Chapiter VII:
Ionization chamber

Types of ionization chambers

- Sensitive volume: gas (most often air → direct measurement of exposure) → ionization chamber
- Sensitive volume: semiconductor (silicon, germanium, diamond) → solid ionization chamber

Advantages and disadvantages of the solid ionization chambers

Advantages:

- Compared to gas detectors → at equal energy, they deliver a more intense pulse (W ≈ 3 eV for semiconductor and W ≈ 30 eV for a gas) → 10 times more electrons → larger accuracy
- Density about 2000 times larger than for a gas (1 atm) → detection efficiency, at equal volume, quite larger than for a gas ionization chamber

Disadvantages:

- Very often, it is necessary to cool it down to obtain optimal performances (Ge)
- Difficult to obtain large size detectors for reasonable price → small size of the sensitive volume
- High price of a diamond detector

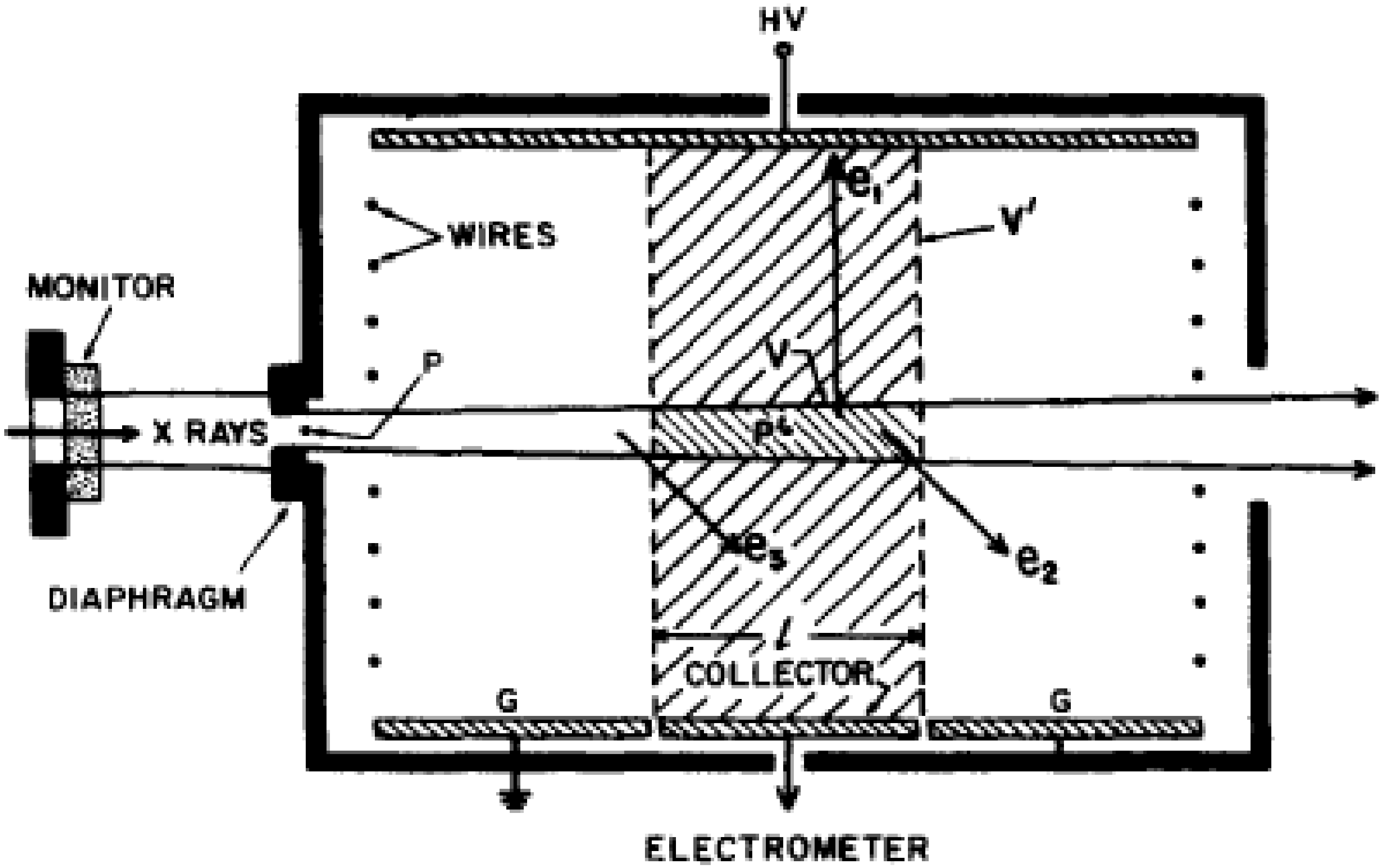
Organization of the chapter

- Free-air ion(ization) chamber

- Cavity ionization chamber

- Recombinations in a ionization chamber

Free-air ion chamber: scheme



Free-air ion chamber: description

- Goal: measure of the exposure for photons with an energy varying between 5 keV and 500 keV
- The chamber is surrounded by a Pb shielding to exclude rays coming from elsewhere
- The central axis of the beam is aligned with the axis of the chamber
- The beam enters the chamber from a tungsten diaphragm
- On one side of the chamber: 3 plates → collector electrode (length *l*) and 2 guard electrodes (to determine the total volume)
- On the other side: a plate relied to high voltage (to obtain a field of $\sim$ 100 V/m between the plates)
- A set of wires encircles the space between the plates to provide a uniform electric field

