Chapter VII:
Beta decay

Summary

1. General principles
2. Energy release in β decay
3. Fermi theory of β decay
4. Selections rules
5. Electron capture decay
6. Other β decays

General principles (1)

- The β decay is due to the weak interaction
- The most basic β process is the conversion of a proton to a neutron or of a neutron to a proton → $Z \rightarrow Z \pm 1$ and $N \rightarrow N \mp 1$ → $A = Z + N$ is constant
- β decay is a convenient way for unstable nuclei to slide down the mass parabola of constant A to reach the stable isobar
- The charge conservation implies the intervention in the process of a charged particle: electron or positron
- If an electron or positron is emitted (i.e. β^- or β^+) → identical to « classical » electron or positron

General principles (2)

- A process of the type $n \rightarrow p + e^-$ is impossible for 3 reasons:
  1. The emitted electron has a continuous distribution of energies (from 0 to an upper limit = to the energy difference between the initial and final states)
  2. There is no conservation of the lepton number
  3. There is no conservation of momentum and angular momentum in the process
- (Wolfgang) Pauli suggests (1930) that an extremely light neutral and highly penetrating particle (which he called neutron) is involved in the process and takes a part of the energy release → Enrico Fermi proposes a full theory (1934) involving this particle recalled neutrino (we use the term « neutrino » for both neutrino and antineutrino) → the existence of the neutrino was experimentally confirmed in 1956

General principles (3)

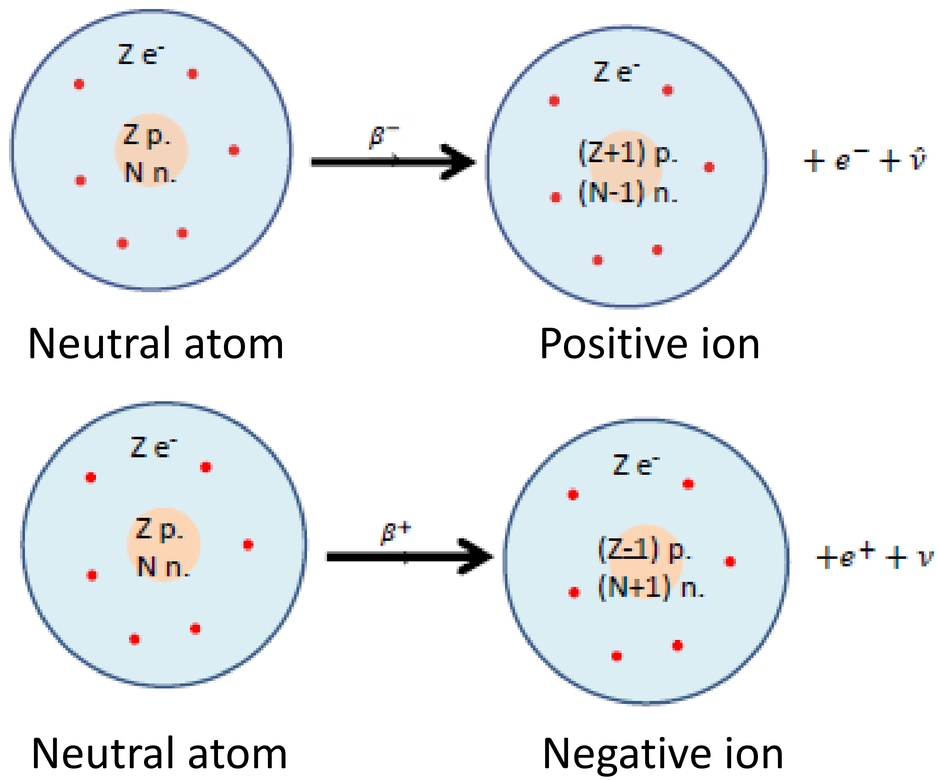
- The possible weak processes are →

$$
\begin{array}{ll}
n \rightarrow p + e^- + \bar{\nu}_e & \beta^- \text{ decay} \\
p \rightarrow n + e^+ + \nu_e & \beta^+ \text{ decay} \\
p + e^- \rightarrow n + \nu_e & \text{electron capture} \\
n + e^+ \rightarrow p + \bar{\nu}_e & \text{positron capture}
\end{array}
$$

- The positron capture is not experimentally observed because of the absence of free positrons is stable matter (possibly present in star explosion)
- The neutrino carries the « missing » energy and allow the conservation of electronic lepton number ($\ell_e(e^-) = +1$; $\ell_e(e^+) = -1$; $\ell_e(\nu_e) = +1$; $\ell_e(\bar{\nu}_e) = -1$)

General principles (4)

- Remark: the β^+ decay and the electron capture occur only for protons bound in nuclei → they are energetically forbidden for free protons or for protons in hydrogen atoms ($m_n - m_p$ = 1.3 MeV → free proton is stable)



General principles (5)

- Examples of decays:

Decay	Type	Q (MeV)	$t_{1/2}$
$^{23}Ne \rightarrow {}^{23}Na + e^- + \bar{\nu}$	β^-	4.38	38 s
$^{99}Tc \rightarrow {}^{99}Ru + e^- + \bar{\nu}$	β^-	0.29	2.1×10^5 y
$^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$	β^+	3.26	7.2 s
$^{124}I \rightarrow {}^{124}Te + e^+ + \nu$	β^+	2.14	4.2 d
$^{15}O + e^- \rightarrow {}^{15}N + \nu$	ε	2.75	1.22 s
$^{41}Ca + e^- \rightarrow {}^{41}K + \nu$	ε	0.43	1.0×10^5 y

- Attention: the decay can populate several states → this is true for majority of decays →this fact is known as the branching in the decay →the relative population of the branches is called the branching ratio

Energy release in β decay: β^- decay (1)

- The β^- disintegration to ground state of daughter nucleus is written:

$$ {}^{A}_{Z}X_N \xrightarrow{\beta^-} {}^{A}_{Z+1}Y_{N-1} + e^- + \overline{\nu}_e $$

- The energy release (Q^N) when only nuclei are considered (no mass for neutrino) is →

$$ Q^N_{\beta^-} = m(A,Z)c^2 - m(A,Z+1)c^2 - m_e c^2 $$

- Considering $m(A,Z) = Zm_p + Nm_n - \mathrm{B}(A,Z)$ →

$$ Q^N_{\beta^-} = B(A,Z+1) - B(A,Z) + (m_n - m_p - m_e)c^2 $$

with $(m_n - m_p - m_e)c^2 \approx 0.782333$ MeV

- These expressions are valid in the absence of electrons (astrophysics)

Energy release in β decay: β^- decay (2)

- Considering now atoms and bound electrons (attention: if nuclei are in form of molecules or crystalline lattice → other corrections) with *I* the (positive) ionization energy of the atom:

$$
\begin{aligned}
Q_{\beta-} &= M({}^A_Z X_N)c^2 - M({}^{A}_{Z+1}Y^+_{N-1})c^2 - m_e c^2 \\
&= M({}^A_Z X_N)c^2 - M({}^{A}_{Z+1}Y_{N-1})c^2 - I
\end{aligned}
$$

- Indeed an e^- is missing in the electron cloud of Y^+ (positive ion) → the electron masses cancel except the value of *I*
- If *I* (a few eV) is neglected →

$$
\begin{aligned}
Q_{\beta-} &\approx M(A,Z)c^2 - M(A,Z+1)c^2 \\
&\approx \Delta(A,Z) - \Delta(A,Z+1)
\end{aligned}
$$

with $\Delta(A,Z) = [M(A,Z) - A]uc^2$

Energy release in β decay: β^- decay (3)

- Atom decay is thus possible if →

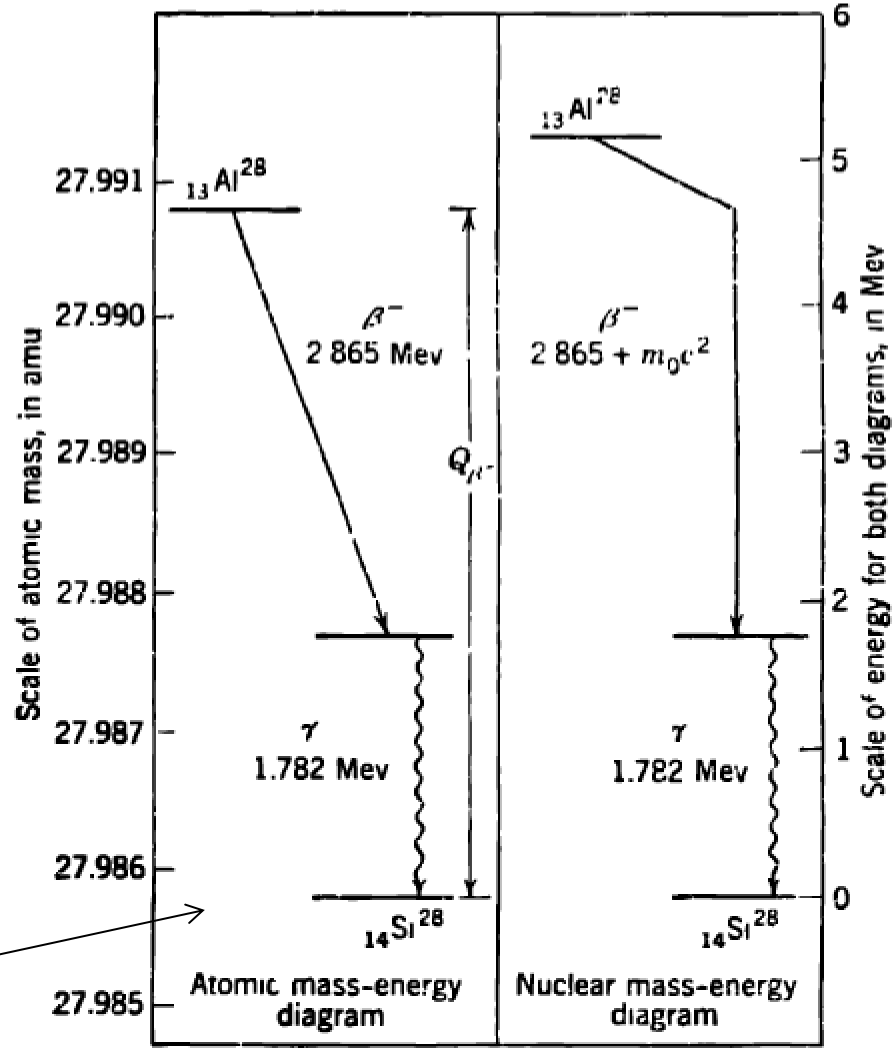
$$M(A, Z) > M(A, Z+1)$$

- **Attention**: if the daughter nucleus is in an excited state → the excitation energy of the nucleus has to subtracted of Q_β
- If the differences of electron binding energies between the final and initial atomic systems (a few keV) can be neglected →

$$Q^N_{\beta-} \approx Q_{\beta-}$$

- In some cases this difference cannot be neglected: ^{187}Re $\rightarrow$ ^{187}Os reaction → $Q^N_{\beta-} = -2.54\ \mathrm{keV}$ → forbidden transition for the ^{187}Re <u>nucleus</u> but if we consider a <u>neutral atom</u> of rhenium → $B_e(^{187}Os^+)$- $B_e(^{187}Re)$ ≈ 5.01 keV → $Q_{\beta-} = 2.47\ \mathrm{keV}$ → spontaneous decay → for neutral atom τ = 4.1 $\times$ 10^{10} years (for ionized atom → τ depends on ionization degree)

Energy release in β decay: β^- decay (4)



Conventional diagram

Energy release in β decay: β^+ decay (1)

- The β^+ disintegration to ground state of daughter nucleus is written:

$$^{A}_{Z}X_N \xrightarrow{\beta^+} {}^{A}_{Z-1}Y_{N+1} + e^+ + \nu_e$$

- Energy release for only nuclei (Q^N) is →

$$Q^N_{\beta+} = m(A,Z)c^2 - m(A,Z-1)c^2 - m_e c^2$$

- Considering binding energies →

$$Q^N_{\beta+} = B(A,Z-1) - B(A,Z) - (m_n - m_p + m_e)c^2$$

with $(m_n - m_p + m_e)c^2 \approx 1.804331$ MeV

Energy release in β decay: β^+ decay (2)

- Initially → formation of a negative ion → loss of an atomic electron
- For a neutral atom →

$$Q_{\beta+} = M(A,Z)c^2 - M(A,Z-1)c^2 - 2m_e c^2$$

- Considering mass excess Δ →

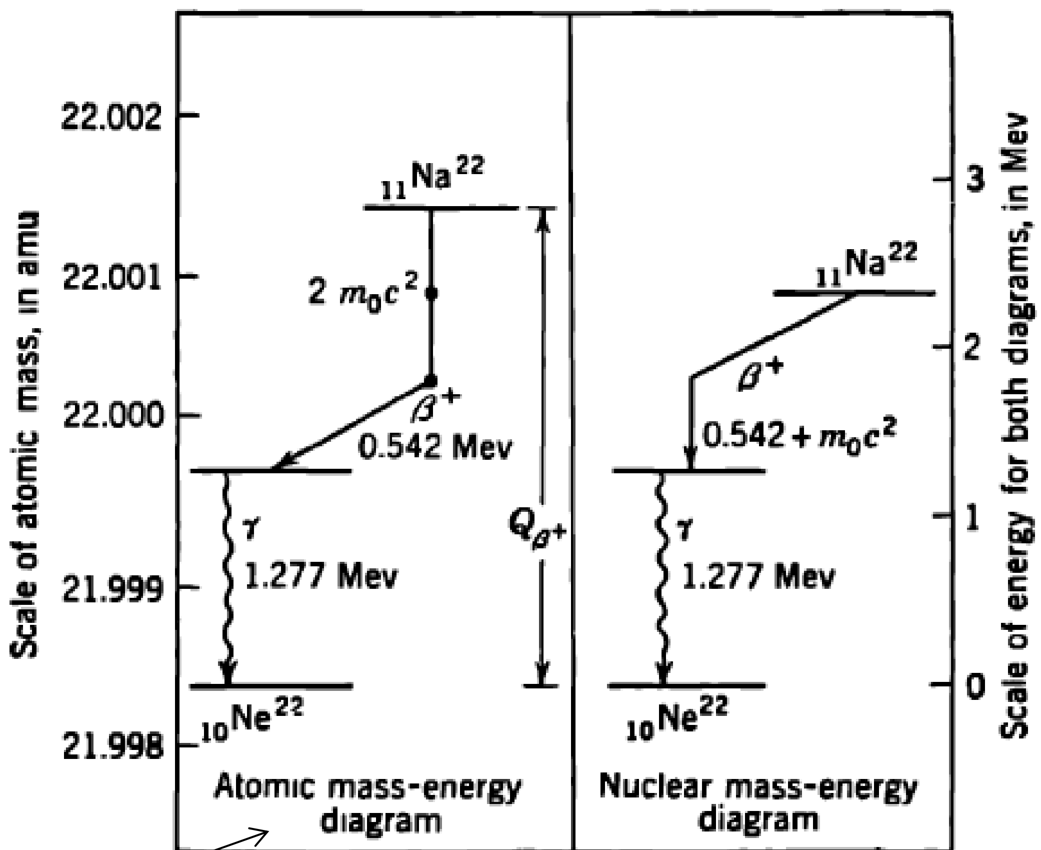
$$Q_{\beta+} = \Delta(A,Z) - \Delta(A,Z-1) - 2m_e c^2$$

