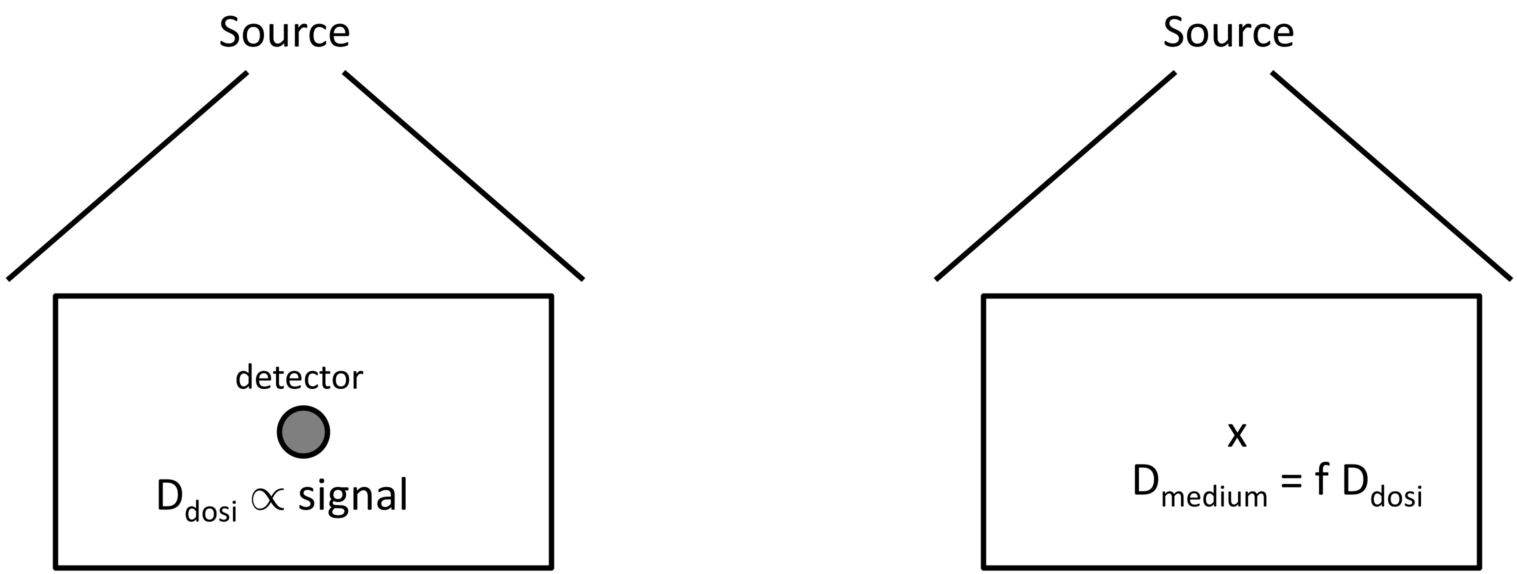
Chapter V:
Cavity theories

Introduction

- Goal of radiation dosimetry: measure of the dose absorbed inside a medium (often assimilated to water in calculations)
- A detector (dosimeter) **never** measures directly the dose absorbed in the medium → measure of the dose **inside** the dosimeter
- Problems: the detector has a composition ≠ from the medium and has a finite volume → correlation between the dose inside the dosimeter and the dose inside the medium
- **Cavity theories** allow the interpretation of the dose read from the dosimeter

Conversion factor



- Conversion factor f

$$\Rightarrow \quad f = \frac{D_{medium}}{D_{dosi}}$$

- In the following → medium = water (w) and detector = gas (g)

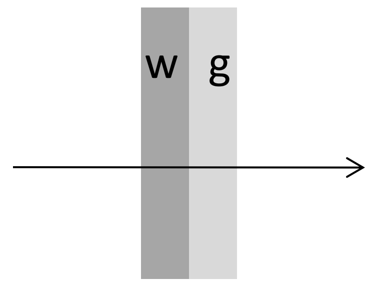
Preliminary 1: Dose in a thin slide (charged particles)

For a beam of charged particles with energy *E* and fluence Φ incident $\perp$ on a medium with atomic number Z, density ρ, thin (thickness *l*) →