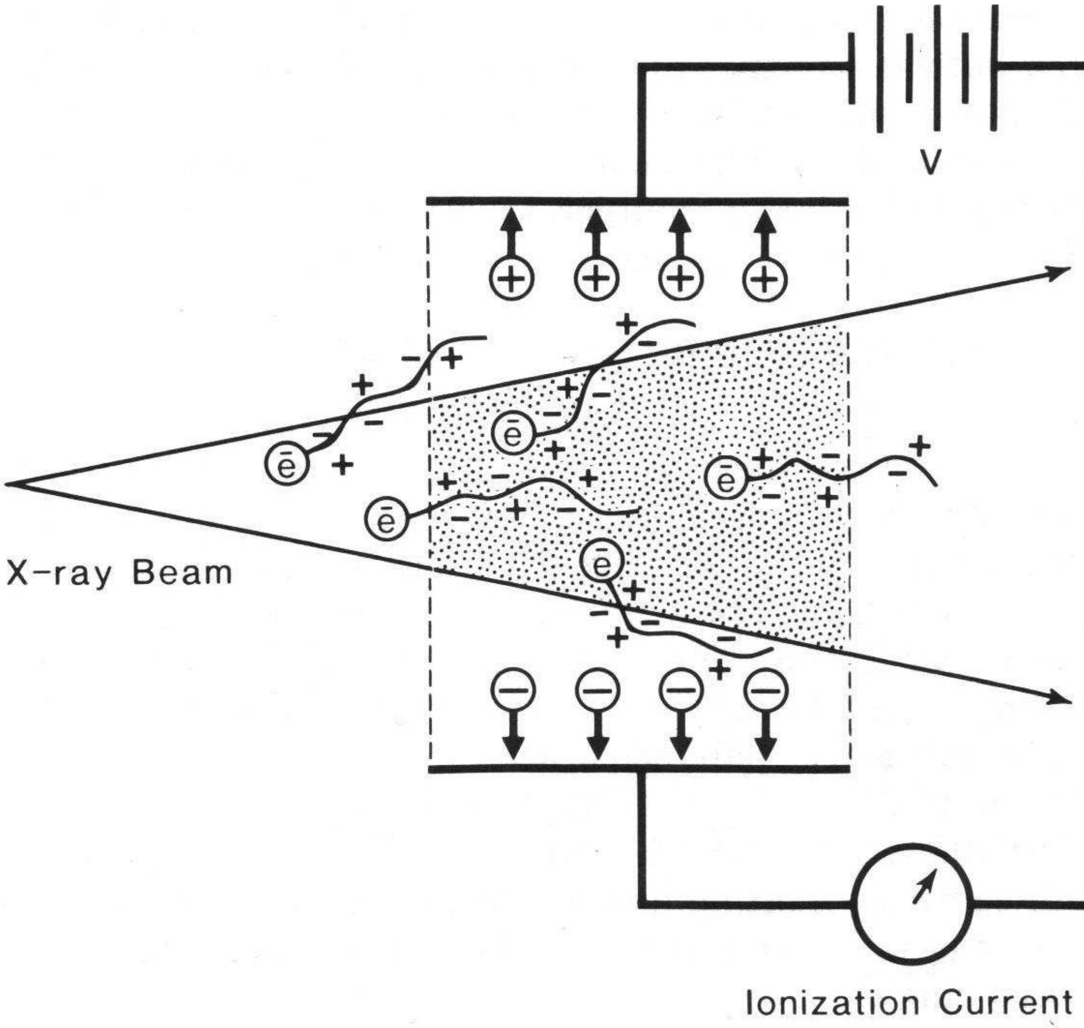
Free-air ion chamber: Principle (1)

- Photons penetrating into the chamber set « high » energy electrons in motion by photoelectric or Compton effect inside *V*
- These « high » energy e^- create secondary ionizations along their path (≈ 40 cm for 500 keV X-rays)
- Due to the electric field → positive charges created inside *V’* move to the negative electrode and negative charges created inside *V’* move to the positive electrode→ presence of a current that can be measured
- V’ is called the collection volume
- Scattered photons, Bremmstrahlung and recombinations are neglected

Free-air ion chamber: Principle (2)

- Reminder of the exposure definition: quotient of dQ by dm, where dQ is the absolute value of the mean total charge of the ions of one sign produced when all the electrons and positrons liberated or created by photons incident on a mass dm of dry air are completely stopped in dry air
- If all e^- created by the photons in *V* lose all their energy by ionization in air included in volume *V'* → the charge can be measured → measure of exposure
- However some of the e^- created in *V* can lose their *E* outside *V'* → non-collected charge
- However some of the e^- created outside *V* can lose their *E* inside *V'* → supplementary collected charge

Free-air ion chamber: Charges collection

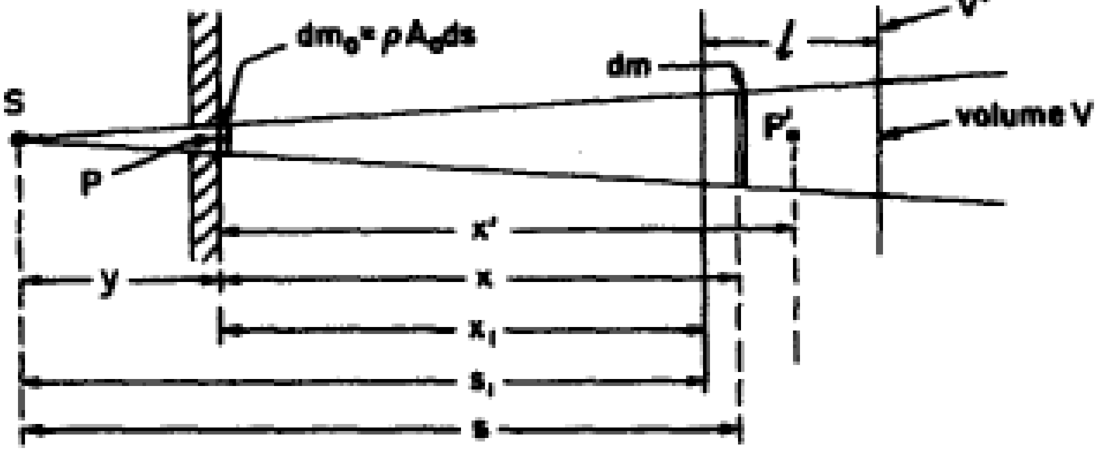


CPE for volume of collection

- To satisfy the exposure definition → CPE is necessary in *V'* → compensation between ingoing and outgoing charges

