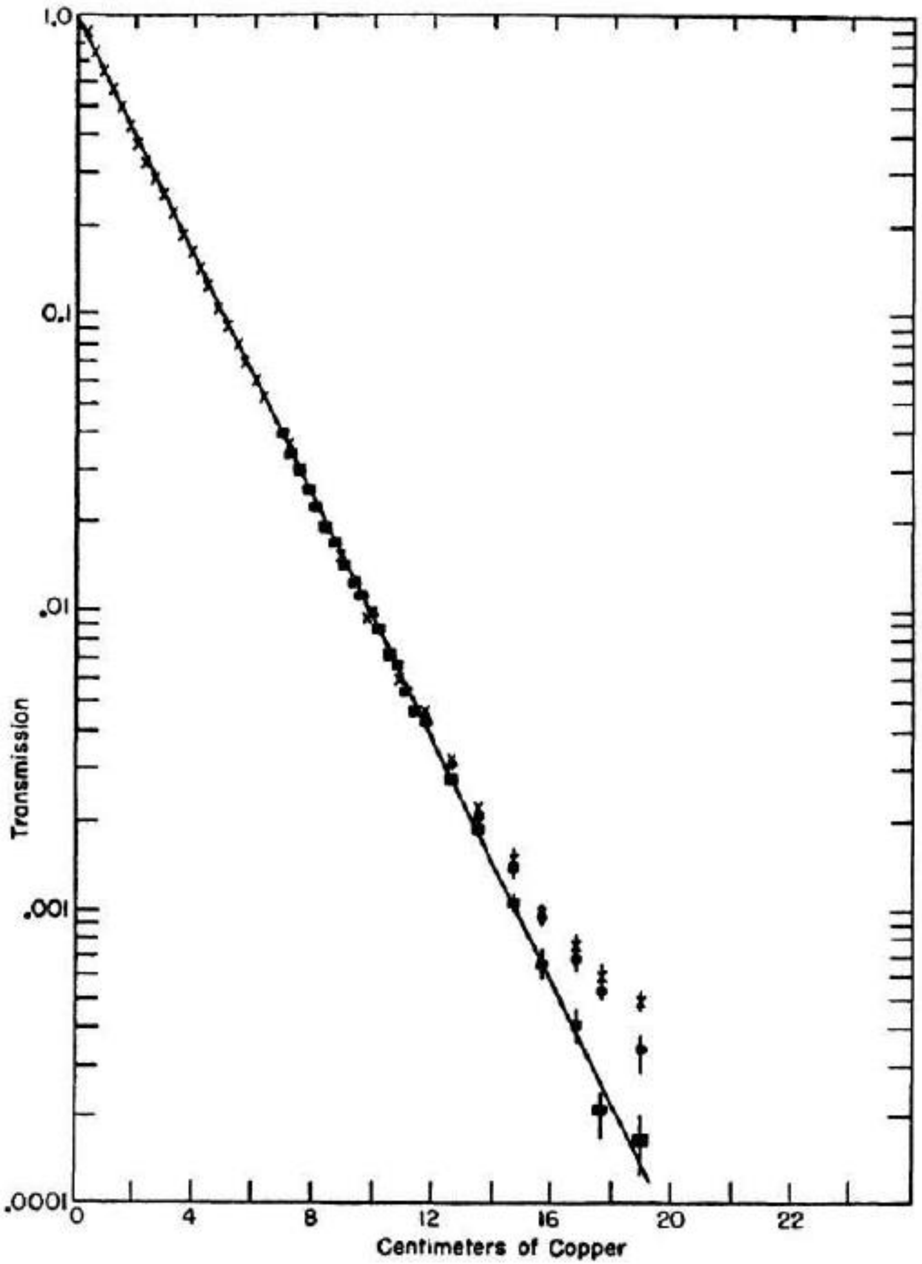
Narrow beam geometry: characteristics

- Large distance between the source and the attenuator → particles perpendicular to the attenuator
- Large distance between attenuator and detector → each particle deflected in the attenuator will miss the detector (intensity of the primary beam in the detector independent of the distance of the attenuator ↔ intensity of the deflected primaries and of the secondaries ↘ with the square of this distance) → the relative intensity of the primary beam ↗ with this distance
- The beam is collimated → it uniformly covers the detector → ↘ of the number of deflected primaries and of secondaries generated inside the attenuator

Narrow beam geometry: shielding

- The shield around the attenuator stops all incident radiations except those passing through the aperture
- The shield around the detector stops all radiations except passing through the aperture ($\theta \approx 0°$) → Pb for X-rays or γ (advantage: small thickness)