1. $S_{elec}(E) \approx$ constant
2. Rectilinear trajectories
3. The kinetic *E* taken away from the film by δ e^- is negligible (CPE or δ-ray equilibrium)

$$D = \Phi \left(\frac{dE}{\rho dx} \right)_{elec}$$

Preliminary 2: Dose at the interface between 2 media (charged particles)

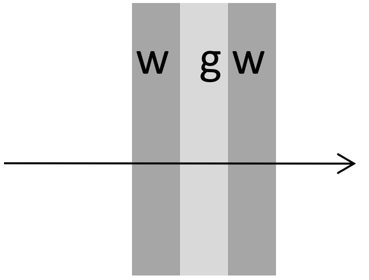


- We consider a small thickness of matter at the interface → Φ is equal across the interface

$$D_w = \Phi \left[\left(\frac{dE}{\rho dx} \right)_{elec,w} \right]_E \Leftrightarrow D_g = \Phi \left[\left(\frac{dE}{\rho dx} \right)_{elec,g} \right]_E$$

$$\Rightarrow \frac{D_w}{D_g} = \frac{(dE/\rho dx)_{elec,w}}{(dE/\rho dx)_{elec,g}}$$

Bragg-Gray cavity theory (B-G)



- We consider a thin layer of medium « g » (cavity) sandwiched between 2 regions containing medium « w » (walls)
- This system is irradiated by a field of particles (**charged or not**) → the B-G theory is applied to charged particles entering into the cavity and coming from either an initial beam of charged particles either from interactions of uncharged particles inside w
- Conditions of Bragg-Gray:

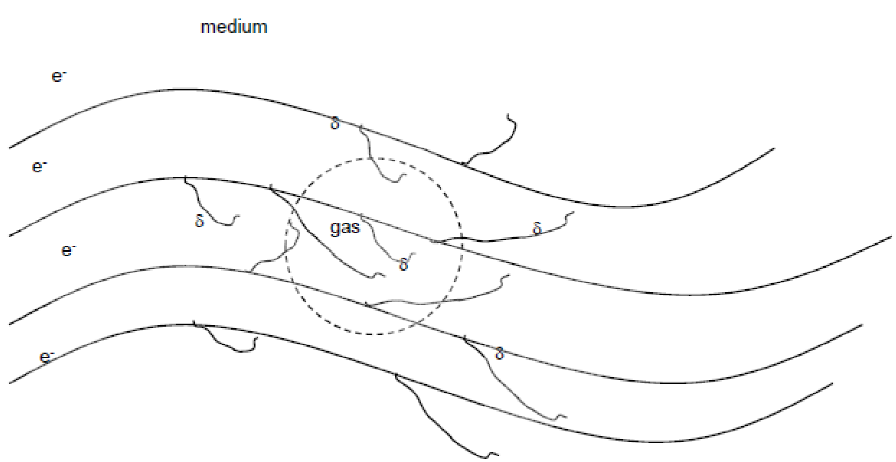
1. *First condition of Bragg-Gray*: The thickness of the g-layer is assumed to be so small in comparison with the range of the charged particles striking it that its presence does not perturb the charged-particle field
2. *Second condition of Bragg-Gray*: The absorbed dose in the cavity is assumed to be deposited entirely by the charged particles crossing it

Comments on B-G conditions (1)

- First condition: no perturbation due to g:
  - For heavy charged particles (primaries or secondaries issue from a primary beam of neutrons) → little scattering → no restrictive condition (at the condition that the cavity is **small** by comparison to the particles range)
  - For electrons (primaries or secondaries issue from a primary beam of γ) → important scattering → even a small cavity can disrupt the field → the cavity **has to be small** (except if the composition of g ≈ w)
- Second condition: dose due to charged particles crossing g:
  - For incident γ → no interaction of the γ rays inside g → no charged particle can be created inside g (they have to come from w) → E > ≈ 1 MeV
  - No charged particle can stop inside g
  - For incident neutrons → no creation of charged particles inside g → can be a problem if the gas inside the cavity is hydrogen

Comments on B-G conditions (2)