- CPE is reached if the range of e^- (≈ 40 cm) created by incident photons is smaller than the distance between each plate and the volume *V* (and also than the distance between V and each end of the box chamber) and if the fluence of the photons beam is constant along the distance *l* (no attenuation)

Determination of X at the plane of the aperture (1)



Definition:

- P: point at the center of the aperture
- A_0: area of the aperture
- y: distance between the source S and the aperture
- dm_0: disc-shaped mass element at P → $dm_0 = \rho A_0 ds$
- dm: disc-shaped mass element at P‘ at a distance s of the source included inside V → $dm = \rho A_0 (s/y)^2 ds$
- Ψ_0: fluence at P

Determination of X at the plane of the aperture (2)

- If all e$^-$ resulting from interactions of photons inside *dm$_0$* dissipate all their *E* in air →

$$dQ_0 = dm_0 \Psi_0 \left(\frac{\mu_{en}}{\rho}\right)_{air} \left(\frac{e}{\overline{\overline{W}}}\right)_{air} = \rho A_0 \Psi_0 \left(\frac{\mu_{en}}{\rho}\right)_{air} \left(\frac{e}{\overline{\overline{W}}}\right)_{air} ds$$

- We consider the energy fluence in *dm* (with μ the attenuation coefficient in air)→

$$\Psi(s) = \Psi_0 \left(\frac{y}{s}\right)^2 e^{-\mu(s-y)} \cong \Psi_0 \left(\frac{y}{s}\right)^2 [1-\mu(s-y)]$$

- The ionization caused by the e$^-$ originating from *dm* →

$$\begin{aligned} dQ &= dm\Psi(s) \left(\frac{\mu_{en}}{\rho}\right)_{air} \left(\frac{e}{\overline{\overline{W}}}\right)_{air} \\ &= \rho A_0 \left(\frac{s}{y}\right)^2 \Psi_0 \left(\frac{y}{s}\right)^2 [1-\mu(s-y)] \left(\frac{\mu_{en}}{\rho}\right)_{air} \left(\frac{e}{\overline{\overline{W}}}\right)_{air} ds \\ &= dQ_0[1-\mu(s-y)] \end{aligned}$$

Determination of X at the plane of the aperture (3)

The total charge generated by the e$^-$ originating from V (with s_1, the distance between the source and V and x_1, the distance between the aperture and V)→

$$\begin{aligned} Q &= \rho A_0 \Psi_0 \left(\frac{\mu_{en}}{\rho}\right)_{air} \left(\frac{e}{\overline{\overline{W}}}\right)_{air} \int_{s_1}^{s_1+l} [1-\mu(s-y)]ds \\ &= \rho A_0 l \Psi_0 \left(\frac{\mu_{en}}{\rho}\right)_{air} \left(\frac{e}{\overline{\overline{W}}}\right)_{air} \left[1-\mu\left(x_1+\frac{l}{2}\right)\right] \end{aligned}$$

with $x_1 + l/2$ the distance between P and P'

Determination of X at the plane of the aperture (4)

- The exposure (for a volume $A_0 l$) at P is given by →

$$\begin{aligned} X_P &= \frac{dQ_0}{dm_0} = \frac{Q}{m}\left[1-\mu\left(x_1+\frac{l}{2}\right)\right]^{-1} \\ &\cong \frac{Q}{m}\left[1+\mu\left(x_1+\frac{l}{2}\right)\right] \end{aligned}$$

- Typically $\mu \cong$ 2-3% → $\mu(x_1+l/2) < 0.01$

Result of a measurement in a free-air ion chamber

$$X_P = \frac{Q}{\rho A_0 L} \prod_i k_i$$

- k_{TP}: correction due to temperature and pressure effects
- k_D: correction due to the aperture estimation
- k_A: correction due to the attenuation of the beam in air
- k_S: correction due to recombinations
- k_H: correction due to humidity
- k_{sc}: correction due to ionizations produced by scattered photons
- …

Remark on the maximal energy of photons

- If E of X-rays ↗ → range of e^- ↗ → to maintain CPE → dimensions of the chamber ↗ → problem of non-uniformity of the field (+ difficult to neglect recombination)

- Solution: ↗ of the air pressure → in that case ↗ of the attenuation and of scattering of photons in air