- Atom decay is thus possible if →

$$M(A,Z) > M(A,Z-1) + 2m_e$$

- $Q^N_{\beta+}$ and $Q_{\beta+}$ are generally very close

Energy release in β decay: β^+ decay (3)



Conventional diagram

Remark: only dominant process is shown

Energy release in β decay: electron capture (1)

- The electron capture ϵ is written:

$$^{A}_{Z}X_N + e^- \xrightarrow{\epsilon} {}^{A}_{Z-1}Y^*_{N+1} + \nu_e$$

- The electron capture preferentially implies an inner-shell electron → presence of vacancy/hole → very excited final atomic state → final atomic mass > mass of the atom in its ground state → * notation
- Electron capture is obviously impossible for fully ionized atom
- Otherwise → competition with β^+ process
- Due to the vacancy in the inner-shell → reorganization of the electrons cloud → emission of X-rays or Auger electrons

Energy release in β decay: electron capture (2)

- The energy release is (often) written with atomic masses because the captured electron was one of the electrons cloud of the initial nucleus X →

$$Q_\epsilon = M(A,Z)c^2 - M(A,Z-1)c^2 - \Delta E_{el}$$

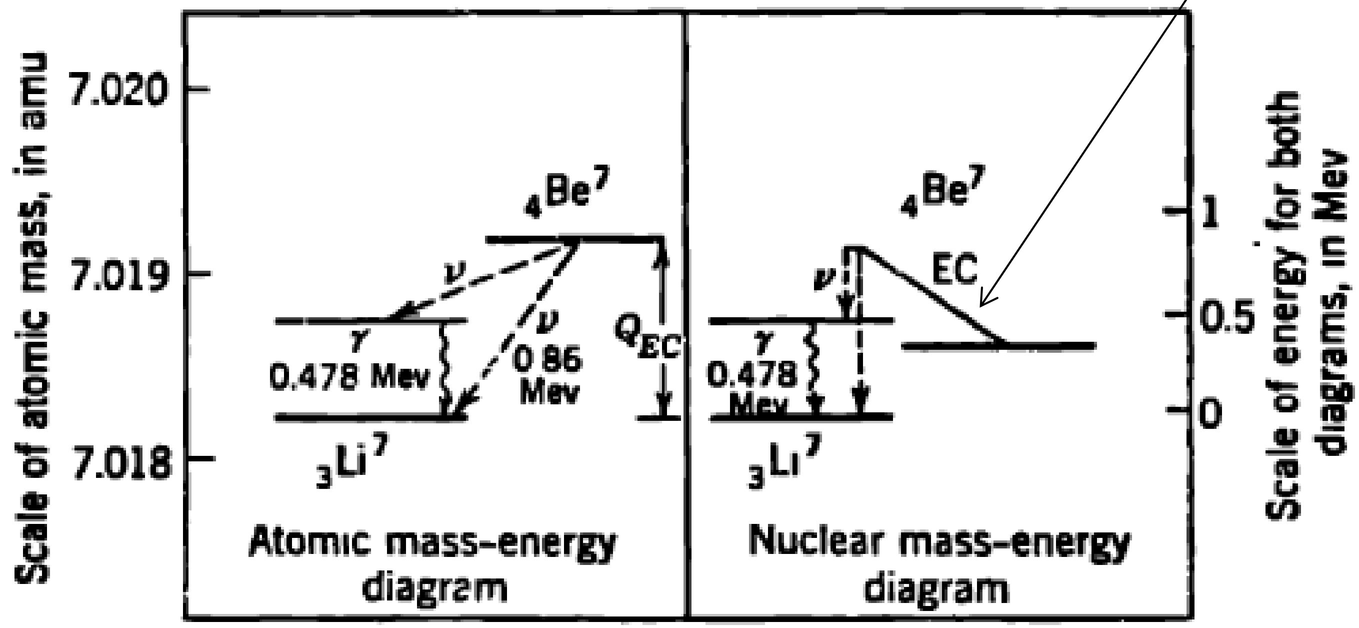
- ΔE_{el} is the excitation energy of the Y* atom → to a good approximation $\Delta E_{el} = B_i$ (binding energy of the electron captured by X ≈ a few eV to a few tens of keV)
- The capture can spontaneously occurs if →

$$M(A,Z) > M(A,Z-1) + B_i/c^2$$

- Electron capture is thus possible in some cases for which β^+ is not possible → transitions between isobars whose mass is nearly the same may therefore take place by ϵ when β^+ decay is excluded energetically

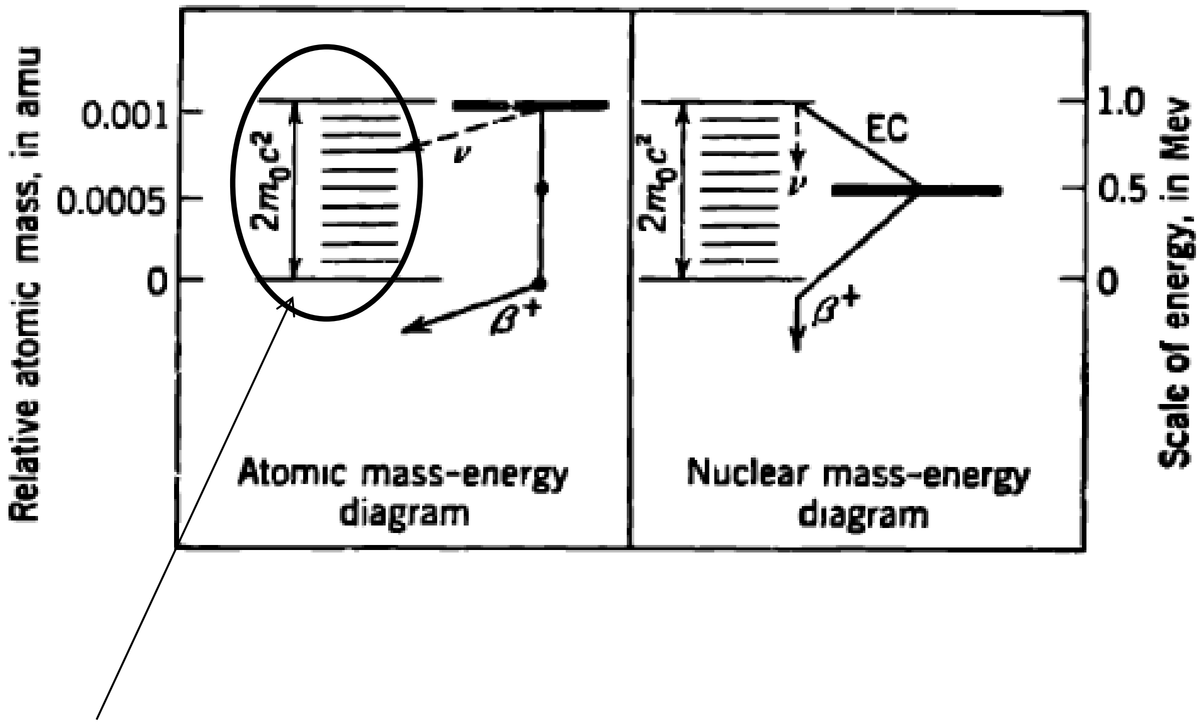
Energy release in β decay: electron capture (3)

The Be^7 nucleus <u>gains</u> the mass m_e of the captured electron



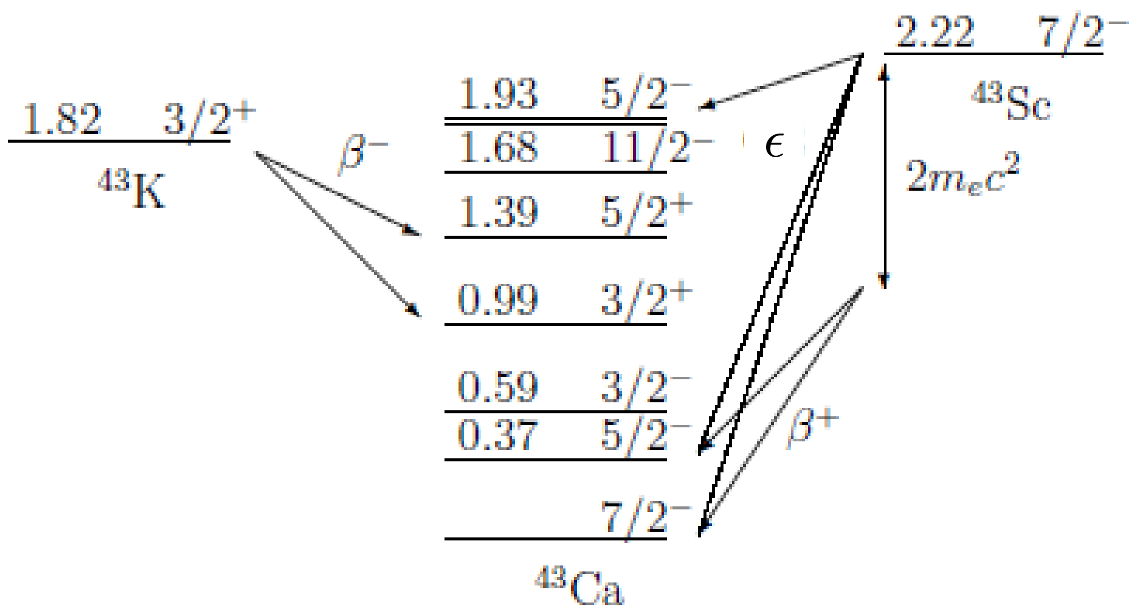
Conventional diagram

Energy release in β decay: electron capture (4)



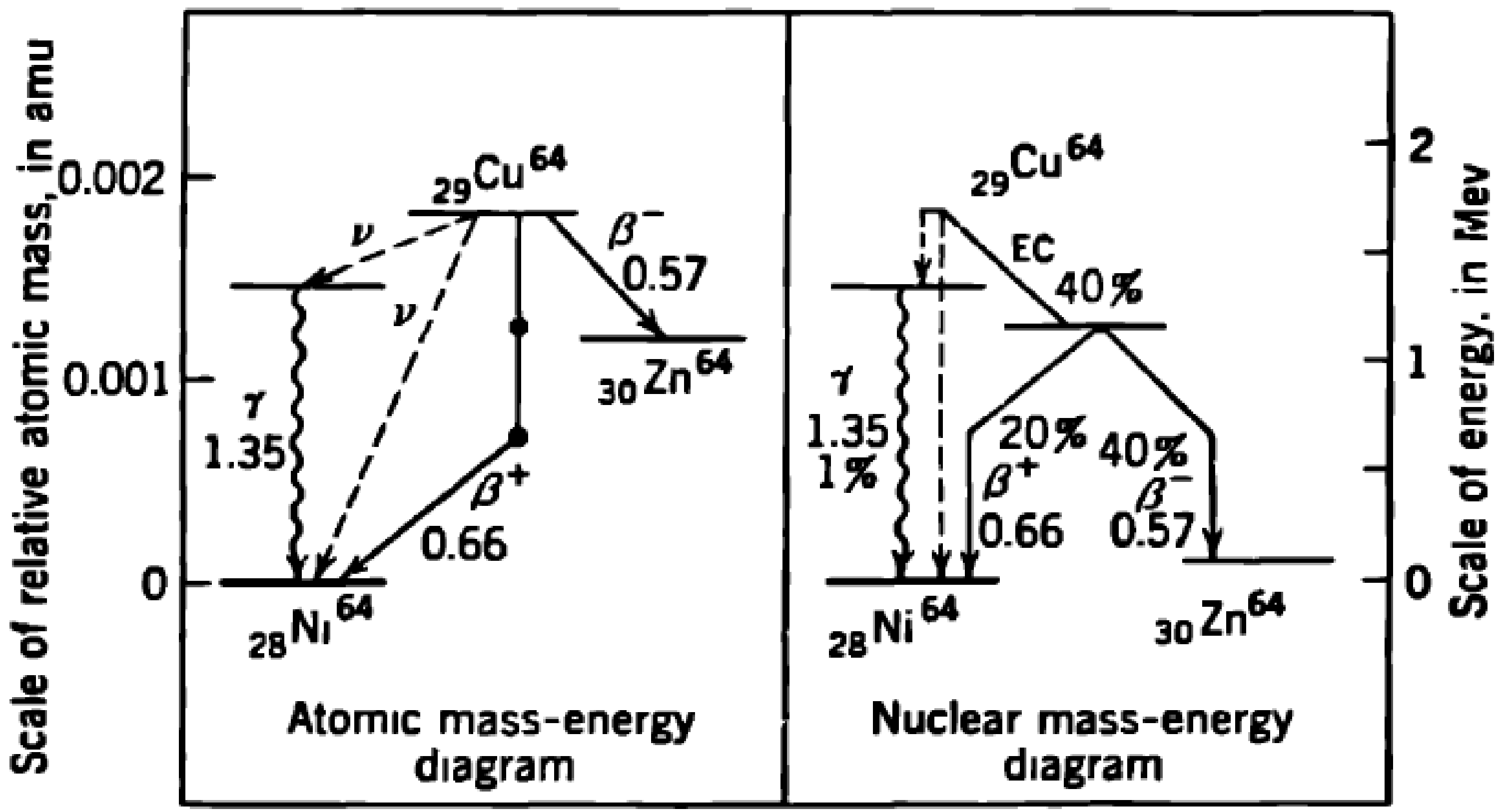
Electron capture is possible in this domain while β^+ is excluded

Energy release in β decay: summary (1)



- The decay threshold differs from $2m_ec^2$ between β^+ and ϵ

Energy release in β decay: summary (2)



Energy release in β decay: summary (3)

- For a nucleus at rest → the sum of the momenta of the emitted particles is zero
- Moreover the recoil energy (kinetic energy of the daughter nucleus) may be neglected → in β^-/β^+ decay the release energy is shared among electron/positron and antineutrino/neutrino (kinetic energy) →

$$Q_\beta = T_e + T_\nu$$

- The energy of each of the particles may vary between 0 and Q_β
- In electron capture only the emitted neutrino takes the energy → the neutrino is monoenergetic →

$$Q_\epsilon = T_\nu$$

Fermi theory of β decay: principle

- β^- and β^+ decays implies a completely different approach for the calculations of transition probabilities
- The electron and neutrino do not exist before the decay process (no preformation as in α decay)
- The electron and neutrino must be treated relativistically (small mass at rest for e^- and very small mass for ν)
- Fermi assumed β decay results from sort of interaction between the nucleons, the electron and the neutrino
- This interaction is expressed as a perturbation to the total Hamiltonian

Fermi theory of β decay: Fermi Golden Rule

- Decay probability per unit time is expressed by Fermi Golden Rule →

$$\lambda = \frac{2\pi}{\hbar}|V_{fi}|^2\rho(E_f) \text{ where } V_{fi} = \int \psi_f^* V \psi_i d\boldsymbol{r}$$