- Problem 1 for δ e^- → the B-G theory assumes that all collisions (with *E* losses) undergone by e^- inside the cavity imply a local energy loss → the loss *E* is deposited inside the cavity → this is not the case for δ e^-
- Problem 2 for δ e^- → for incident e^- the fluence only takes into account the primaries e^- → no δ e^- → not obvious



Comments on B-G conditions: Practically

- For incident photons → the cavity must be small (ionization chamber only) and the photons $E >\approx 1$ MeV (no X-rays of only a few keV)

- For incident e^- → If $E >\approx 1$ MeV → all detectors practically used agree with the B-G theory because of the large range of primaries e^- (if g ≈ w and if we neglect δ e^- – see further)

Bragg-Gray relation (1)

- With the 2 conditions → for a beam of fluence Φ made of identical charged particles with kinetic energy *E* crossing the whole w-g-w→

$$\frac{D_w}{D_g} = \frac{(dE/\rho dx)_{elec,w}}{(dE/\rho dx)_{elec,g}}$$

- We consider now the differential distribution in *E* of the fluence (spectric fluence) Φ_E (which can depend on *E* in an arbitrarily way but with E_{max} as maximum energy) → we define ${}_m\overline{S}_g$ and ${}_m\overline{S}_w$ the mean values of the mass collision stopping power in g and w weighted by the spectric fluence of the particles

Bragg-Gray relation (2)

$$ {}_m\overline{S}_g \equiv \frac{\int_0^{E_{max}} \Phi_E \left(\frac{dE}{\rho dx}\right)_{elec,g} dE}{\int_0^{E_{max}} \Phi_E dE} \qquad {}_m\overline{S}_w \equiv \frac{\int_0^{E_{max}} \Phi_E \left(\frac{dE}{\rho dx}\right)_{elec,w} dE}{\int_0^{E_{max}} \Phi_E dE} $$

$$ D_w = \int_0^{E_{max}} \Phi_E \left(\frac{dE}{\rho dx}\right)_{elec,w} dE $$

$$ D_g = \int_0^{E_{max}} \Phi_E \left(\frac{dE}{\rho dx}\right)_{elec,g} dE $$

$$ \Phi = \int_0^{E_{max}} \Phi_E dE $$

$$ \Rightarrow \quad {}_m\overline{S}_g = \frac{D_g}{\Phi}, \qquad {}_m\overline{S}_w = \frac{D_w}{\Phi} \quad \Rightarrow \quad \frac{D_w}{D_g} = \frac{{}_m\overline{S}_w}{{}_m\overline{S}_g} = {}_m\overline{S}_g^w $$

Bragg-Gray relation (3)

On the two conditions of Bragg-Gray, the ratio of the doses in the walls and in the cavity is equal to the ratio of the mean values of the mass collision stopping power weighted by the spectric fluence of the particles

Bragg-Gray relation (4)

- If medium g is gaseous (with mass *m*) in which a charge *Q* is produced (with each sign) by the incident particles and $(\overline{W}/e)_g$ is the mean energy needed per unit of produced charge →