- Practically → free-air ionization chamber limited to $E < 500$ keV

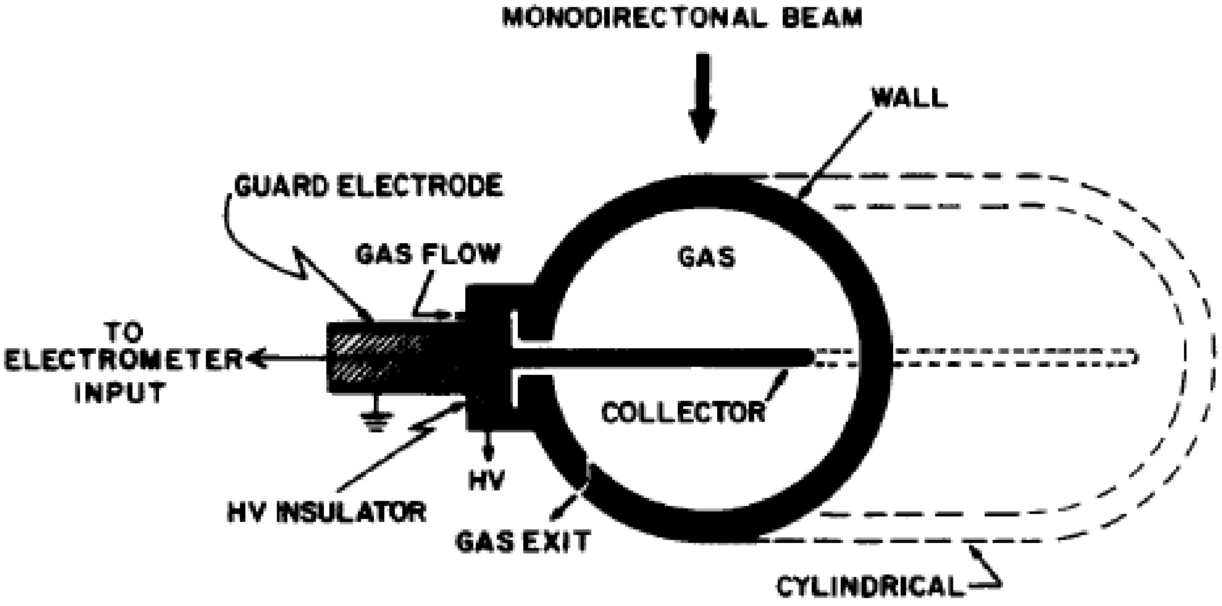
Example of free-air ionization chamber

National Radiation Standard Laboratory (NRSL), Taipei, Taiwan

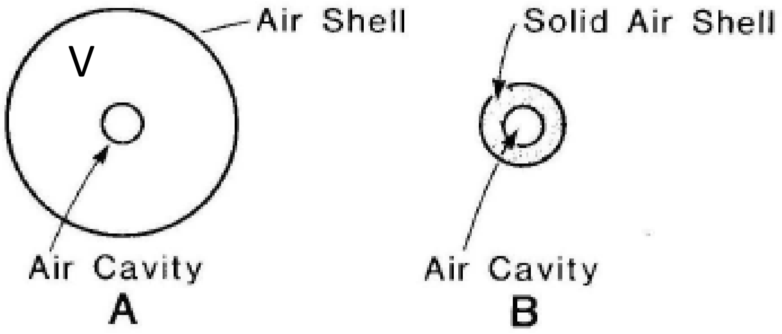
Standardization laboratories → calibration

Cavity ionization chamber (with « equivalent-air » walls)

- Large number of ≠ designs → always the same principle
- A solid envelope surrounds a gas –filled cavity (air,…) and an electric field is applied outside the cavity → collection of ions formed by radiation



Cavity ionization chamber: Principle for γ (1)



- **A**: A spherical air volume *V* surrounds a air cavity at the center → we assume a uniform photons radiation field in *V* → the distance between extern and intern walls of *V* ≥ range of e^- in air → the number of e^- entering or outgoing the cavity are = → CPE → measure of *Q* produced (in the cavity) by e^- created in *V* → with known *m* → calculation of *X*

Cavity ionization chamber: Principle for γ (2)

- **B**: The air wall is « compressed » to obtain a solid wall → ↗ density but condition on the range of the e^- in air always respected → CPE always respected with a largely smaller wall thickness (ex: 1.25 MeV-γ of ^{60}Co → ≈ 5 mm)
- Practically → an « equivalent-air » medium is chosen for the wall (equivalence of μ_{en} for incident photons but also of $dE/\rho dx$ for created e^- → conditions OK when Compton effect is dominating) → polystyrene, polyethylene, PMMA, Teflon, graphite,…
- The intern wall of the cavity is covered with a conducting medium (colloidal graphite in a alcohol base) → electrode

$$X_P = \frac{Q}{\rho_{air} V} \prod_i k_i$$

Examples of thicknesses required for CPE

Energie du photon (MeV)	Epaisseur (g/cm^2)
0.02	0.0008
0.05	0.0042
0.1	0.014
0.2	0.044
0.5	0.17
1	0.43
2	0.96
5	2.5
10	4.9

The thickness calculation is based on the range of e^- in H_2O → for ionization chamber with tissue-equivalent wall and air cavity → corrections are needed

Cavity ionization chamber: Principle for e^-

- If we consider incident charged particles → V has to be small → the wall has to be thin compared to the range of the incident des particles
- If Z equivalence between the wall and the gas of the cavity → equilibrium between the δ e^- going from the cavity to the wall and vice-versa if the wall is thick enough
- Practically → for incident e^- producing δ e^- with $E < 100$ keV → a wall thickness of $\cong 15$ mg/cm^2 is enough (range of a 100 keV e^-)
- Application of the cavity theory to obtain the dose in the medium

Cavity (c) ↔ free-air (f) ion chambers

- c: allows the measure of the dose for all types of incident particles and for a large range of *E*; f: only for photons with $E <$ 500 keV
- c: very compact (ratio of 1000 between the ranges of an e^- in a solid and in a gas) → allows measures in ≠ points of an irradiated medium
- c: measure of multidirectional radiations; f: direction of the beam ⊥ to the aperture
- c: via cavity theory → the dose can be determined for all media constituting the wall