Example of attenuation experiment



Transmission of γ from ^{60}Co (1.17 and 1.33 MeV) though a Cu target

Alternative coefficients

- We van write (with *M,* the molar masse of the medium, ρ its density and N_A the Avogadro number):

$$\mu l = \left({}_a\mu \frac{N_A}{M}\right)(\rho l)$$

- (ρl): **Area density** (unit: kg m^{-2})
- μ/ρ: **Mass attenuation coefficient** (unit: m^2 kg^{-1}) →

$$\left(\frac{\mu}{\rho}\right) = \left({}_a\mu \frac{N_A}{M}\right)$$

- $\lambda = 1/\mu$: **Mean free path** (unit: m) → mean distance travelled by a photon between two collisions
- ρ/μ: **Mass attenuation length** (unit: kg m^{-2})

Mass attenuation coefficient (1)

- μ/ρ: **Mass attenuation coefficient** (unit: m^2kg^{-1}) → ratio of dI/I by ρdl with dI/I, the fraction of indirectly ionizing radiations which undergo interactions along the distance dl travelled inside a medium of density ρ
- Global coefficient global → takes into account the interactions of particles in matter regardless of the nature of the interaction
- The mass coefficients are directly proportional to the cross section and do not depend on the physical nature of the target → these coefficients are displayed in databases

Mass attenuation coefficient (2)

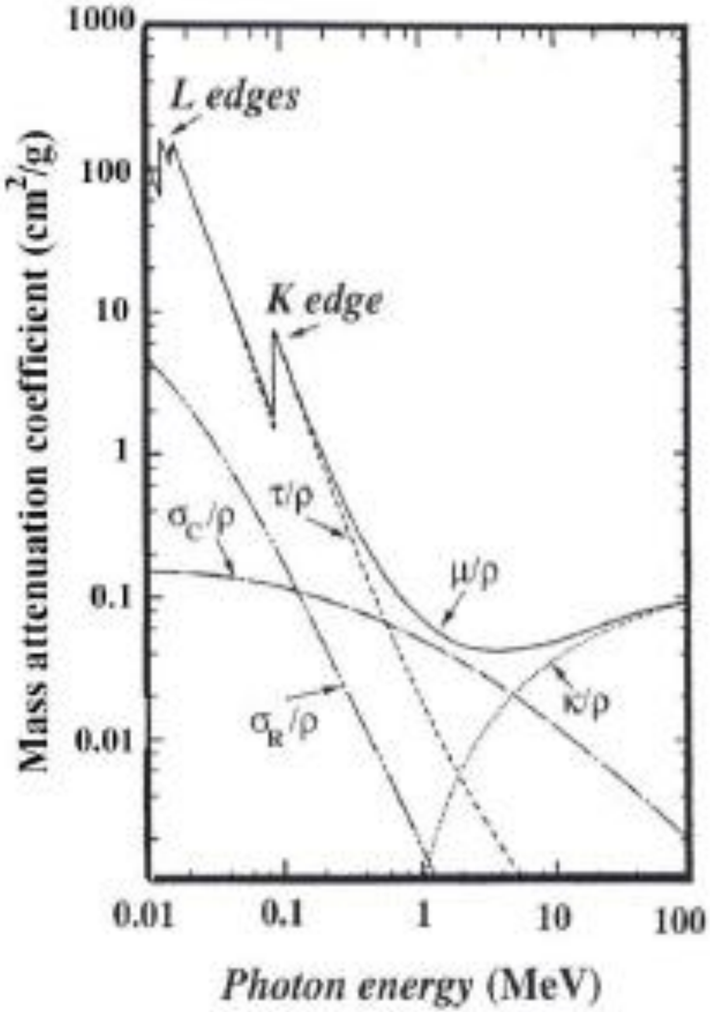
- In a medium with various atom species → the interaction probability is the sum of interaction probabilities with each atom specie (since the molecular binding energies are weak compared to energies of γ rays)