- $\rho(E_f)$ is the density of final states = dn_f/dE_f
- The matrix element V_{fi} is the integral of the interaction V between the initial and final quasi-stationary states of the system → V_{fi} reflects the probability of going from state i ($\Psi_{i,N}$: nucleus) to state f ([$\Psi_{f,N}$ Ψ_e Ψ_ν] : [nucleus electron neutrino]) in β^- decay
- Fermi did not know the mathematical form of V (weak interaction) → he supposed an operator analog to the operators used for electromagnetic transitions: $V \rightarrow G_F O_\beta$ with G_F the coupling constant called the Fermi constant

Fermi theory of β decay: Fermi constant

- G_F has a role analog to the term $e^2/4\pi\epsilon_0$ in the electromagnetic processes
- $G_F = 1.41 \times 10^{-56}$ J cm^3 (empiric determination) → very small → justify the use of the perturbation method
- G_F is sometimes replaced by a the dimensionless constant G_β →

$$G_F = \frac{\hbar^3}{m_e^2 c} G_\beta$$

- $G_\beta = 3.002 \times 10^{-12}$
- G_β is a dimensionless constant characteristic of the beta interaction as the fine-structure constant α is characteristic of the Coulomb interaction
- The electroweak theory links together G_β and α → $G_\beta \approx 10.1\,\alpha \times (m_e/m_W)^2$

Fermi theory of β decay: shape of β spectrum (1)

- We will show that the density of final states determines the shape of the energy distribution → we need to know the number of final states accessible to the decay products
- We consider an electron (or positron) emitted with momentum ***p*** and a neutrino (or antineutrino) with momentum ***q***
- The locus of momenta points in the range *dp* at *p* is a spherical shell of radius *p* and thickness *dp* → volume = $4\pi p^2 dp$
- If the e^- is confined to a box of volume *V* → the number of final e^- states dn_e corresponding to momenta in [*p*, *p* + *dp*] in the phase space (six-dimensional space: x, y, z, p_x, p_y, p_z) is →

$$dn_e = \frac{4\pi p^2 dp V}{h^3}$$

constant included to have a number = unit volume in phase space (with $\Delta p_i \Delta r_i = h$)

Fermi theory of β decay: shape of β spectrum (2)

- Similarly the number of neutrino is →

$$dn_\nu = \frac{4\pi q^2 dqV}{h^3}$$

- Then the number of final states which have simultaneously an electron and a neutrino with proper momenta is →

$$dn^2 = dn_e dn_\nu = \frac{(4\pi)^2 p^2 dp q^2 dq V^2}{h^6}$$

Fermi theory of β decay: shape of β spectrum (3)

- The e$^-$ and ν wave functions have the usual free-particle form (normalized to V) →

$$\Psi_e(\boldsymbol{r}) = \frac{1}{\sqrt{V}} \exp\left(i\boldsymbol{p}\boldsymbol{r}/\hbar\right) \text{ and } \Psi_\nu(\boldsymbol{r}) = \frac{1}{\sqrt{V}} \exp\left(i\boldsymbol{q}\boldsymbol{r}/\hbar\right)$$

- Consider an e$^-$ with typical kinetic energy T_e = 1 MeV → $E_e = m_e c^2 + T_e \approx 1.5$ MeV → as $E_e^2 = p^2c^2 + (m_e c^2)^2$ → $pc \approx 1.4$ MeV → $p/\hbar \approx 0.007$ fm^{-1} → over the nuclear volume: $pr \ll 1$ (creation at $r = 0$) →

$$\begin{aligned} \exp\left(i\boldsymbol{p}\boldsymbol{r}/\hbar\right) &= 1 + \frac{i\boldsymbol{p}\boldsymbol{r}}{\hbar} + \ldots \cong 1 \\ \exp\left(i\boldsymbol{q}\boldsymbol{r}/\hbar\right) &= 1 + \frac{i\boldsymbol{q}\boldsymbol{r}}{\hbar} + \ldots \cong 1 \end{aligned}$$

- This approximation is called **allowed approximation**

Fermi theory of β decay: shape of β spectrum (4)

- In the allowed approximation the partial decay rate can thus be written →

$$d\lambda = \frac{2\pi}{\hbar} G_F^2 |M_{fi}|^2 (4\pi)^2 \frac{p^2 dp q^2}{h^6} \frac{dq}{dE_f}$$

- $M_{fi} = \int \Psi_{f,N} O_\beta \Psi_{i,N}\, d\mathbf{r}$ is the **nuclear matrix element** → in the allowed approximation M_{fi} is assumed to be independent of p
- The final energy $E_f = E_e + E_\nu = E_e + qc$ → $dq/dE_f = 1/c$ at fixed E_e
- If only the shape of the distribution is concerned → previous equation may be written →

$$N(p)dp = Cp^2 q^2 dp$$

Fermi theory of β decay: shape of β spectrum (5)

- With Q the decay energy and neglecting the recoil energy →

$$q = \frac{Q - T_e}{c} = \frac{Q - \sqrt{p^2c^2 + m_e^2c^4} + m_ec^2}{c}$$

- The spectrum is given by →

$$\begin{aligned} N(p) &= \frac{C}{c^2}p^2(Q - T_e)^2 \\ &= \frac{C}{c^2}p^2\left[Q - \sqrt{p^2c^2 + m_e^2c^4} + m_ec^2\right]^2 \end{aligned}$$

- This function vanishes at $p = 0$ and when $T_e = Q$
- $p^2(Q - T_e)^2$ is called the **statistical factor** (associated with the density of final states)

Fermi theory of β decay: shape of β spectrum (6)

- For energy spectrum →

$$E_e^2 = p^2c^2 + m_e^2c^4 = (T_e + m_ec^2)^2 = T_e^2 + 2m_ec^2T_e + m_e^2c^4$$

$$\rightarrow \quad p^2c^2 = T_e^2 + 2m_ec^2T_e$$

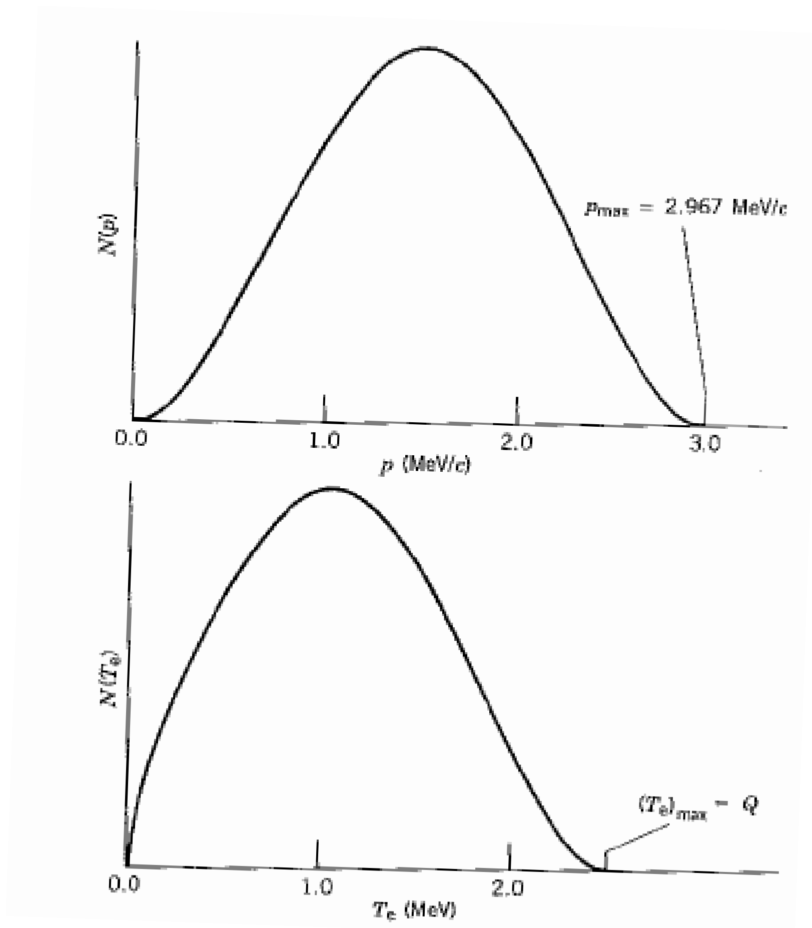
$$\rightarrow \quad c^2pdp = (T_e + m_ec^2)dT_e$$

$$\rightarrow \quad N(T_e) = \frac{C}{c^5}(T_e^2 + 2m_ec^2T_e)^{1/2}(Q - T_e)^2(T_e + m_ec^2)$$

- This function vanishes at $T_e = 0$ and at $T_e = Q$

Fermi theory of β decay: shape of β spectrum (7)

- Expected momentum and energy distribution For Q = 2.5 MeV



Shape of β spectrum: difference due to Fermi function (1)

- A difference arises from Coulomb interaction between β and daughter nucleus → simple point of view → Coulomb repulsion (→ acceleration) for β^+ (giving fewer low-energy positrons) and Coulomb attraction for β^- (giving more low-energy electrons)
- More correctly → due to Coulomb potential → modification of the wave function of the electron/positron → correction factor to be introduced in the equation: the **Fermi function** $F(Z',p)$ or $F(Z',T_e)$ with Z' the atomic number of the daughter nucleus → non-relativistic ($Q \ll m_e c^2$) quantum calculations give for point nucleus →

$$F(Z',p) = \frac{|\Psi_e(0)|^2}{|\Psi_{e,free}(0)|^2} \Rightarrow F(Z',p) = \frac{2\pi\eta}{\exp(2\pi\eta) - 1}$$

- η is the Sommerfeld parameter (with $Z' = Z + 1$ for β^- emission and $Z' = Z - 1$ for β^+ emission) →

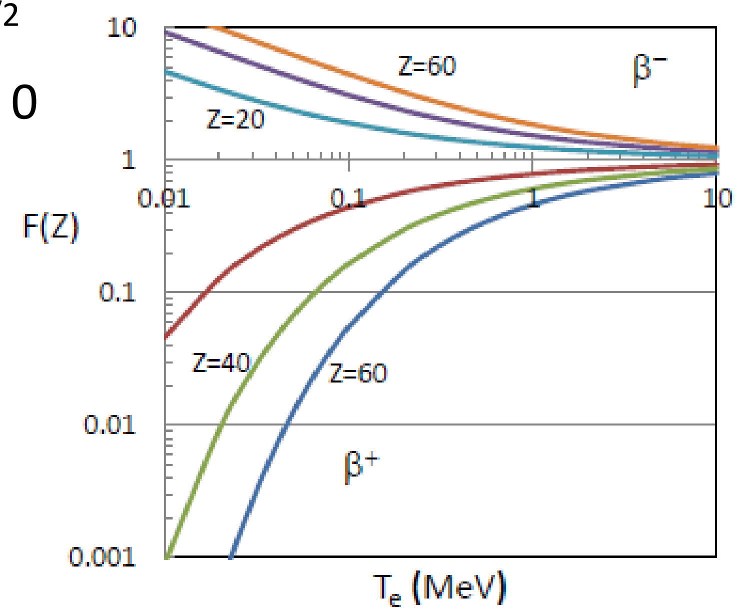
$$\eta = \mp \frac{\alpha(Z \pm 1)c}{v_e}$$

Shape of β spectrum: difference due to Fermi function (2)

- $F(Z',p)$ is always > 0 , strongly dependent on Z
- β^- : $\eta < 0$, $F(Z',p) > 1$ → attraction between nucleus and e^-
- β^+ : $\eta > 0$, $F(Z',p) < 1$ → repulsion between nucleus and e^+
- For large p (or T_e) → $\eta \rightarrow 0$ → F $\rightarrow$ 1
- For small p (or T_e) → large difference between $\eta < 0$ or > 0

→ for β^- : $\eta < 0$ → F $\rightarrow |2\pi\eta| \rightarrow T_e^{-1/2}$

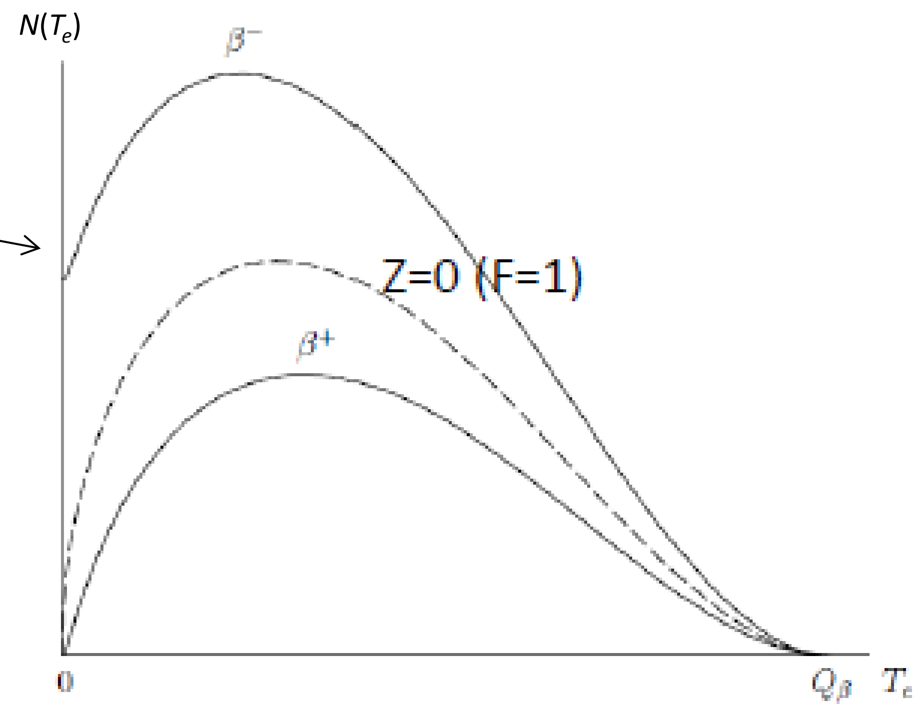
→ for β^+ : $\eta > 0$ → F $\rightarrow \exp(-2\pi\eta) \rightarrow 0$



Shape of β spectrum: difference due to Fermi function (3)

- Finally → difference between β^- and β^+ emission spectra →

non-zero for $T_e = 0$



Shape of β spectrum: difference due to Fermi function (4)

- When relativistic calculations are made (with a non-zero extension for the nucleus) →

$$F(Z', p) = 2(1+\gamma_0)(2p_e R/\hbar)^{-2(1-\gamma_0)} e^{-\pi\eta} \frac{|\Gamma(\gamma_0 + i\eta)|^2}{\Gamma(2\gamma_0+1)^2}$$

- *R* is the nucleus radius and the dimensionless parameter γ_0 is given by →

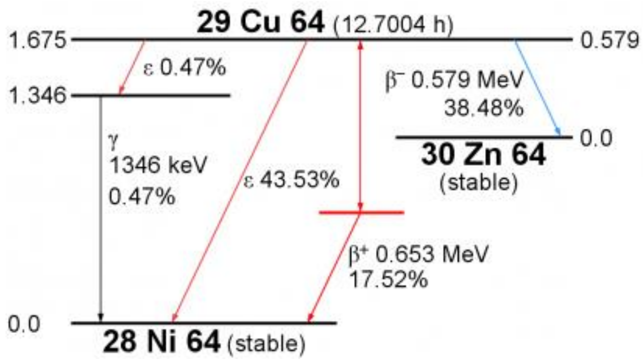
$$\gamma_0 = \sqrt{1-\alpha^2(Z\pm 1)^2}$$