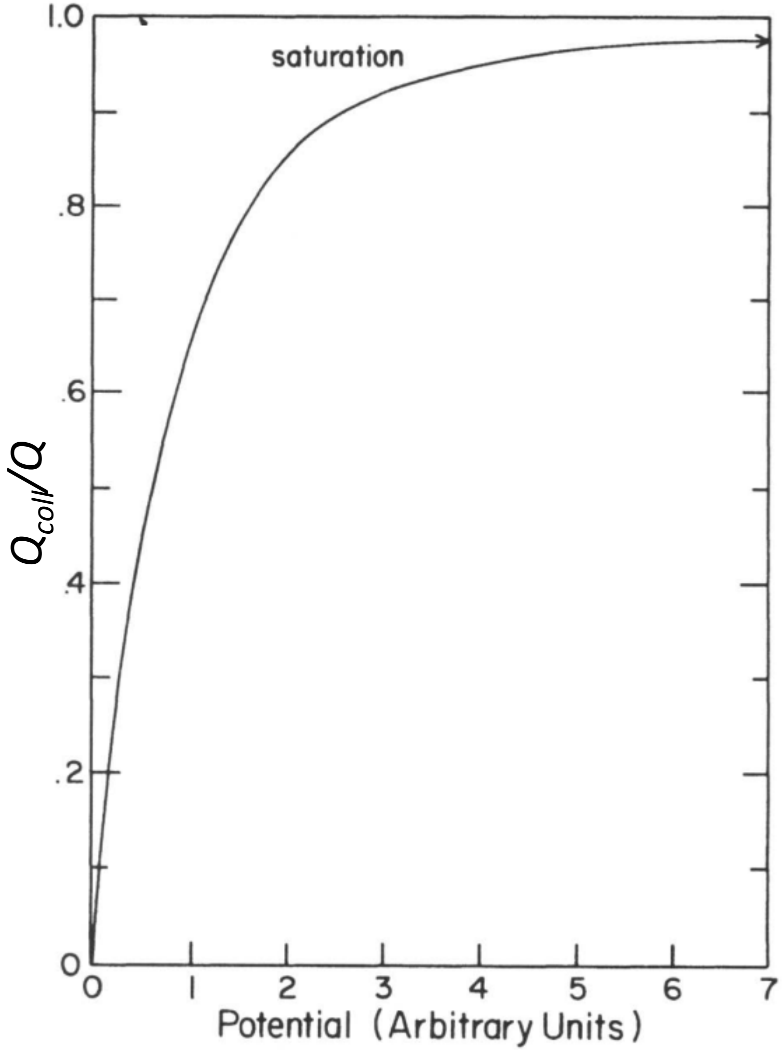
Application: Direct-reading dosimeter

- Charged condenser → constant potential at the condenser contacts
- When ionizing radiation crosses the dosimeter → air ionization → displacement of the charges towards the electrodes → potential decrease → discharge of the condenser
- Potential difference $\propto$ to exposure
- Use:
  - Alarm
  - Continuous measurement of the dose

Recombinations

- The dose deposited in a gas by ionizing radiation is $\propto$ to the charge Q produced in the gas
- In practical case the measured collected charge Q_{coll} at the electrodes of the chamber is $< Q$ → recombination of ions in the gas
- Recombinations occur between positive ions and e^- or between positive and negative ions
- A ion chamber is said to be saturated if $Q_{coll} \rightarrow Q$
- Generally if the applied potential U_0 ↗ → we asymptotically approach saturation
- Attention: if U_0 ↗ too much → multiplication

Variation of Q_{coll} as a function of the applied potential



Recombinations in an ion chamber

Definition of a recombination collision rate R ($m^{-3}s^{-1}$) and of a recombination coefficient α (m^3s^{-1}) such as:

$$R = -\frac{dn^+}{dt} = -\frac{dn^-}{dt} = \alpha n^+ n^-$$

with n^+ and n^-, the volume densities of charge

Types of recombinations

- Columnar (or initial) recombination: ion pairs are formed into a column along the track of the ionizing particle → density of pairs is high along the track until pairs diffuse → columnar recombination is most severe for densely ionizing particles (as α) compared to electrons that deposit their energy over a much longer track → independent on the irradiation rate and on *D*

- Volume recombination: due to encounters between ions and/or electrons after they left the location of the track → since ions slowly drift to electrodes → ions and/or electrons from independent tracks can recombine → increases with irradiation rate and with *D*

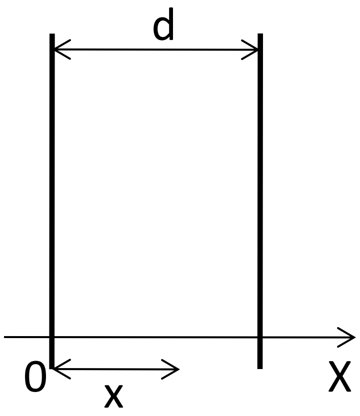
Type of gas and recombinations

- An e^- created during an ionization can be fixed (attachment) on a neutral atom of the gas → negative ion
- Attachment occurs in electronegative gases (O_2, air, …)
- The drift velocity of an ion is $v_d \approx 1$ cm/s per applied V/cm and for an e^- $v_d \approx 1000$ cm/s per V/cm
- The e^- are quickly cleared out of the ion chamber → little chance to recombine
- On the contrary ions being slowly collected have more chance to recombine
- Recombinations are particularly important for electronegative gases and thus for air

General theory of volume recombination in an ion chamber (Mie's theory)