- The total mass attenuation coefficient is given by →

$$\left(\frac{\mu}{\rho}\right) = \left(\frac{\mu}{\rho}\right)_1 w_1 + \left(\frac{\mu}{\rho}\right)_2 w_2 + \ldots$$

with w_i, the mass fractions of the various atom species

Attenuation coefficients: example of lead



Attenuation coefficients: practical examples

- 1 MeV photons in air: μ/ρ=0.064 cm^2/g with ρ(air)=0.001205 g/cm^3 → μ=7.71 10^{-5} cm^{-1} → after 1m → I/I_0= 99.2%
- 10 keV photons in air: μ/ρ=5.1 cm^2/g with ρ(air)=0.001205 g/cm^3 → μ=6.15 10^{-3} cm^{-1} → after 1m → I/I_0= 54.1%
- 1 MeV photons in lead: μ/ρ=0.070 cm^2/g with ρ(lead)=11.35 g/cm^3 → μ=7.95 10^{-1} cm^{-1} → after 1m → $I/I_0 \approx$0%
  → after 1cm → $I/I_0 \approx$45.2%
- 10 keV photons in lead: μ/ρ=130.6 cm^2/g with ρ(lead)=11.35 g/cm^3 → μ=1.48 10^{3} cm^{-1} → after 1cm → $I/I_0 \approx$0%

http://www.nist.gov/pml/data/xraycoef/index.cfm

Mass energy-transfer coefficient (1)

- The mass attenuation coefficient μ/ρ is a measurement of the mean number of interactions between a photon and matter → it allows to evaluate frequency of collisions
- For frequent applications → important parameter is the energy transferred « locally » in the medium i.e. the energy transferred to electrons → effects of photons in matter are due (almost) exclusively to electrons → see « Radiation protection »
- Definition of another quantity more adapted to this aspect → mass energy-transfer coefficient μ_{tr}/ρ

Mass energy-transfer coefficient (2)

- μ_{tr}/ρ: **Mass energy-transfer coefficient** (unit: m^2kg^{-1}) → quotient of $dE_{tr}/(EN)$ (with *E* the energy of all particles excluding rest energy) by ρdl where $dE_{tr}/(EN)$ is the fraction of energy of the incident particles transformed in kinetic energy of charged particles by interactions in a depth *dl* of the medium of density *ρ* → also: $\mu_{tr} = (E_{tr}/E)\mu$
- Also defined as →

$$\frac{\mu_{tr}}{\rho} = f_{ph}\frac{\tau}{\rho} + f_C\frac{\sigma}{\rho} + f_{pn}\frac{\kappa_n}{\rho} + f_{pe}\frac{\kappa_e}{\rho}$$

with f_i, the fractions of photon energy transferred to kinetic energy of charged particles for all processes

Fractions of energy transferred

- Photoelectric effect →

$$f_{ph} = 1 - \frac{E_X}{E}$$

  with E_X, the mean energy of fluorescence photons
- Compton effect →

$$f_C = 1 - \frac{\langle E_1 \rangle + E_X}{E}$$

  with $\langle E_1 \rangle$, the mean energy of scattered photon → remark: formally X-rays have to be considered → practically they can be neglected
- Pair production (in the field of nucleus and of electron) →

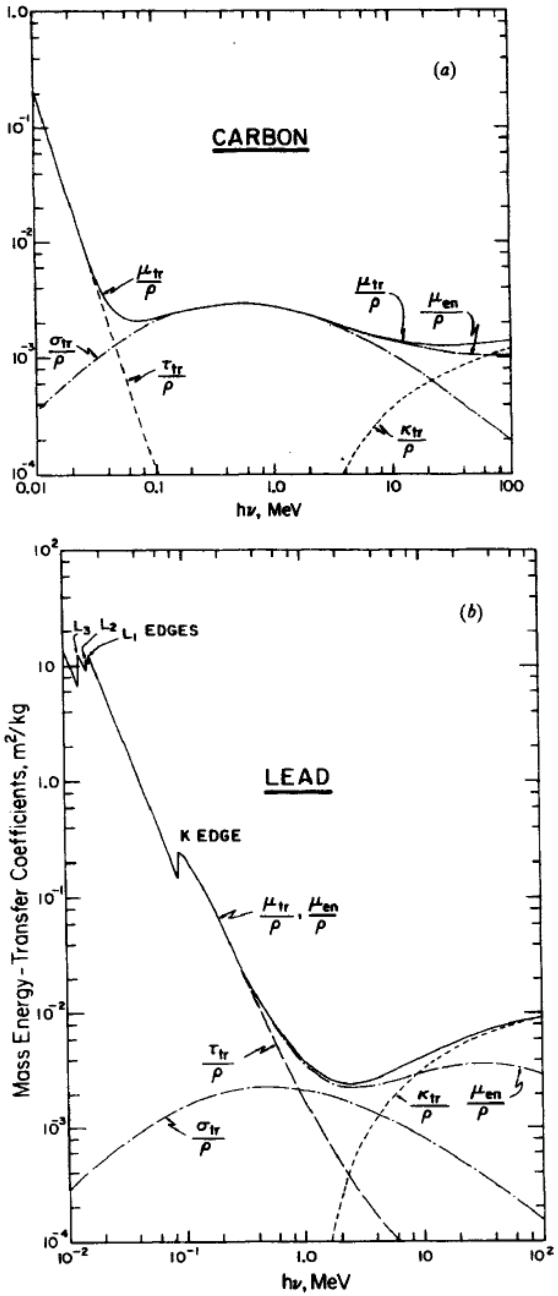
$$f_{pn} = 1 - \frac{2mc^2}{E} \qquad f_{pe} = 1 - \frac{2mc^2 + E_X}{E}$$