- If $(\alpha Z')^2$ can be neglected → $\gamma_0 = 1$ and $F(Z',p)$ takes its non-relativistic form

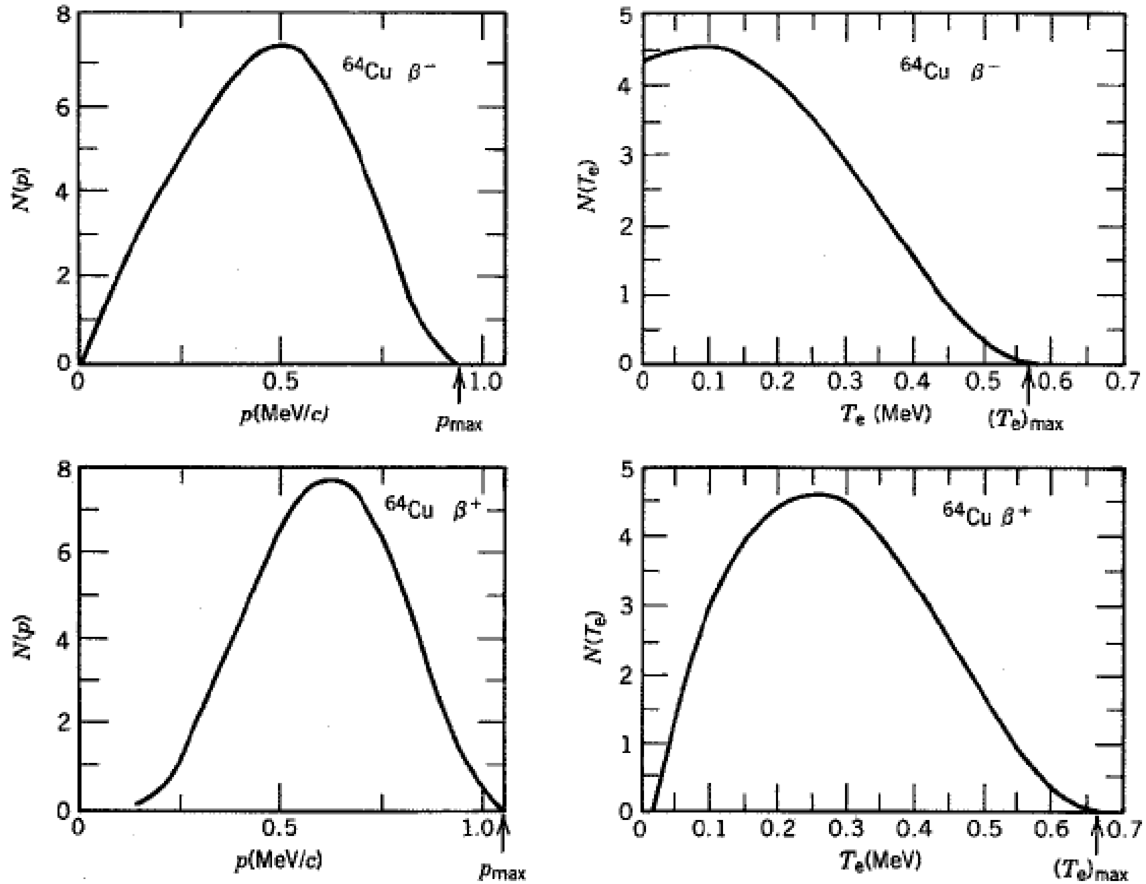
Shape of β spectrum: difference due to $|M_{fi}|^2$

- Up to now → M_{fi} was assumed to have no influence of the spectrum shape (allowed approximation) → often very good but in some cases very bad
- In some cases M_{fi} = 0 in the allowed approximation → no spectrum → in such cases the next terms of the plane wave expansion have to be considered → introduction of another momentum dependence → introduction of a $S(p,q)$ term (« **shape factor** »)
- Such cases are called (incorrectly) **forbidden** decays → not forbidden but they occur less likely than allowed decays → longer half-life
- The degree to which a transition is forbidden depends on how far the plane wave expansion to give first-forbidden decays is taken → first term beyond the 1 $\rightarrow$ first-forbidden decay → the next term $\rightarrow$ second-forbidden …
- Attention → angular momentum and parity selections rules restrict the kinds of decay

Shape of β spectrum: real spectra



$$N(p) \propto p^2(Q - T_e)^2 F(z', p)|M_{fi}|^2 S(p, q)$$



Shape of β spectrum: Fermi-Kurie plot (1)

- In the allowed approximation we write →

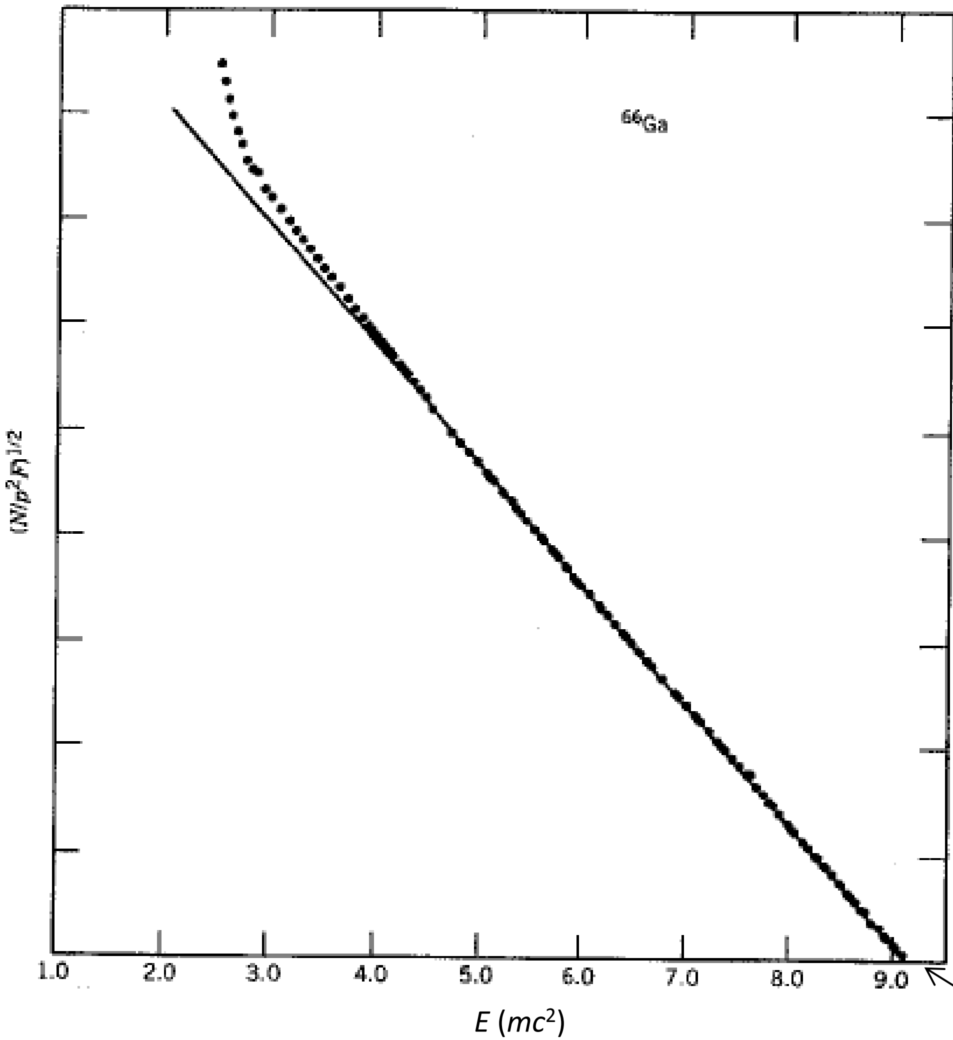
$$(Q - T_e) \propto \sqrt{\frac{N(p)}{p^2 F(z', p)}}$$

- Plotting this square root as a function of T_e → straight line intercepting the x-axis at Q → (Fermi-)Kurie plot
- If forbidden transition →

$$(Q - T_e) \propto \sqrt{\frac{N(p)}{p^2 F(z', p) S(p, q)}}$$

- For first forbidden case → $S(p,q) = p^2 + q^2$
- For second forbidden case → $S(p,q) = p^4 + 10/3\ p^2 q^2 + q^4$
- For third forbidden case → $S(p,q) = p^6 + 7\ p^4 q^2 + 7\ p^2 q^4 + q^6$

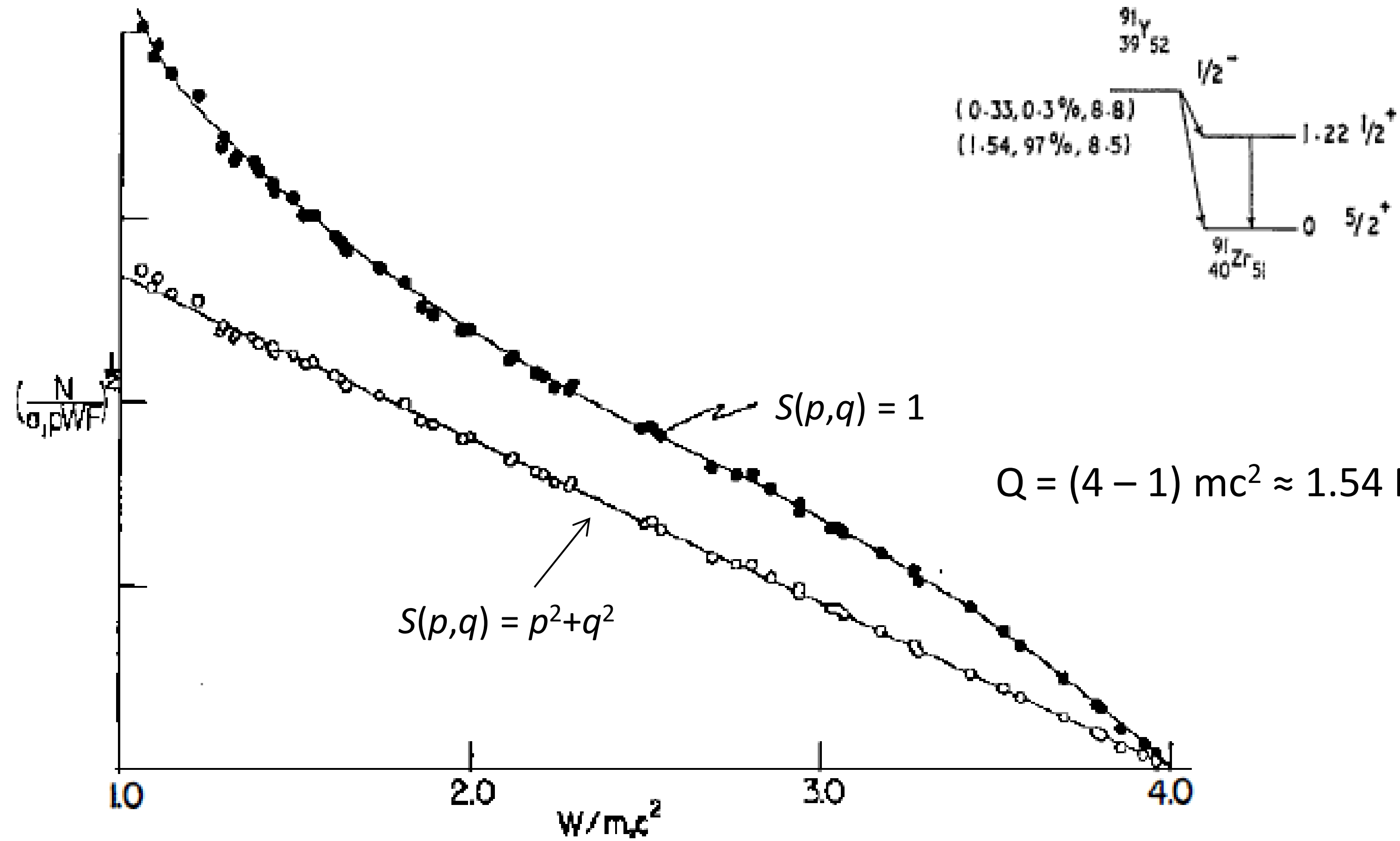
Shape of β spectrum: Fermi-Kurie plot (2)



Fermi-Kurie plot of allowed $0^+ \rightarrow 0^+$ β^+ decay of ^{66}Ga (deviation at low energies comes from electrons scattering within the source)

Q = (9.1 – 1) $mc^2 \approx$ 4.1 MeV

Shape of β spectrum: Fermi-Kurie plot (3)



$Q = (4 - 1)\ mc^2 \approx 1.54$ MeV

Uncorrected and corrected Fermi-Kurie plots
of first-forbidden $1/2^- \rightarrow 5/2^+$ β^- decay of ^{91}Y

Fermi theory of β decay: total decay rate (1)

- The integration of the partial decay rate gives →

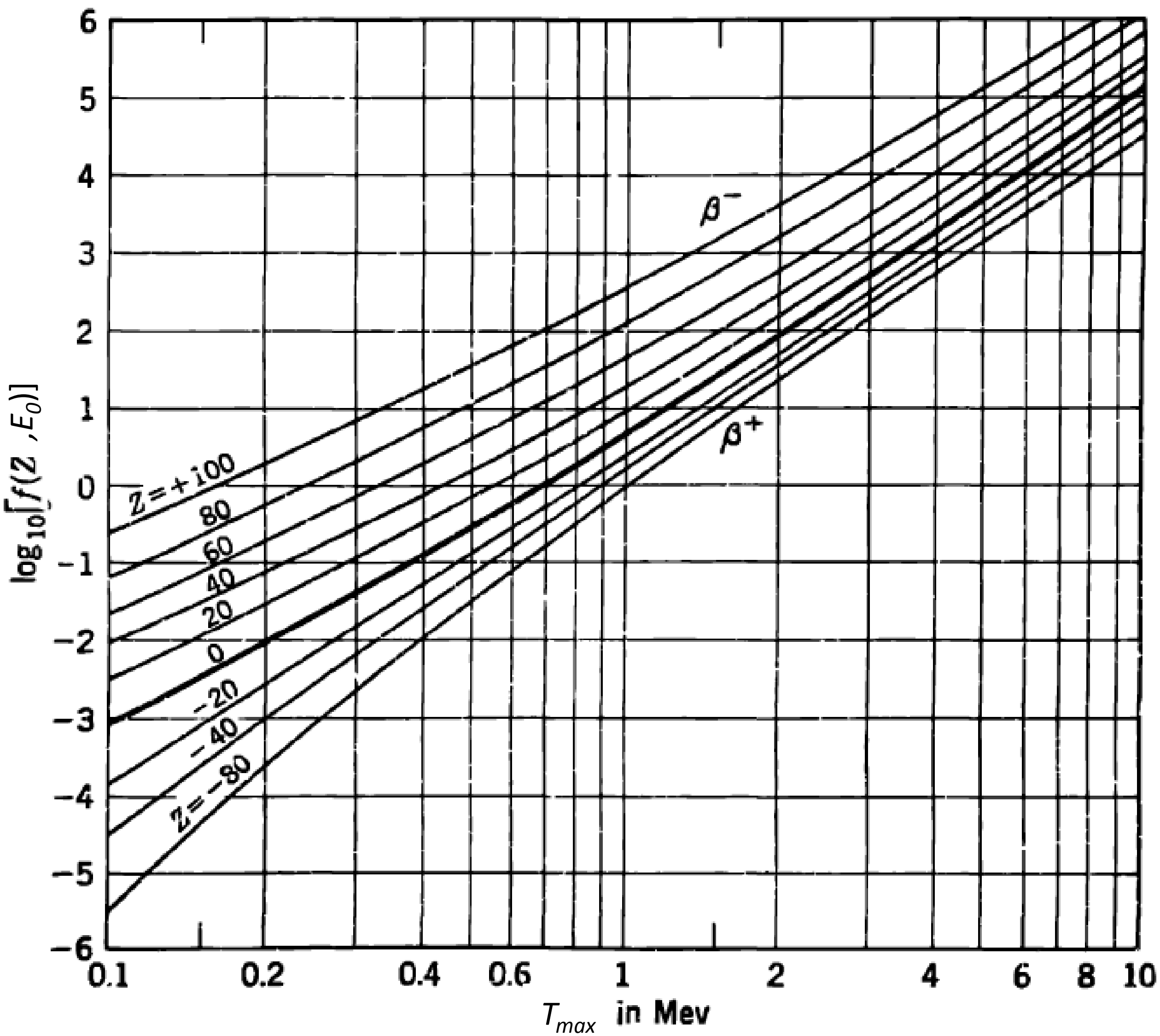
$$\lambda = \frac{G_F^2 |M_{fi}|^2}{2\pi^3 \hbar^7 c^3} \int_0^{p_{max}} F(Z', p) p^2 (Q - T_e)^2 dp$$