- Theory valid for a uniform ionization rate in the chamber
- Ion chamber with plane-parallel electrodes
- Cylindrical ion chamber
- Spherical ion chamber
- General extrapolation

Ion chamber with plane-parallel electrodes (1)



- We consider an ion chamber with ∥ electrodes (area *S)* separated by a distance *d*
- The OX axis is ⊥ to the electrodes and oriented from the positive electrode to the negative electrode → anode (+) is at $x = 0$ and cathode (−) at $x = d$
- The electric field produced by the ions is negligible (at first approximation) compared to the applied field E_0
- As E_0 is uniform, the drift velocities of positive (u_+) and negative (u_-) ions are constant
- The ionization rate *q* (number of ionizations per unit of time and volume) is assumed to be constant and uniform

Ion chamber with plane-parallel electrodes (2)

- The quantity of + charges produced between *0* and *x* in the time interval *dt* is equal to the quantity of charges crossing the plane *X = x* in the same time interval → with $n^+(x)$ and $n^-(x)$ the bulk densities of charges at the distance *x* of the anode →

$$qSxdt = n^+(x)Su_+dt$$

$$qS(d-x)dt = n^-(x)Su_-dt$$

⇨

$$n^+(x) = \frac{qx}{u_+}$$

$$n^-(x) = \frac{q(d-x)}{u_-}$$

Ion chamber with plane-parallel electrodes (3)

- The recombination rate R' (per unit of surface and time - $m^{-2}s^{-1}$) is:

$$R' = \int_0^d \alpha n^+(x) n^-(x) dx = \frac{\alpha q^2 d^2}{u_- u_+} \int_0^d \frac{x}{d}\left(1 - \frac{x}{d}\right) dx = \frac{1}{6}\frac{\alpha q^2 d^3}{u_- u_+}$$

- With *qd*, the number of ionizations per unit of surface and time, we define the collection efficiency of charges *f* as the ratio between Q_{coll}, the collected charge at the electrodes and *Q*, the charge produced in the chamber by a constant radiation field →

$$f = \frac{Q_{coll}}{Q} = 1 - \frac{R'}{qd} = 1 - \frac{1}{6}\frac{\alpha q d^2}{u_- u_+} = 1 - \frac{1}{6}\xi^2$$

Ion chamber with plane-parallel electrodes (4)

- The drift velocity of a charge *u* is linked to the applied electric field *E* by $u = \mu E$ (with μ, the mobility – $\mathrm{m^2V^{-1}s^{-1}}$)
- In this case the applied potential $U_0 = Ed$ →

$$\xi = \sqrt{\frac{\alpha q d^2}{u_- u_+}} = \sqrt{\frac{\alpha}{\mu_+ \mu_-}} \frac{d^2 \sqrt{q}}{U_0} = m^2 \frac{d^2 \sqrt{q}}{U_0}$$

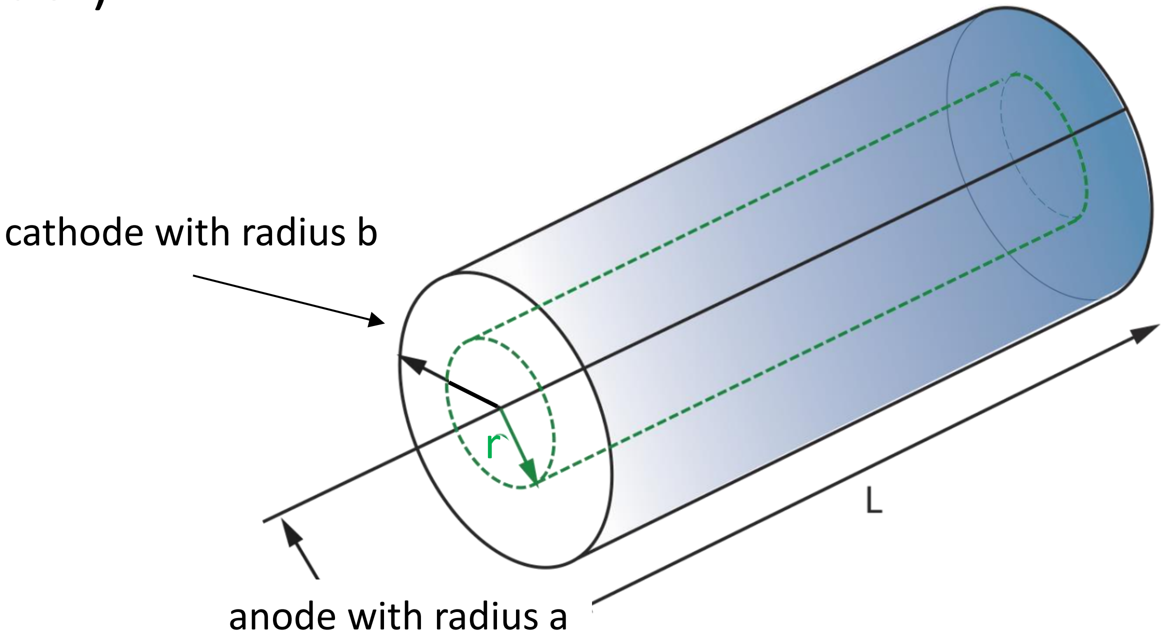
with m^2 a constant of the gas (for air NCTP $m^2 = 6.47 \times 10^{-5}$ $\mathrm{V^2 s m^{-1}}$)

- Calculation accounting for the electric field produced by the space charges (→ non-uniform *E*) gives →

$$f = \frac{1}{1 + \frac{1}{6}\xi^2}$$

Cylindrical ion chamber (1)

Same arguments can be applied (with *u* and μ dependent on the position *r*) to a cylindrical ion chamber (with *a* and *b*, the intern and extern radii of the electrodes and *L*, the length of the cylinder):