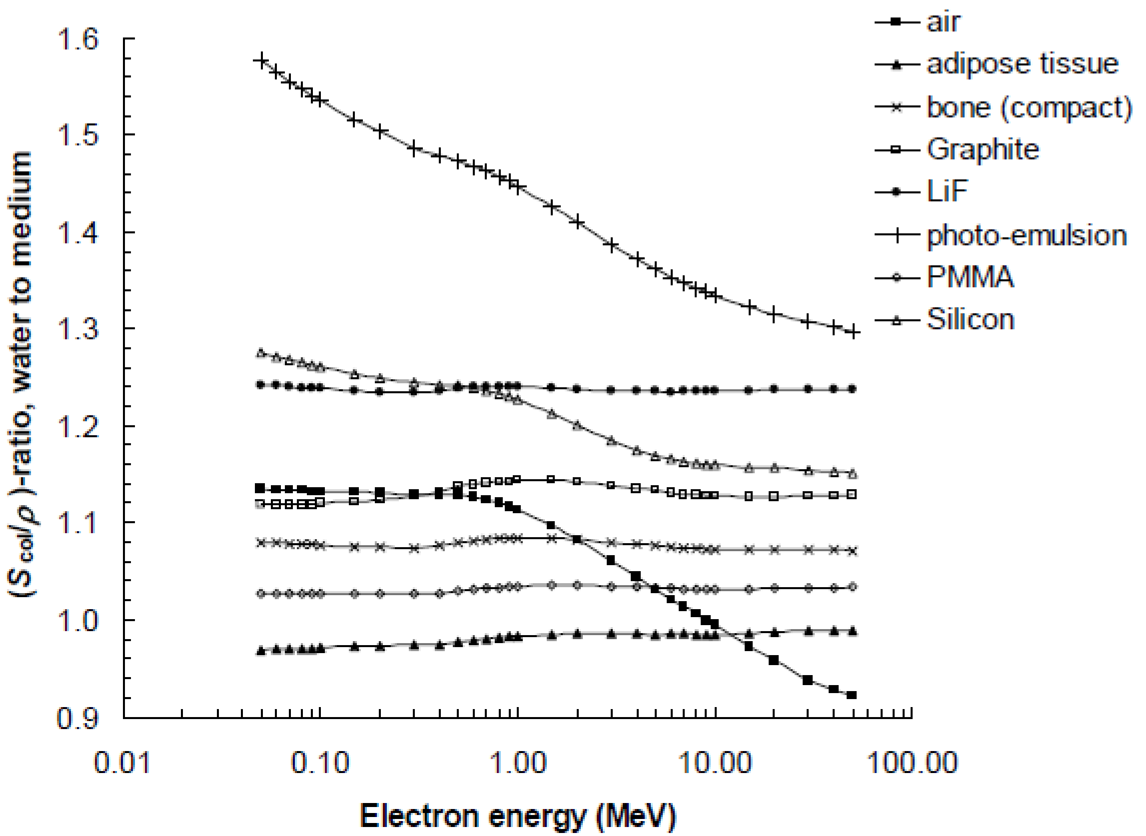
$$D_g = \frac{Q}{m}\left(\frac{\overline{W}}{e}\right)_g$$

- We obtain the **Bragg-Gray relation** →

$$D_w = \frac{Q}{m}\left(\frac{\overline{W}}{e}\right)_g {}_m\overline{S}_g^w$$

- This relation allows to calculate the absorbed dose inside a medium surrounding directly a B-G cavity as a function of the charge produced inside the cavity (at the condition that $(\overline{W}/e)_g$ and ${}_m\overline{S}_g^w$ are known)

Ratio of stopping powers



Remarks on the B-G relation

- CPE **is not** required in the B-G theory → but Φ_E must be equal in g and w (where D_w is evaluated) → in old publications it is written that « CPE has to exist in the absence of cavity » → False but previously problem for the evaluation of Φ_E → now with MC methods no more problems
- Charge *Q'* collected in the ionization chamber is generally < than Q (because of recombinations) → requires a correction
- The mass *m* is generally < than the mass of gas *m'* in the ionization chamber (because of non-active volumes) → *m* has to be determined via a calibration
- The medium surrounding the gas inside the ionization chamber is generally the wall of the chamber itself
- The B-G theory can be applied to solids → ratio of 1000 for the density → size of the cavity 1000 x smaller (1 mm of gaseous cavity is comparable to1 μm of solid cavity)

First B-G corollary (1)

- A cavity with volume V is sandwiched between walls made with medium w → the cavity is first filled by gas g_1 with density ρ_1, then by gas g_2 with density ρ_2
- In the two cases the cavity is irradiated by identical radiation producing charges Q_1 and Q_2 respectively →

The dose in g_1 → $$D_1 = D_w \times {}_m\overline{S}_w^{g_1} = \frac{Q_1}{\rho_1 V}\left(\frac{\overline{W}}{e}\right)_1$$

The dose in g_2 → $$D_2 = D_w \times {}_m\overline{S}_w^{g_2} = \frac{Q_2}{\rho_2 V}\left(\frac{\overline{W}}{e}\right)_2$$

⇨ $Q_1 = D_w \ {}_m\overline{S}_w^{g_1} \rho_1 V / \left(\overline{W}/e\right)_1$ and $Q_2 = D_w \ {}_m\overline{S}_w^{g_2} \rho_2 V / \left(\overline{W}/e\right)_2$

First B-G corollary (2)