- We have a dependence on *Z'* and on the maximum electron total energy E_0 (since $cp_{max} = [E_0^2 - (m_e c^2)^2]^{1/2}$) → we represent it with the Fermi integral $f(Z', E_0)$ which is tabulated with the use of approximate forms for $F(Z',p)$ for values of *Z'* and E_0 →

$$f(Z', E_0) = \frac{1}{(m_e c)^3 (m_e c^2)^2} \int_0^{p_{max}} F(Z', p) p^2 (E_0 - E_e)^2 dp$$

To be dimensionless

Fermi theory of β decay: total decay rate (2)



Fermi integral with $Z < 0$ for β^+ and $Z > 0$ for β^-

Fermi theory of β decay: total decay rate (3)

- With λ = ln2/$T_{1/2}$ →

$$fT_{1/2} = \ln 2 \frac{2\pi^3\hbar^7}{G_F^2 m_e^5 c^4 |M_{fi}|^2}$$

- $fT_{1/2}$ is called the comparative half-life or *ft* value ($T_{1/2}$ is always in s)
- *ft* gives a way to compare the decay probabilities
- We can also write →

$$ft = f(Q)\frac{T_{1/2}}{BR} = \frac{K}{G_F^2 |M_{fi}|^2}$$

- Measure gives access to Q (and thus to f(Q)), $T_{1/2}$ and branching ratio (BR) → to the left-hand side of the equation
- If M_{fi} is known → G_F can be calculated → determination of M_{fi} ?

Selections rules: Fermi decay (1)

- If exp(i$\boldsymbol{pr}$/ħ) = 1 and exp(i$\boldsymbol{qr}$/ħ) = 1 → allowed approximation → electron and neutrino are created at r = 0 → they cannot carry orbital angular momentum (ℓ = 0) → the only change in the nucleus angular momentum results from their spin (each of them s = 1/2)
- These two spins can be parallel (S = 1) or antiparallel (S = 0)
- If S = 0 → transition called **Fermi** (F) decay → in that case the O_β (in $M_{fi} = \int \Psi_{f,N} O_\beta \Psi_{i,N}$) takes the form →

$$O_\beta = O_F = \sum_{j=1}^{A} t_{j\pm} = T_\pm$$

- Where t_{j+} and t_{j-} are respectively the raising and lowering isospin operators for nucleon j → they transform proton/neutron j into neutron/proton → t_{j+}/t_{j-} corresponds to β^+/β^-
- $T_\pm$ correspond to raising and lowering operators of the total isospin

Selections rules: Fermi decay (2)

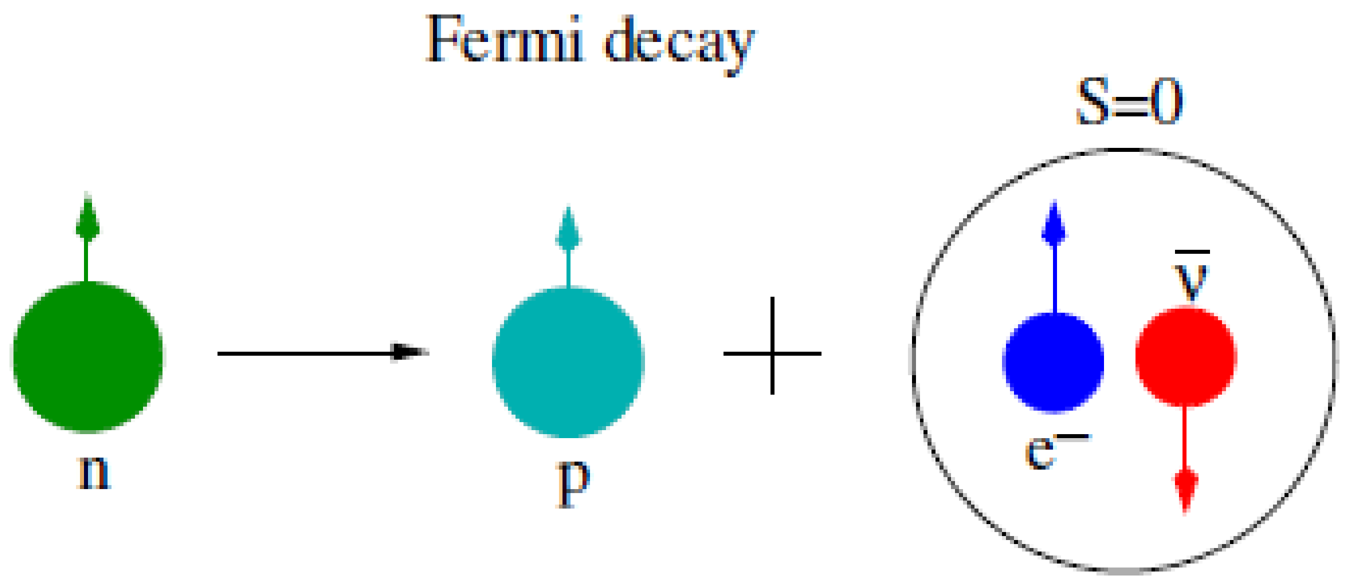
- For S = 0 → there is no change in the total angular momentum → $J_i = J_f$ (Wigner-Eckart theorem for an irreducible tensor operator of rank 0 in position and spin space)
- As electron and neutrino carries no orbital angular momentum (ℓ = 0) → the parities of initial and final states must be identical since the parity associated with orbital angular momentum is $(-1)^\ell$ → $\pi_i = \pi_f$
- Finally due to the property of raising and lowering operators →

$$\langle T' M'_T | T_\pm | T M_T \rangle = \sqrt{(T \mp M_T)(T \pm M_T + 1)}\delta_{T'T}\delta_{M'_T M_{T\pm 1}}$$

→ $T_i = T_f$

$$J_i = J_f, \;\; \pi_i = \pi_f, \;\; T_i = T_f$$

Selections rules: Fermi decay (3)



The spin of the baryons to point in the same direction before and after the decay

Selections rules: Gamow-Teller decay (1)

- If $S = 1 \rightarrow \rightarrow$ transition called **Gamow-Teller** (GT) decay → in that case the O_β takes the form →

$$O_\beta = O_{GT} = \sum_{j=1}^{A} S_j t_{j\pm}$$

- S_j is the spin operator of nucleon j (irreducible tensor operator of rank 1 in position and spin space and in isospin space)
- For $S = 1$ → electron and neutrino carry a total angular momentum of 1 unit → J_i and J_f must be coupled through a vector of length 1 → $\boldsymbol{J_i = J_f + 1}$ → only possible if $\Delta J = 0$ or 1 except for $J_i = J_f = 0$ in which case only Fermi transition can contribute (Wigner-Eckart theorem for an irreducible tensor operator of rank 1 in position and spin space)

$$|J_i - J_f| \leq 1 \leq J_i + J_f$$

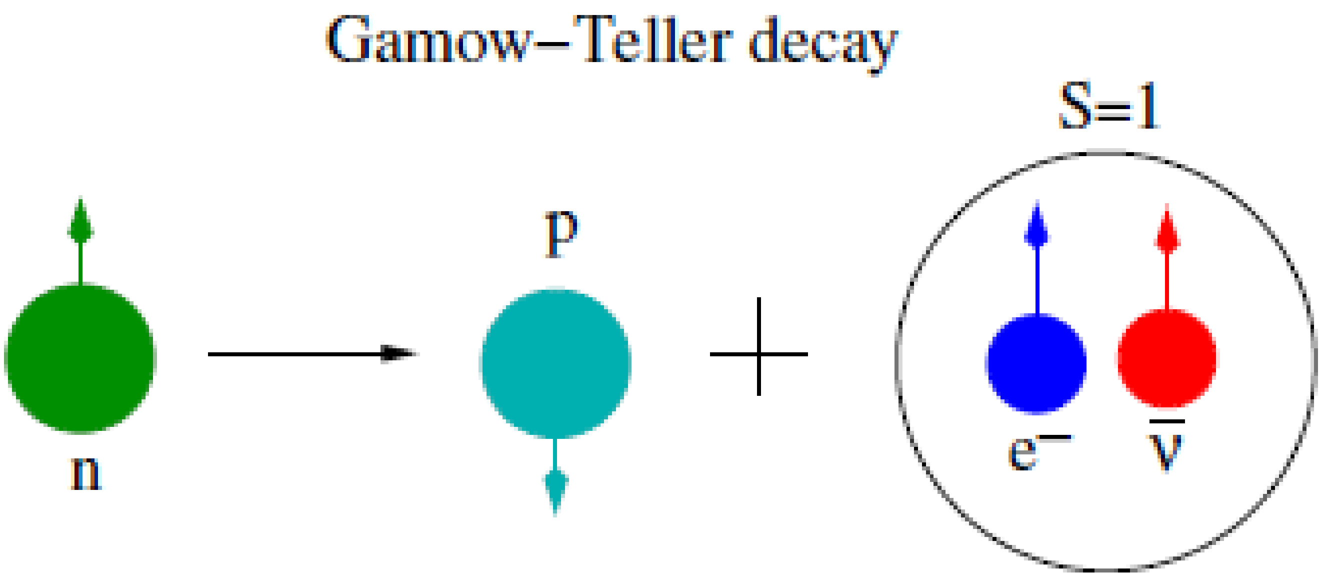
Selections rules: Gamow-Teller decay (2)

- As for Fermi decay → the parities of initial and final states must be identical → $\pi_i = \pi_f$
- And finally from Wigner-Eckart theorem for an irreducible tensor operator of rank 1 in isospin space →

$$|T_i - T_f| \leq 1 \leq T_i + T_f$$

$$|J_i - J_f| \leq 1 \leq J_i + J_f, \quad \pi_i = \pi_f, \quad |T_i - T_f| \leq 1 \leq T_i + T_f$$

Selections rules: Gamow-Teller decay (3)



The spin of the baryons to point in the opposite direction before and after the decay → the GT decay mode is sometimes called the spin-flip mode

Selections rules: Mixed F + GT decay

- When the rules:

$$J_i = J_f \neq 0, \;\; \pi_i = \pi_f, \;\; T_i = T_f \neq 0$$

are satisfied → the 2 types of decay are simultaneously possible

- We have in this case a mixed Fermi-Gamow-Teller decay in which the exact proportion of F and GT are determined by the initial and final nuclear wave functions

Selections rules: examples of allowed decays

- Pure Fermi decays: $^{14}O \rightarrow {}^{14}N^*$ ($0^+ \rightarrow 0^+$), $^{34}Cl \rightarrow {}^{34}S$ ($0^+ \rightarrow 0^+$), $^{10}C \rightarrow {}^{10}B^*$ ($0^+ \rightarrow 0^+$) ($0^+ \rightarrow 0^+$ cannot decay via GT transition which must carry 1 unit of angular momentum)

- Pure GT decays: $^{6}He \rightarrow {}^{6}Li$ ($0^+ \rightarrow 1^+$), $^{13}B \rightarrow {}^{13}C$ ($3/2^- \rightarrow 1/2^-$), $^{230}Pa \rightarrow {}^{230}Th^*$ ($2^- \rightarrow 3^-$), $^{111}Sn \rightarrow {}^{111}In$ ($7/2^+ \rightarrow 9/2^+$)

- Mixed F + GT (both F and Gt rules are satisfied): $n \rightarrow p$ ($1/2^+ \rightarrow 1/2^+$) (82% GT and 18% F), $^{3}H \rightarrow {}^{3}He$ ($1/2^+ \rightarrow 1/2^+$) (81% GT and 19% F), $^{13}N \rightarrow {}^{13}C$ ($1/2^- \rightarrow 1/2^-$) (24% GT and 76% F)

Selections rules: forbidden decays (1)

- The designation «forbidden decay » is a misnomer → these decays are less probable than allowed decays but if the allowed M_{fi} matrix vanishes → only forbidden decays occur in that case
- When previous rules are not fulfilled (as a change in parity) → forbidden decays occur → conditions of the allowed approximation are not fulfilled →

$$\begin{aligned}\exp\left(i\boldsymbol{p}\boldsymbol{r}/\hbar\right) &= 1+\frac{i\boldsymbol{p}\boldsymbol{r}}{\hbar}+\frac{1}{2}(\frac{i\boldsymbol{p}\boldsymbol{r}}{\hbar})^2+\cdots\neq 1\\ \exp\left(i\boldsymbol{q}\boldsymbol{r}/\hbar\right) &= 1+\frac{i\boldsymbol{q}\boldsymbol{r}}{\hbar}+\frac{1}{2}(\frac{i\boldsymbol{q}\boldsymbol{r}}{\hbar})^2+\cdots\neq 1\end{aligned}$$

Selections rules: forbidden decays (2)

- If we consider again an e^- with typical kinetic energy T_e = 1 MeV and a a typical nuclear radius of $R \simeq 6$ fm → $pR/\hbar \approx 0.04$ → transition for ℓ = 1 has an intensity 0.04 smaller than ℓ = 0 and transitions for ℓ > 1 are even more unlikely
- If the first term in the development given a non-zero matrix element corresponds to ℓ = 1 → **first-forbidden** decay
- If the first term in the development given a non-zero matrix element corresponds to ℓ = 2 → **second-forbidden** decay
- If the first term in the development given a non-zero matrix element corresponds to ℓ = 3 → **third-forbidden** decay
- …