Cylindrical ion chamber (2)

$$q\pi(r^2 - a^2)Ldt = n^+(r)2\pi r L u_+(r)dt$$
$$q\pi(b^2 - r^2)Ldt = n^-(r)2\pi r L u_-(r)dt$$

$$\Rightarrow \quad n^+(r) = \frac{q(r^2 - a^2)}{2ru_+(r)}$$
$$n^-(r) = \frac{q(b^2 - r^2)}{2ru_-(r)}$$

For cylindrical chamber →

$$E(r) = \frac{U_0}{r\ln(b/a)} \quad \Rightarrow \quad u_\pm(r) = \mu_\pm E(r) = \mu_\pm \frac{U_0}{r\ln(b/a)}$$

Cylindrical ion chamber (3)

$$n^+(r) = \frac{qa^2 \ln(b/a)}{2\mu_+ U_0}\left[\left(\frac{r}{a}\right)^2 - 1\right]$$

$$n^-(r) = \frac{qa^2 \ln(b/a)}{2\mu_- U_0}\left[\left(\frac{b}{a}\right)^2 - \left(\frac{r}{a}\right)^2\right]$$

The recombination rate R’’ (per unit of length and time - $m^{-1}s^{-1}$) is:

$$\begin{aligned}
R'' &= \int_0^d \alpha n^+(r) n^-(r) 2\pi r dr \\
&= \frac{\pi\alpha}{2\mu_-\mu_+}\left(\frac{qa^3 \ln(b/a)}{U_0}\right)^2 \int_a^b \frac{r}{a}\left[\left(\frac{b}{a}\right)^2 - \left(\frac{r}{a}\right)^2\right]\left[\left(\frac{r}{a}\right)^2 - 1\right]\frac{dr}{a} \\
&= \frac{\pi\alpha}{4\mu_-\mu_+}\left(\frac{qa^3 \ln(b/a)}{U_0}\right)\frac{[(b/a)^2 - 1]^3}{6}
\end{aligned}$$

Cylindrical ion chamber (4)

$$\begin{aligned}
f &= \frac{Q_{coll}}{Q} = 1 - \frac{R''}{\pi(b^2-a^2)q} \\
&= 1 - \frac{1}{6}\frac{\alpha q}{4\mu_-\mu_+}\left(\frac{a^2\ln(b/a)^2}{U_0}\right)^2\left[(b/a)^2-1\right]^2 \\
&= 1 - \frac{1}{6}\frac{\alpha q}{\mu_-\mu_+}\left[\frac{(b-a)^4}{U_0^2}\right]\left[\frac{b+a}{b-a}\frac{\ln(b/a)}{2}\right]^2 \\
&= 1 - \frac{1}{6}\xi^2
\end{aligned}$$

with

$$\begin{aligned}
\xi &= \sqrt{\frac{\alpha}{\mu_+\mu_-}}\frac{\left[\kappa_{cyl}(b-a)\right]^2\sqrt{q}}{U_0} \\
\kappa_{cyl} &= \sqrt{\frac{b/a+1}{b/a-1}\frac{\ln(b/a)}{2}}
\end{aligned}$$

Spherical ion chamber

Same calculation as for a cylinder → with ξ and κ values corresponding to a sphere →

$$\begin{aligned}\xi &= \sqrt{\frac{\alpha}{\mu_+\mu_-}}\frac{\left[\kappa_{sph}(b-a)\right]^2\sqrt{q}}{U_0}\\ \kappa_{sph} &= \sqrt{\frac{1}{3}\left(\frac{b}{a}+1+\frac{a}{b}\right)}\end{aligned}$$

General extrapolation

- Previous relations can be extrapolated and written as →

$$\frac{1}{Q_{coll}} = \frac{1}{Q} + \frac{c}{U_0^2}$$

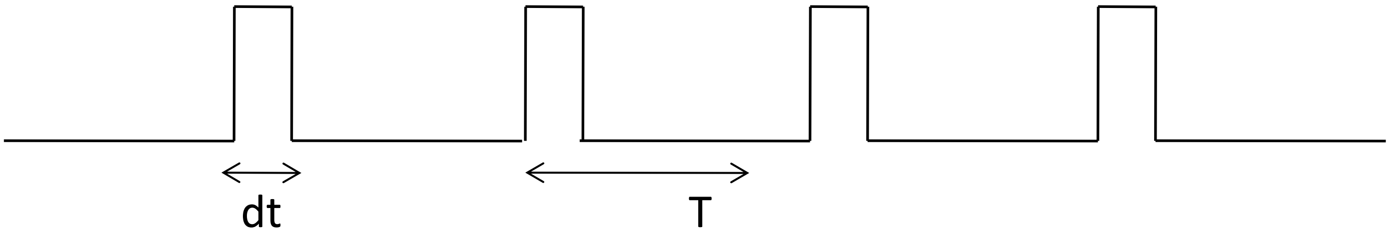
  with c a constant depending on the detector geometry and on the nature of the gas
- ≠ values of $1/Q_{coll}$ as a function of U_0→ extrapolation for $1/U_0^2 = 0$ → $1/Q$ correct for a continuous irradiation