Mass energy-absorption coefficient

- A part of kinetic energy of charged particles set in motion can be absorbed no locally → a part of the energy can be lost in radiative processes (especially Bremsstrahlung but also in-flight annihilation or fluorescence radiations)
- μ_{en}/ρ: **Mass energy-absorption coefficient** (unit: m^2kg^{-1}) → product of the mass energy-transfer coefficient by *(1-g), with* g the fraction of energy lost on average in radiative processes as the charged particles slow to rest in the material
- *g* is specific to the material

$$\frac{\mu_{en}}{\rho} = (1 - g)\frac{\mu_{tr}}{\rho}$$

Comparison $\mu_{tr} \leftrightarrow \mu_{en}$ (1)



Significant difference only for high energies of the γ rays → when the charged particles produced by the interaction have enough energy to be characterized by an important Bremsstrahlung (especially for high Z materials)

Comparison $\mu_{tr} \leftrightarrow \mu_{en}$ (2)

γ-ray Energy (MeV)	$100\,(\mu_{tr} - \mu_{en})/\mu_{tr}$		
	$Z = 6$	29	82
0.1	0	0	0
1.0	0	1.1	4.8
10	3.5	13.3	26

Mass absorption coefficient

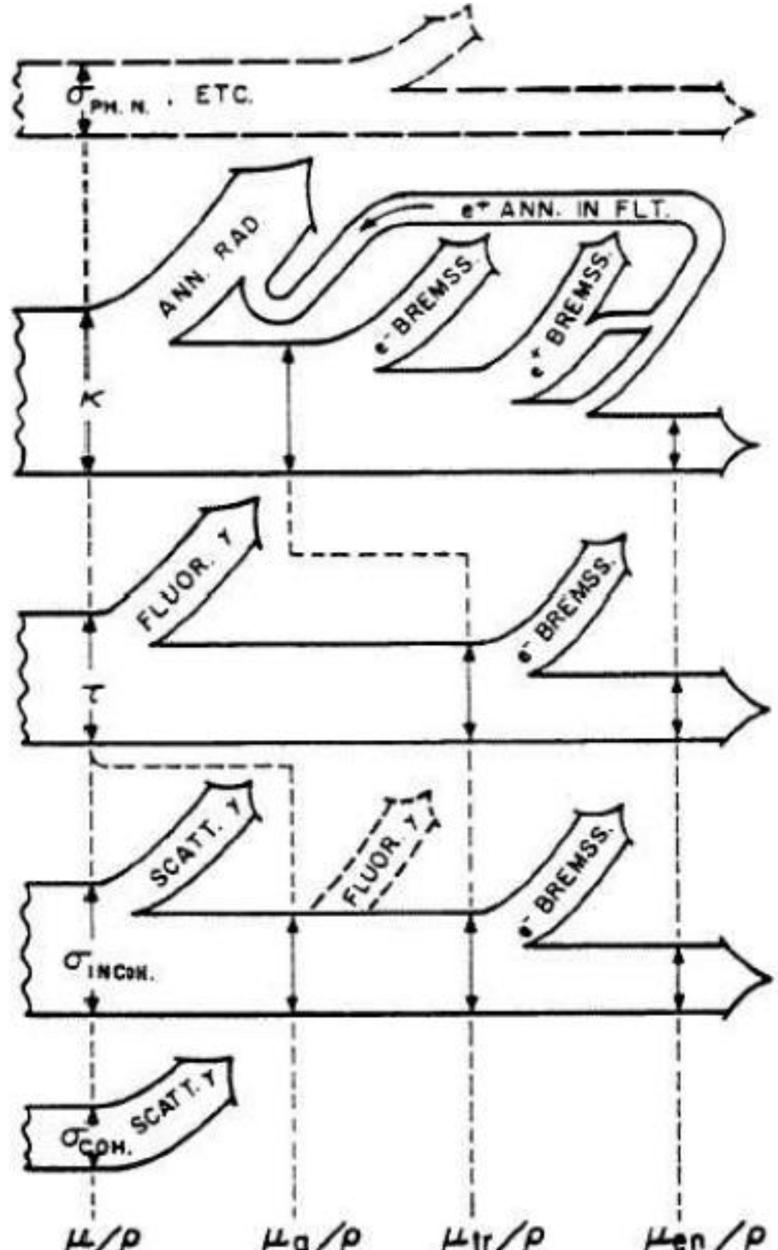
- μ_a/ρ: **Mass absorption coefficient** (unit: m^2kg^{-1}) → coefficient for which only we suppose that only scattered photons (coherently or incoherently) take energy away

- Coefficient rarely used (never!)