Selections rules: first-forbidden decay

- As the allowed decays → they can be on Fermi type (electron an neutrino spins opposite → S = 0) and Gamow-Teller type (parallel spins → S = 1)
- The coupling of S = 0 with ℓ = 1 for the Fermi decay gives total angular momentum of 1 unit carried by the beta decay → ΔJ = 0 or 1 (but not 0 $\rightarrow$ 0)
- Coupling S = 1 with ℓ = 1 for the Gamow-Teller decay gives 0, 1 or 2 units of total angular momentum → ΔJ = 0, 1 or 2
- Selections rules for first-forbidden decay are →

$$\Delta J = 0, 1, 2 \quad \text{and} \quad \pi_i \neq \pi_f$$

Selections rules: examples of : first-forbidden decays

- $^{17}N \rightarrow {}^{17}O$ ($1/2^- \rightarrow 5/2^+$)
- $^{76}Br \rightarrow {}^{76}Se$ ($1^- \rightarrow 0^+$)
- $^{122}Sb \rightarrow {}^{122}Sn^*$ ($2^- \rightarrow 2^+$)

Selections rules: second-forbidden decay

- When $S = 0$ or 1 is coupled with $\ell = 2$ → we can in principle have $\Delta J = 0, 1, 2$ or 3 but the $\Delta J = 0, 1$ cases fall within the selection rule for allowed decays (with no parity change) → the contribution of the second-forbidden term to those decays is negligible
- Excepting these cases the selections rules are →

$$\Delta J = 2, 3 \quad \text{and} \quad \pi_i = \pi_f$$

- Examples of second-forbidden decays are ^{22}Na $\rightarrow$ ^{22}Ne ($3^+ \rightarrow 0^+$) or ^{137}Cs $\rightarrow$ ^{137}Ba ($7/2^+ \rightarrow 3/2^+$)

Selections rules: other forbidden decays

- Third-forbidden decay ($\ell = 3$) → considering selections rules not satisfied by the first-forbidden decay →

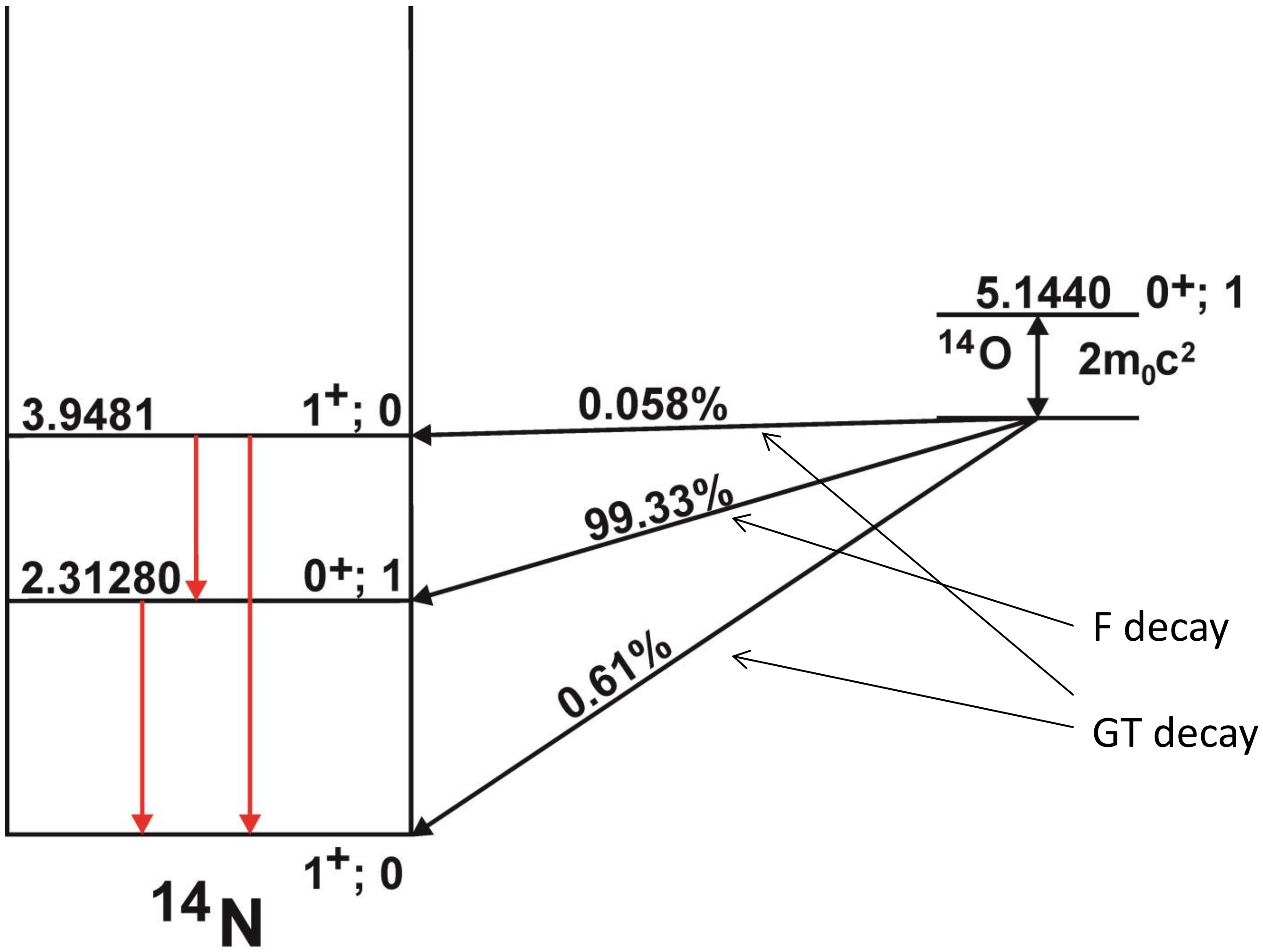
$$\Delta J = 3, 4 \quad \text{and} \quad \pi_i \neq \pi_f$$

- Examples of third-forbidden decay → ^{87}Rb $\rightarrow$ ^{87}Sr ($3/2^- \rightarrow 9/2^+$) or ^{40}K $\rightarrow$ ^{40}Ca ($4^- \rightarrow 0^+$)
- In very unusual circumstances → fourth-forbidden decay ($\ell = 4$) →

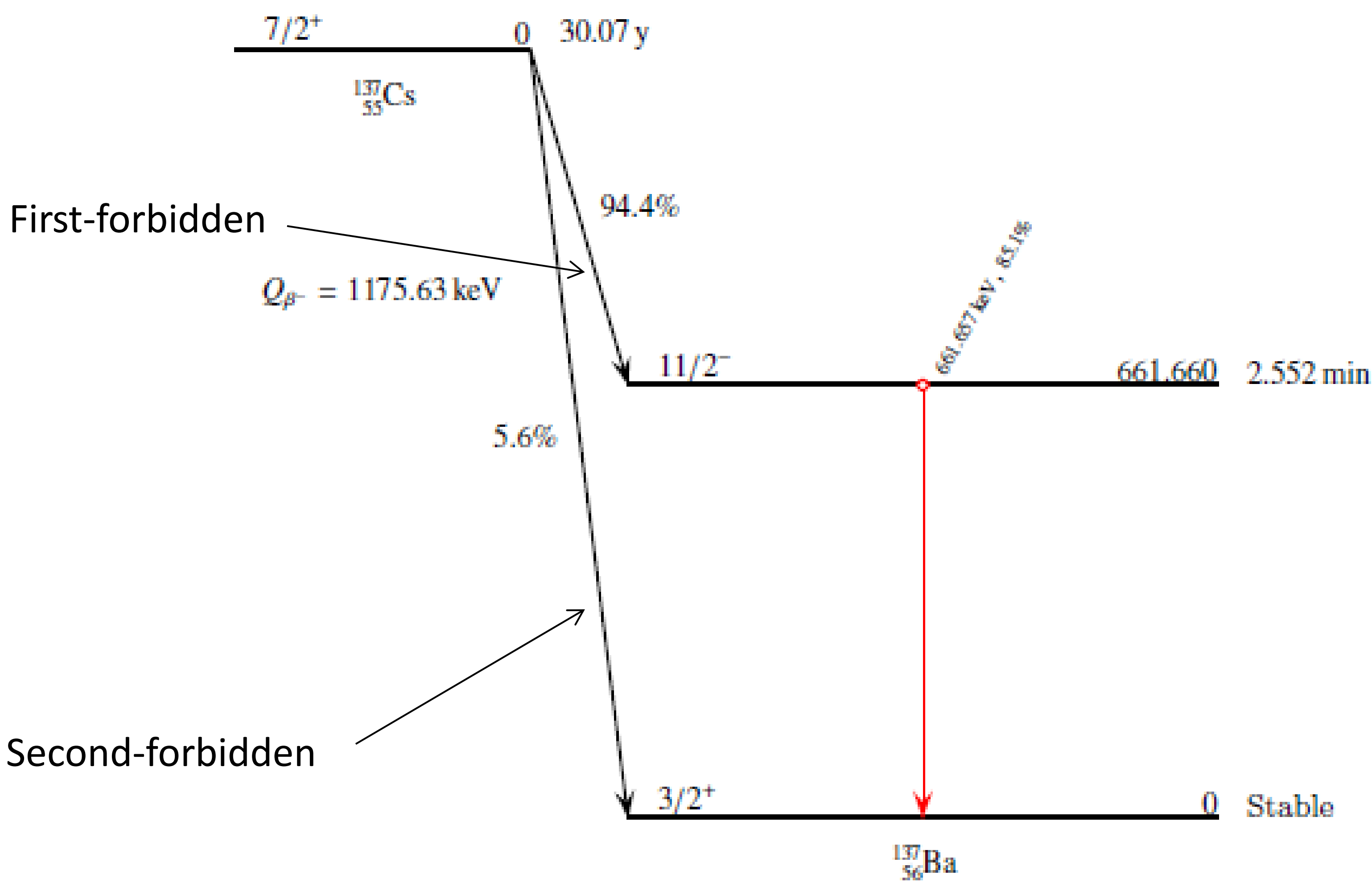
$$\Delta J = 4, 5 \quad \text{and} \quad \pi_i = \pi_f$$

- Example of fourth-forbidden decay → ^{115}In $\rightarrow$ ^{115}Sn ($9/2^+ \rightarrow 1/2^+$)

Selections rules: example of ^{14}O



Selections rules: example of ^{137}Cs



Comparative half-lives

- β decay half-lives varies from the order of ms to about 10^{16} years → ft values from about 10^3 to 10^{20} s → one of the reasons is the difficulty to create β particle and neutrino with $\ell > 0$
- The decays with shortest half-lives ($\log ft \simeq$ 3-4) are called superallowed decays (particular case of allowed decays)
- Superallowed decays are pure Fermi transitions → they occur between analog isobaric states (AIS) i.e. states having the same isospin and configuration → the matrix element M_{fi} reaches its maximum values because the spatial component of the initial and final wave functions are strongly similar
- The energy difference between mother and daughter AIS is due to the difference of Coulomb energies between the 2 nuclei → this kind of transition can only occur for neutron-deficient nuclei → β^+ or electron capture

Comparative half-lives: superallowed decays (1)

- A particular case of superallowed decay has 0^+ and $T = 1$ initial and final states (with $M_T = -1$ and $M'_T = 0$) → for this case M_{fi} can be calculated easily →

$$M_{fi} = \langle T' M'_T | \, T_\pm \, | T M_T \rangle = \sqrt{(T \mp M_T)(T \pm M_T + 1)} \delta_{T'T} \delta_{M'_T M_{T\pm 1}}$$

$$M_{fi} = \langle T = 1, M_T = 0 | \, T_+ \, | T = 1, M_T = -1 \rangle = \sqrt{2}$$

- The ft values are identical for all $0^+ \rightarrow 0^+$ transitions $ft_{\text{theo}} = 3073$ s
- Moreover this transition gives the theoretical value of G_F → large success of the Fermi theory

Comparative half-lives: superallowed decays (2)

Decay	ft (s)
$^{10}C \rightarrow {}^{10}B$	3100 ± 31
$^{14}O \rightarrow {}^{14}N$	3092 ± 4
$^{18}Ne \rightarrow {}^{18}F$	3084 ± 76
$^{22}Mg \rightarrow {}^{22}Na$	3014 ± 78
$^{26}Al \rightarrow {}^{26}Mg$	3081 ± 4
$^{26}Si \rightarrow {}^{26}Al$	3052 ± 51
$^{30}S \rightarrow {}^{30}P$	3120 ± 82
$^{34}Cl \rightarrow {}^{34}S$	3087 ± 9
$^{34}Ar \rightarrow {}^{34}Cl$	3101 ± 20
$^{38}K \rightarrow {}^{38}Ar$	3102 ± 8
$^{38}Ca \rightarrow {}^{38}K$	3145 ± 138
$^{42}Sc \rightarrow {}^{42}Ca$	3091 ± 7
$^{42}Ti \rightarrow {}^{42}Sc$	3275 ± 1039
$^{46}V \rightarrow {}^{46}Ti$	3082 ± 13
$^{46}Cr \rightarrow {}^{46}V$	2834 ± 657
$^{50}Mn \rightarrow {}^{50}Cr$	3086 ± 8
$^{54}Co \rightarrow {}^{54}Fe$	3091 ± 5
$^{62}Ga \rightarrow {}^{62}Zn$	2549 ± 1280

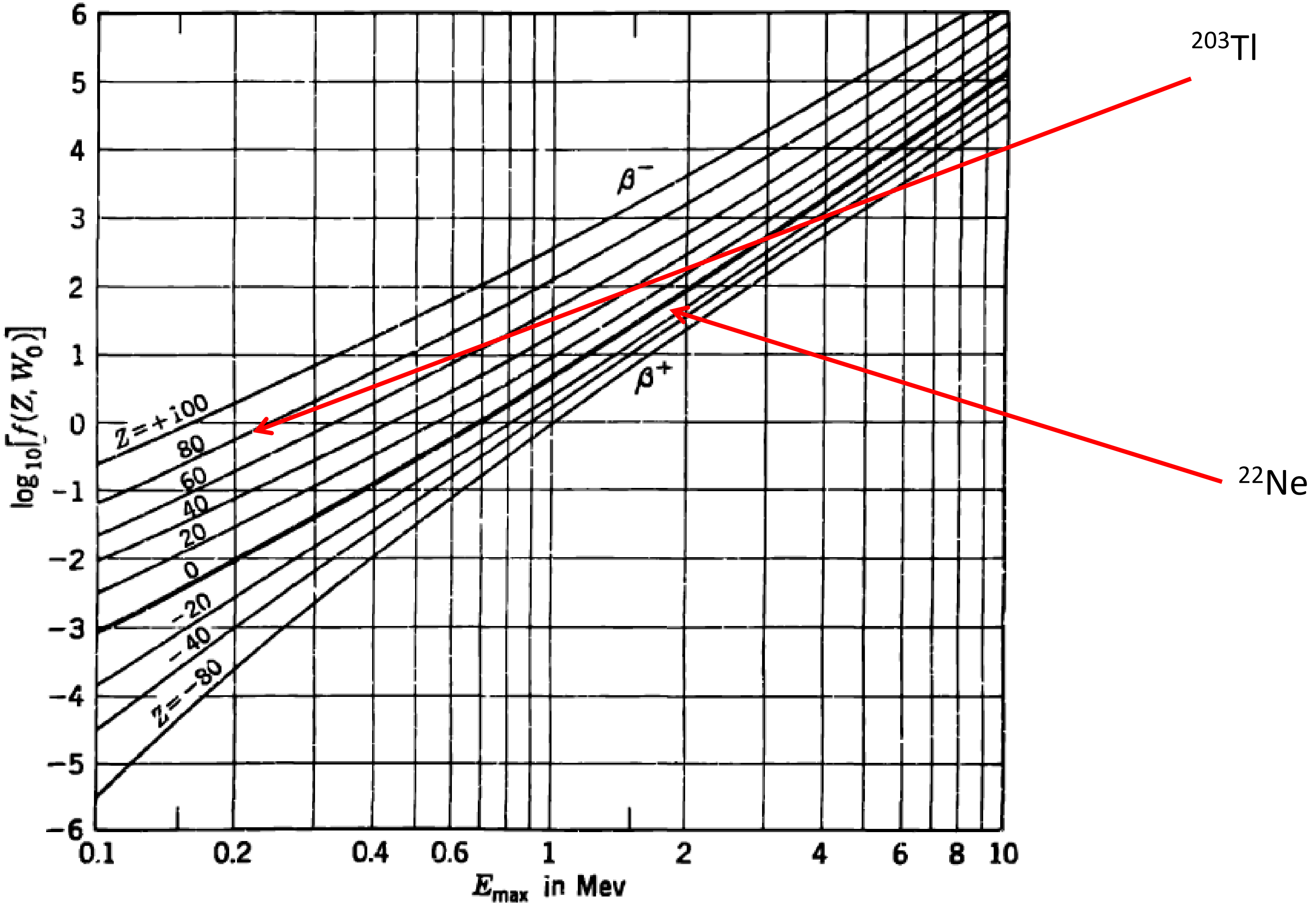
Comparative half-lives: calculation of *ft* (1)