Columnar recombination

- If volume recombinations are negligible and only columnar recombinations have to be considered (particles with high ionization density or high pressure gas) → empirical demonstration of Jaffe-Zanstra →

$$\frac{1}{Q_{coll}} = \frac{1}{Q} + \frac{c'}{U_0}$$

Recombination for pulsed radiation

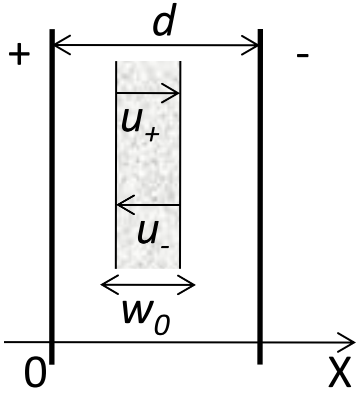


- We choose *dt:* width of radiation pulses and *T*: charges collection time

- If a lot of pulses during *T* or *dt* > *T* → theory of continuous irradiation

- If $dt \ll T$ ($dt \approx \mu s$ and $T \approx ms$) → Boag's theory

Boag's theory for pulse irradiation (1)



- w_0: initial width of the irradiated area → evolution of w with time t: $w(t) = w_0 - (u_+ + u_-)t$ with $w(T) = 0$ → $T = w_0/(u_+ + u_-)$
- Hypothesis: $n^+ = n^- = n$ →

$$\begin{aligned} \frac{dn}{dt} &= -\alpha n^2 \quad \text{with} \quad n_0 = n(0) \\ n(t) &= \frac{n_0}{1 + \alpha n_0 t} \end{aligned}$$

Boag’s theory for pulse irradiation (2)

Integration of the recombination rate during time *T* → R_T: recombination rate per unit of surface - m^{-2}) →

$$\begin{aligned} R_T &= \frac{1}{S}\int_0^T V\,dn = \frac{1}{S}\int_0^T wS\alpha n^2 dt \\ &= \alpha n_0^2 \int_0^T \frac{(w_0 - (u_+ + u_-)t}{(1+\alpha n_0 t)^2} dt \\ &= n_0 w_0 [1 - \frac{1}{\alpha n_0 T}\ln(1+\alpha n_0 T)] \end{aligned}$$

Boag’s theory for pulse irradiation (3)

- The collection efficiency of charges *f* →

$$f = 1 - \frac{R}{n_0 w_0} = \frac{1}{\alpha n_0 T} \ln\left(1 + \alpha n_0 T\right)$$

- If $w_0 = d$ →

$$\alpha n_0 T = \alpha n_0 \frac{d}{u_+ + u_-} = \frac{\alpha n_0}{\mu_+ + \mu_-} \frac{d^2}{U_0}$$

- If small α →

$$f \simeq 1 - \frac{1}{2}\alpha n_0 T \rightarrow \frac{1}{Q_{coll}} = \frac{1}{Q} + \frac{c''}{U_0}$$

Boag’s theory for pulse irradiation (4)

- For a cylindrical chamber → *d* must be replaced by

$$d \rightarrow (a-b)\kappa_{cyl}$$

- For a spherical chamber → *d* must be replaced by

$$d \rightarrow (a-b)\kappa_{sph}$$

Atmospheric corrections: Air density (1)

- The collected charge depends on the type and the mass of gas inside the chamber
- Most of the time → chamber open to ambient atmosphere → equilibrium of the gas with environment → density of the gas (with [T] = K and [P] = Pa) →

$$\rho = \rho_{CNTP} \left(\frac{273.15}{T} \times \frac{P - 0.3783 P_w}{101325} \right)$$

- P_w is the water vapor partial pressure in air
- P_w depends the relative humidity (RH) of air defined as the ratio of water vapor partial pressure in air on the saturation vapor pressure at the same T → measure of the link between the content of water vapor in air and its maximal capacity in the same conditions

Atmospheric corrections: Air density (2)

- Examples of P_w:
  - at T = 295.15 K and RH = 100% → P_w = 2643.33 Pa
  - at T = 295.15 K and RH = 50% → P_w = 1321.73 Pa

- Examples of modifications of the density ρ: If P = 101325 Pa and P_w = 1321.73 Pa → ρ_h (humid air) < by 0.5% compared to ρ_a of dry air (P_w = 0 Pa)

- All these parameters have to be frequently measured during a long series of measurements

- The humidity effect is often neglected because it also implies a correction on *W/e* varying in the opposite direction with the density correction → net error extremely small (< 0.3 %)

Atmospheric corrections: Effect of humidity on *W/e* (1)

- For dry air →

$$(\overline{W}/e)_a = 33.97 \ \ \text{J/C}$$

- For humid air $(W/e)_h < (W/e)_a$ but the ρ of air ↘ for humid air and as

$$D = \Phi \left(\frac{dE}{\rho dx} \right) = \frac{Q}{\rho V} \left(\frac{\overline{W}}{e} \right)$$

  → the effects counterbalance

- However humidity of air also influences on *dE/dx*

Atmospheric corrections: Effect of humidity on *W/e* (2)

- We consider a cavity ionization chamber under a flux of X-rays (for instance) constant, Φ = c
- We assume that it is filled successively with dry air sec and with humid air →

$$\Phi = \frac{Q_a}{\rho_a V}\left(\frac{\overline{W}}{e}\right)_a \frac{1}{(dE/\rho dx)_a} = \Phi = \frac{Q_h}{\rho_h V}\left(\frac{\overline{W}}{e}\right)_h \frac{1}{(dE/\rho dx)_h}$$

➡

$$\frac{Q_h}{Q_a} = \frac{\left(\frac{\overline{W}}{e}\right)_a}{\left(\frac{\overline{W}}{e}\right)_h} \times \frac{\left(\frac{dE}{dx}\right)_h}{\left(\frac{dE}{dx}\right)_a}$$

- 1st term > 1 and 2nd term < 1 → Q_h/Q_a is relatively « flat » → 1.0028 ± 0.0003 (15% < *RH* < 75%)

Effects of humidity