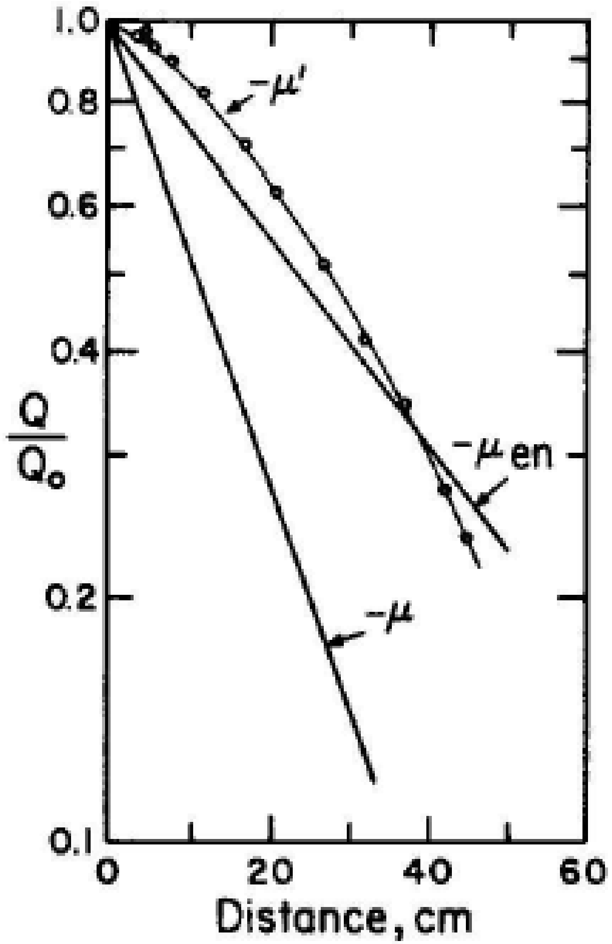
- Finally →

$$\mu \geq \mu_a \geq \mu_{tr} \geq \mu_{en}$$

Schematic overview of the coefficients



Example of the application of these coefficients



Energy deposited inside water by γ rays from a ^{60}Co point-source put at the center of a water sphere as a function of the distance between the source and the detector

Hounsfield units (1)

- Numerical values of μ not very meaningful in clinical routine for practitioners during CT-SCAN (computerized tomography) → introduction of another unit easier to handle
- The attenuation values during a CT are expressed in Hounsfield units (HU) according to a linear scale of densities
- In the Hounsfield scale → the value of 0 HU is arbitrarily attributed to water and the value of -1000 UH to air (NCTP) → 1 HU is defined as the thousandth part of the difference between these two values→

$$1 \text{ UH} = \frac{\mu_{water} - \mu_{air}}{1000} \approx \frac{\mu_{water}}{1000}$$

- The values for tissues with attenuation μ_i are given by:

$$\text{CT Value (HU)} = 1000 \times \frac{\mu_i - \mu_{water}}{\mu_{water}}$$

Hounsfield units (2)

Substance	HU
Air	−1 000
Lung	−500
Fat	−70 à −30
Water	0
Cerebrospinal fluid	15
Kidney	30
Blood	+30 à +45
Muscle	+10 à +40
Grey matter	+37 à +45
White matter	+20 à +30
Liver	+40 à +60
Soft tissue	+100 à +300
Bones	+700 (spongy) to +3 000 (dense)

Hounsfield units (3)

- Interest of this scale in computing → coding of information in 12 bits per pixel → presence of numbers between -1024 for vacuum and 3071 for bones (0 is a number) → 2^{12} = 4096 levels of tissue densities (remark: for metals (implants) → attribution of the max value 3071)
- Windowing representation = application of values in Hounsfield units on the grey scale
- UH values for each pixel converted into a digital image → attribution of a grey scale intensity to each value (for ↗ number → ↗ the brightness intensity of the pixel) → increase in contrast on the family of tissues that constitute the region of interest (soft tissue)
- Example: fat is less dense than water (HU value between -30 and -70) → fat always appears darker than water in CT images

Hounsfield units (4)