- As written above →

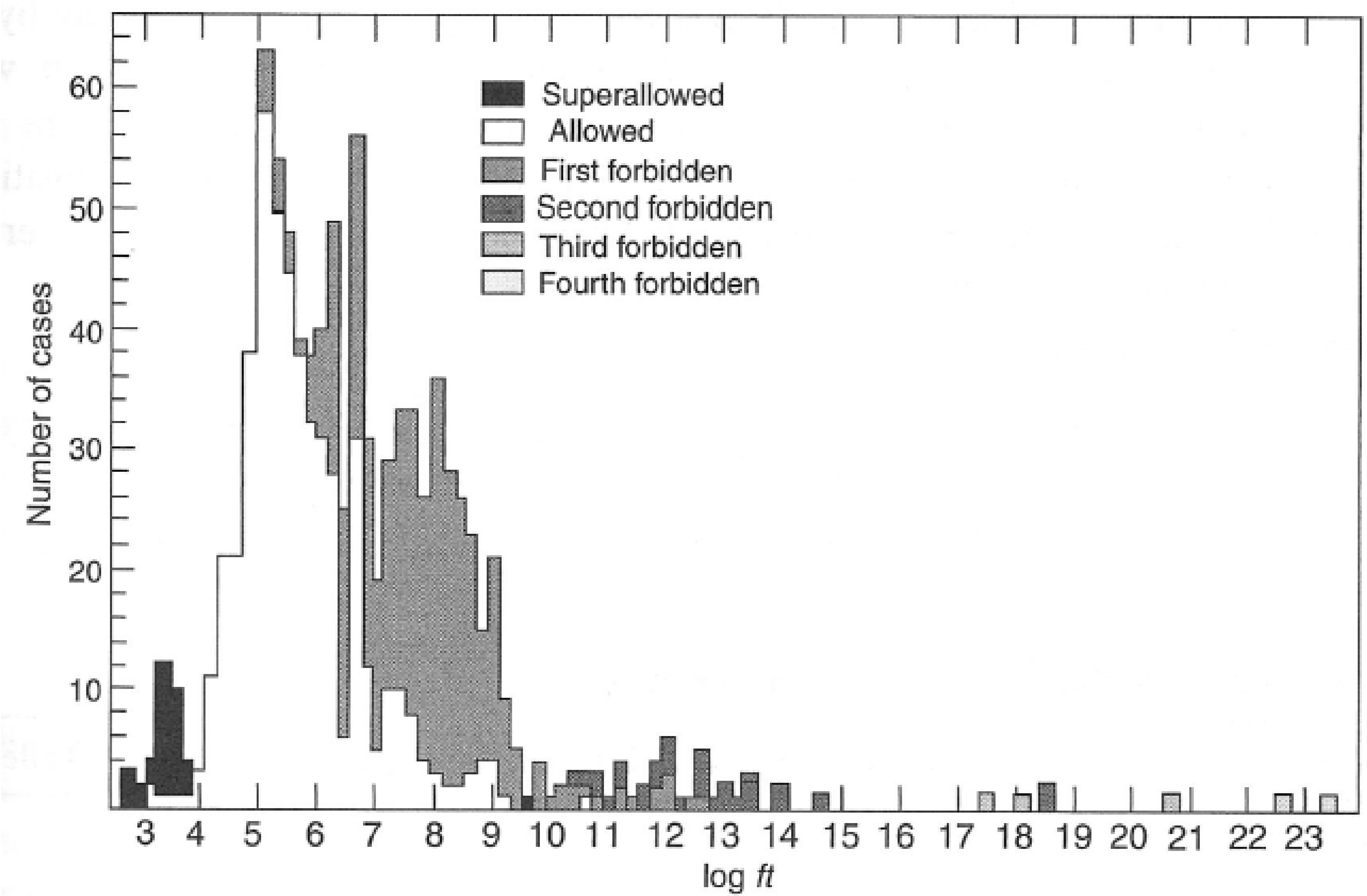
$$ft = f(Q)\frac{T_{1/2}}{BR}$$

- Example 1 → β^- of ^{203}Hg $\rightarrow$ ^{203}Tl (first-forbidden transition)
  - $T_{1/2}$ = 46.8 days = 4043520 s → $\log_{10} T_{1/2}$ = 6.6
  - Q = 0.491 MeV but 100% of the decay goes to the 279 keV excited state of Tl → T_{max} of β = 0.212 MeV → from Fermi integral figure → $\log_{10} f$ = -0.1
  - Finally $\log_{10} ft = \log_{10} f + \log_{10} T_{1/2}$ = -0.1 + 6.6 = 6.5
- Example 2 → β^+ of ^{22}Na $\rightarrow$ ^{22}Ne (second-forbidden transition)
  - $T_{1/2}$ = 2.6 years → $\log_{10} T_{1/2}$ = 7.9
  - Q = 1.8 MeV → from Fermi integral figure → $\log_{10} f$ = 1.6
  - The branching ratio to the ground state is 0.06% → $\log_{10} BR$ = -3.2
  - Finally $\log_{10} ft = \log_{10} f + \log_{10} T_{1/2} - \log_{10} BR$ = 7.9 + 1.6 + 3.2 = 12.7

Comparative half-lives: calculation of *ft* (2)



Comparative half-lives: measurement of *ft*



Comparative half-lives: summary

Type	$\log(ft)$	L	$\Delta\pi$	$\Delta\vec{J}$ $\vec{S}=\vec{0}$ Fermi	$\Delta\vec{J}$ $\vec{S}=\vec{1}$ Gam-Tel
super-allowed	2.9-3.7	0	+	0	0
allowed	4.4-6.0	0	+	0	0,1
first forbidden	6-10	1	-	0,1	0,1,2
second forbidden	10-13	2	+	1,2	1,2,3
third forbidden	> 15	3	-	2,3	2,3,4

Electron capture decay (1)

- In electron capture decay →

$$ {}^{A}_{Z}X_{N} + e^{-} \xrightarrow{\epsilon} {}^{A}_{Z-1}Y^{*}_{N+1} + \nu_{e} $$

- Use of the Fermi Golden Rule as previously but now a bound electron is involved in the transition (large probability to have a K (1s) electron) → 2 main ≠ →
  - the phase-space volume is determined entirely by the energy of the emitted neutrino because the electron is in a definite quantum state before its capture
  - The wave function of the electron at the origin is given by

$$ \varphi_K(0) = \frac{1}{\sqrt{\pi}} \left(\frac{Z m_e e^2}{4\pi\epsilon_0 \hbar^2} \right)^{3/2} $$

Electron capture decay (2)

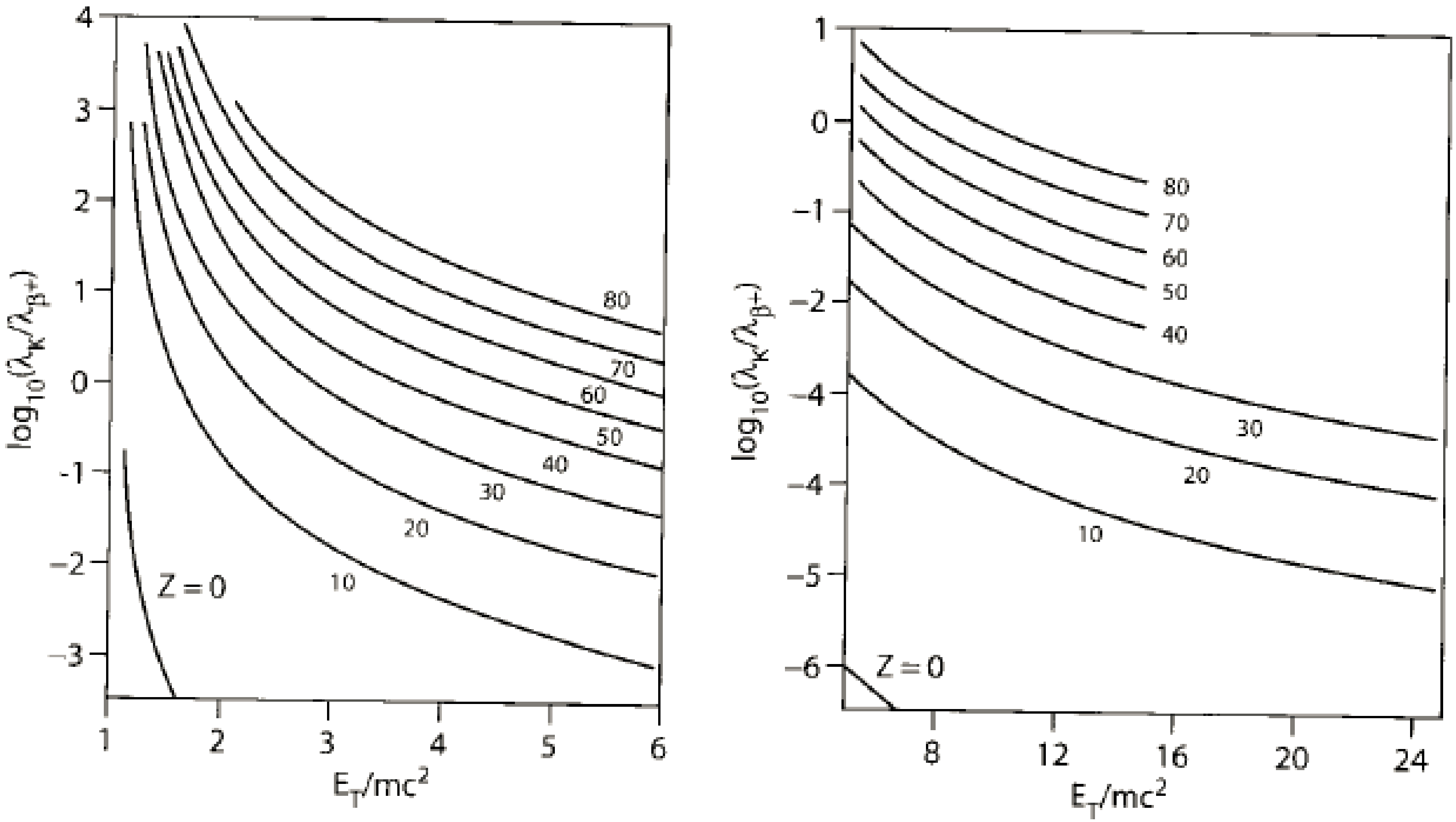
- The decay probability becomes →

$$\lambda_\epsilon = \frac{G_F^2 |M_{fi}|^2 T_\nu^2}{2\pi^2 c^3 \hbar^3} |\varphi_K(0)|^2$$
$$\propto G_F^2 Z^3 |M_{fi}|^2 T_\nu^2$$

- $T_\nu = Q_\epsilon$ is the energy of the neutrino (the recoil energy is neglected)
- It is possible to compare the transition probabilities for β^+ and ϵ decay for $Q_\epsilon > 2m_e c^2$ (for $Q_\epsilon < 2m_e c^2$ only electron capture is possible) →

$$\frac{\lambda_\epsilon}{\lambda_{\beta+}} \propto \frac{Z^3 T_\nu^2}{f(Z', E_0)}$$

Electron capture decay (3)



E_T is the total energy available, Z corresponds to the target

Other processes: double-β decay (1)

- The double-β ($\beta\beta$) decay corresponds to the transition in which two protons are simultaneously transformed into two neutrons, or vice versa, inside an atomic nucleus → emission of 2 β particles (e^- or e^+)

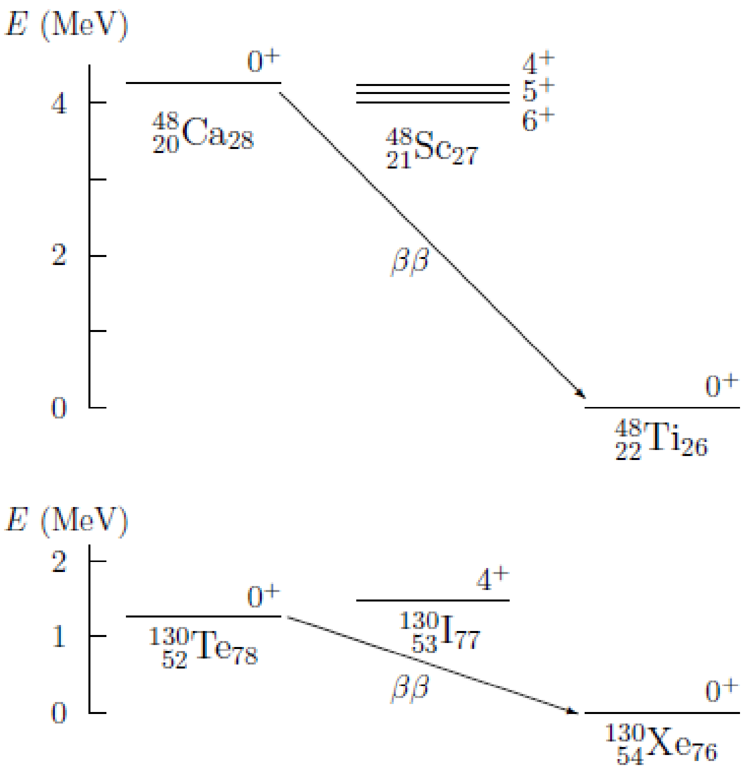
$$ {}^{A}_{Z}X_{N} \quad \xrightarrow{\beta^-\beta^-} \quad {}^{A}_{Z+2}Y_{N-2} + 2\beta^- + 2\bar{\nu}_e $$

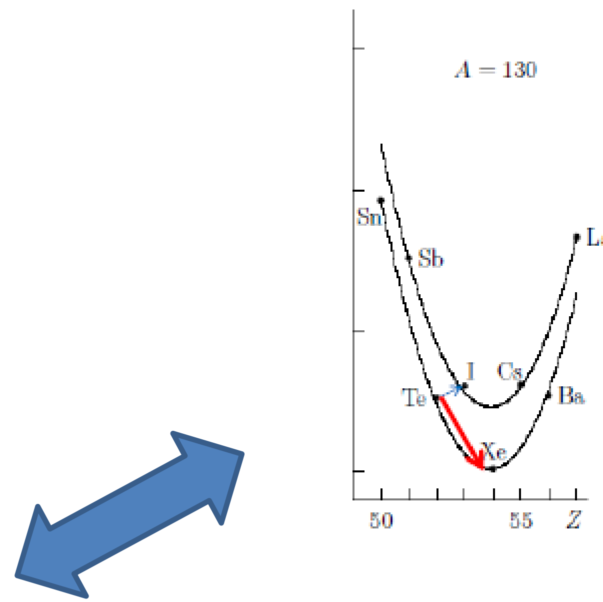
$$ {}^{A}_{Z}X_{N} \quad \xrightarrow{\beta^+\beta^+} \quad {}^{A}_{Z-2}Y_{N+2} + 2\beta^+ + 2\nu_e $$

- Direct process between 2 successive even-even nuclei (which are more stable due to spin-coupling) for which the β transitions with an intermediate odd-odd nucleus is either impossible either very unlikely
- Example 1: ^{48}Ca $\rightarrow$ ^{48}Ti ($0^+ \rightarrow 0^+$) → the *Q* value for the β decay to ^{48}Ti is 0.281 MeV but the only accessible states are 4^+, 5^+ and 6^+ → requiring fourth- or sixth forbidden decays → enormous mean life time → the $\beta\beta$-decay is rare but more probable: 25% β^- and 75% $\beta^-\beta^-$ → $T_{1/2} = 6.4 \times 10^{19}$ years (remark: double magic nucleus: 20 and 28)

Other processes: double-β decay (2)

- Example 2: ^{130}Te $\rightarrow$ ^{130}Xe ($0^+ \rightarrow 0^+$) → ^{130}Te cannot decay to ^{130}I because of negative *Q* value → $\beta\beta$-decay to ^{130}Xe ($T_{1/2}$ = 0.8 $\times$ 10^{21} years!)





Other processes: double-β decay (3)

- One of the goals of this kind of experiments is to observe a decay without neutrino →

$$ {}^{A}_{Z}X_{N} \xrightarrow{\beta^{-}\beta^{-}0\nu} {}^{A}_{Z+2}Y_{N-2} + 2\beta^{-} $$

$$ {}^{A}_{Z}X_{N} \xrightarrow{\beta^{+}\beta^{+}0\nu} {}^{A}_{Z-2}Y_{N+2} + 2\beta^{+} $$

- In this process the two neutrinos annihilate each other or equivalently a nucleon absorbs the neutrino emitted by another nucleon
- Not observed yet

Other processes: neutrino capture

- Processes →

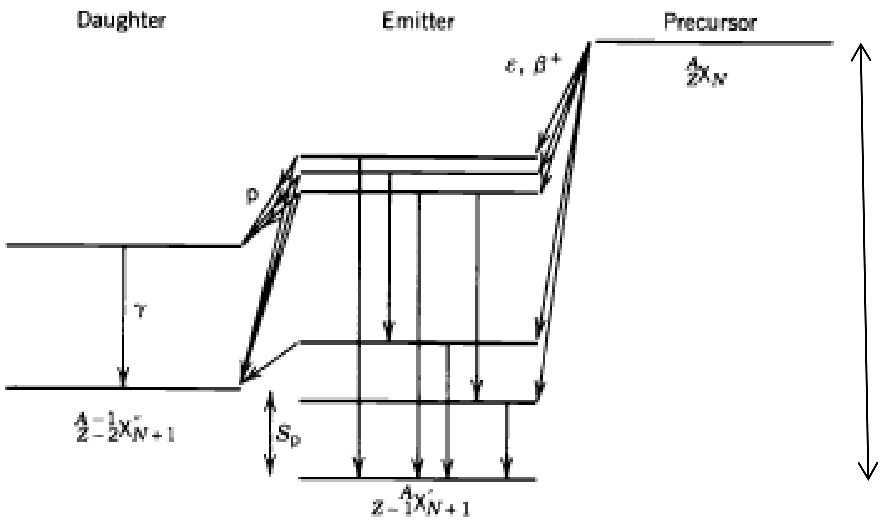
$$
\begin{aligned}
{}^{A}_{Z}X_N + \nu \quad &\rightarrow \quad {}^{A}_{Z+1}Y_{N-1} + e^- \\
{}^{A}_{Z}X_N + \nu \quad &\rightarrow \quad {}^{A}_{Z-1}Y_{N+1} + e^+
\end{aligned}
$$

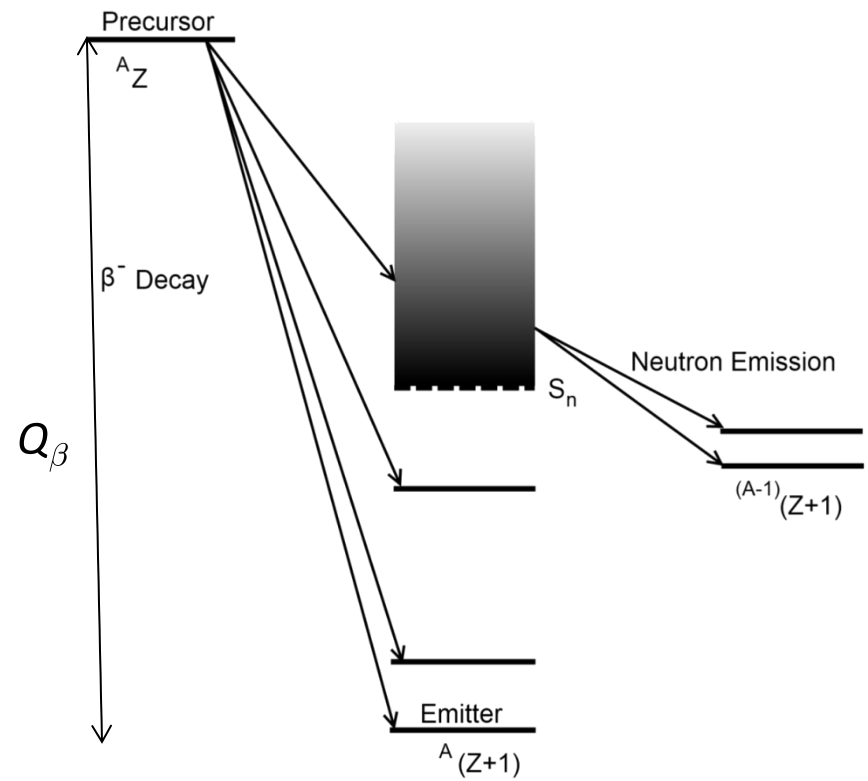
- Small probability of occurrence because of very small cross sections
- Example: $^{37}Cl + \nu \rightarrow {}^{37}Ar + e^-$ → ^{37}Ar is unstable ($T_{1/2}$ = 37 days) → method used for the detection of solar neutrinos (big detector volume)

Other processes: β-delayed nucleon emission (1)

- Following β decay → excited states → generally γ decay
- Occasionally states are unstable against emission of one or more nucleons (proton, neutron, α)
- Nucleon emission occurs rapidly → competition with γ decay → nucleon emission occurs with a half-life characteristic of β decay
- Original β-decay parent is called precursor ↔ nucleon comes from the emitter ↔ final state is the daughter
- Interests in delayed emission
  - → study of nuclei far from stability
  - → importance of delayed neutrons in the control of nuclear reactors

Other processes: β-delayed nucleon emission (2)





Schemes of β-delayed proton and neutron emissions

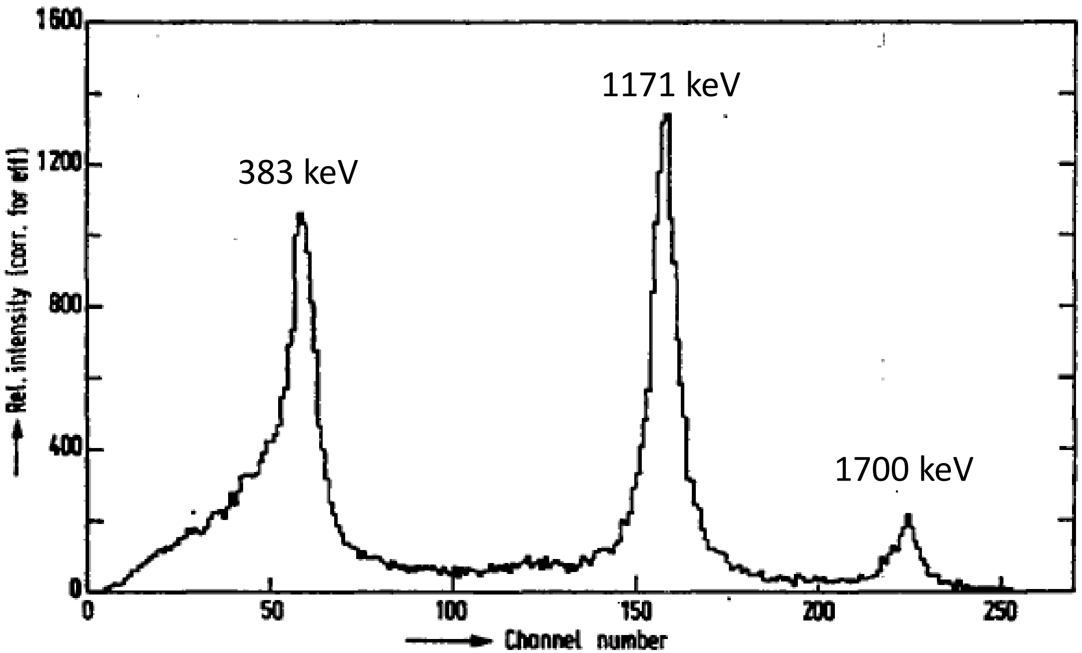
Other processes: β-delayed nucleon emission (3)

- Calculation of energy spectra of emitted nucleon is complicate

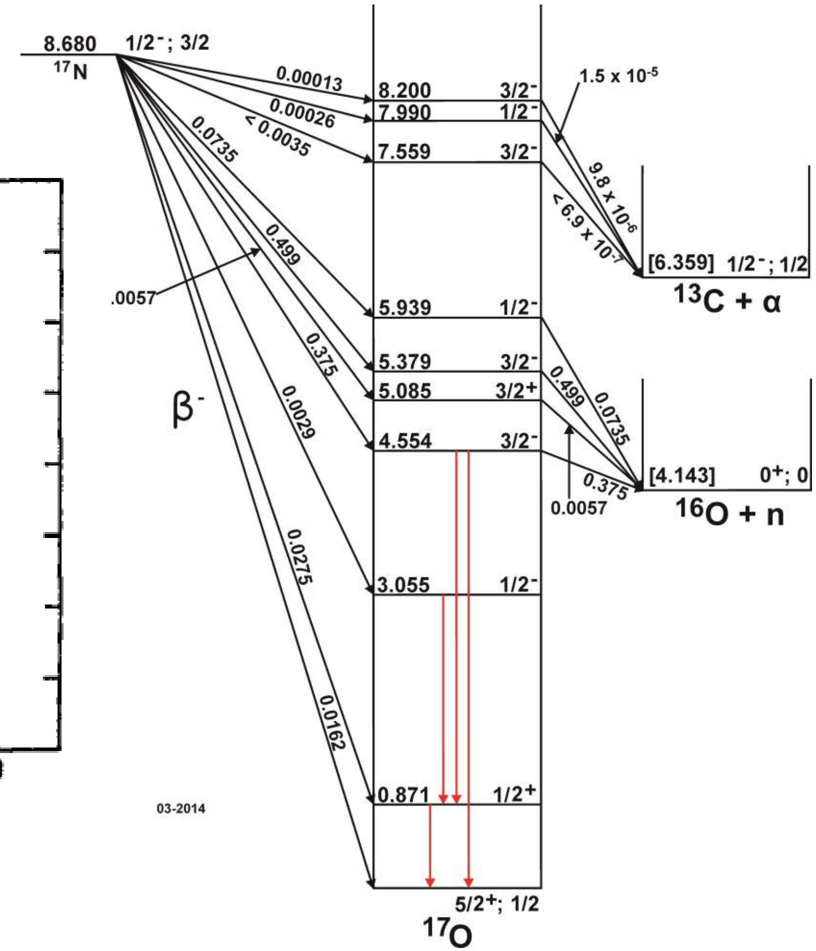
  → requiring knowledge of the spectrum of excited state, probability of β-decay, probability of nucleon decay

  → in heavy nuclei → large density of excited states → broad distribution ($\sim$ continuous)
- One easy point → the β-decay energy must be larger than the nucleon separation energy → $Q_\beta > S_N$ (N for nucleon)
- No discussion of the theory of β-delayed nucleon emission → only one example → β-delayed neutron emission from ^{17}N

Other processes: β-delayed neutron emission from ^{17}N (1)



Neutron spectrum



Other processes: β-delayed neutron emission from ^{17}N (2)

- 3 excited states of ^{17}O are populated in the β^- decay from ^{17}N and emit a neutron to form ^{16}O → 3 neutron groups
- In this particular case (not true in general) → decay to the ground state of ^{16}O ↔ first excited state of ^{16}O at more than 6 MeV → impossible to reach this state before neutron emission
- Determination of the neutron separation energy of ^{17}O →

$$
\begin{aligned}
S_n &= [m(^{16}O) - m(^{17}O) + m_n]c^2 \\
&= (15.99491 \text{ u} - 16.99913 \text{ u} + 1.00866 \text{ u})931.502 \text{ MeV/u} \\
&= 4.144 \text{ MeV}
\end{aligned}
$$

Other processes: β-delayed neutron emission from ^{17}N (3)

- E_X is the excited energy of ^{17}O → the initial energy is $m(^{17}O)c^2 + E_X$
- The final energy is $m(^{16}O)c^2 + E'_X + m_n c^2 + T_n + T_R$ with E'_X a possible excitation energy (= 0 in this case), T_n the neutron kinetic energy and T_R the ^{16}O recoil energy
- Energy conservation gives →

$$m(^{17}O)c^2 + E_x = m(^{16}O)c^2 + E'_x + m_n c^2 + T_n + T_R$$

$$\Rightarrow \quad E_x = E'_x + T_n + T_R + S_n$$

- The recoil energy is obtained from the conservation of momentum → $T_R = T_n(m_n/m_R) \approx T_n/(A\text{-}1)$ →

$$E_x = E'_x + \frac{A}{A-1} T_n + S_n$$

- Assuming $E'_X = 0$ → the 3 E_X are 4.551, 5.388 and 5.950 MeV (corresponding to experiment) ↔ E'_X = 6.049 MeV is impossible → 6.049 MeV + 4.144 MeV > 8.680 MeV

Other processes: β-delayed nucleon emission from heavy nuclei

- In heavy nuclei → large density of excited states → broad distribution ↔ no individual peaks in the nucleon spectrum as for ^{17}N