- The charges ratio →

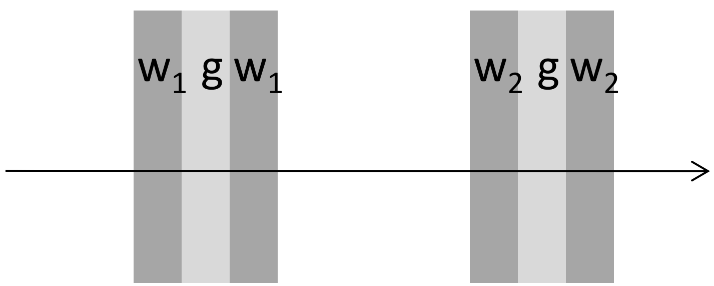
$$\frac{Q_2}{Q_1} = \frac{\rho_2}{\rho_1} \frac{\left(\overline{W}/e\right)_1}{\left(\overline{W}/e\right)_2} \frac{{}_m\overline{S}_w^{g_2}}{{}_m\overline{S}_w^{g_1}}$$

$$\Rightarrow \quad \frac{Q_2}{Q_1} = \frac{\rho_2}{\rho_1} \frac{\left(\overline{W}/e\right)_1}{\left(\overline{W}/e\right)_2} {}_m\overline{S}_{g_1}^{g_2}$$

- No dependency on w → the same Q_2/Q_1 value has to be observed for ≠ w → true if Φ_E of charged particles does not significantly depend on the type of w (true for secondaries e^- produced by γ interacting with Compton effect inside w)

Second B-G corollary (1)

- We consider 2 B-G cavities with an identical gas g (density ρ) but with ≠ volumes (V_1 and V_2) and with walls made with ≠ materials (w_1 and w_2)



- They are irradiated with identical γ or X rays (with energy fluence Ψ)→ Doses D_1 and D_2 in the gas of cavities 1 and 2
- We consider the thickness of the walls of the 2 cavities as strictly larger than the maximal range of the charged particles (« thick walls »)→ CPE

Second B-G corollary (2)

CPE →

$$D_{w_1} \stackrel{\text{CPE}}{=} (K_c)_{w_1}$$

$$D_{w_2} \stackrel{\text{CPE}}{=} (K_c)_{w_2}$$

With: $(K_c)_{w_i} = \Psi \left(\frac{\overline{\mu_{en}}}{\rho} \right)_{w_i}$

$$D_{w_1} \stackrel{\text{CPE}}{=} \Psi \left(\frac{\overline{\mu_{en}}}{\rho} \right)_{w_1}$$

$$= D_1 \; {}_m\overline{S}_g^{w_1} = \frac{Q_1}{\rho V_1} \left(\frac{\overline{W}}{e} \right)_g {}_m\overline{S}_g^{w_1}$$

Second B-G corollary (3)

$$D_{w_2} \overset{\text{CPE}}{=} \Psi \left(\overline{\frac{\mu_{en}}{\rho}} \right)_{w_2}$$

$$= D_2 \; {}_m\overline{S}_g^{w_2} = \frac{Q_2}{\rho V_2} \left(\overline{\frac{W}{e}} \right)_g {}_m\overline{S}_g^{w_2}$$

with Q_1 and Q_2 the charges created inside 1 et 2 and $\left(\overline{\mu_{en}/\rho}\right)_{w_1,w_2}$ the mean mass energy-absorption coefficients weighted on the photons spectra

Second B-G corollary (4)

- The ratio of the charges Q_1 and Q_2 gives thus (with $(W/e)_g$ constant for energies ≈ keV) →

$$\frac{Q_2}{Q_1} = \frac{V_2}{V_1} \frac{\overline{(\mu_{en}/\rho)}_{w_2}}{\overline{(\mu_{en}/\rho)}_{w_1}} \frac{{}_m\overline{S}_g^{w_1}}{{}_m\overline{S}_g^{w_2}}$$

- If Φ_E is identical for the 2 chambers (Compton interaction as noted above) →

$$\frac{Q_2}{Q_1} = \frac{V_2}{V_1} \frac{\overline{(\mu_{en}/\rho)}_{w_2}}{\overline{(\mu_{en}/\rho)}_{w_1}} {}_m\overline{S}_{w_2}^{w_1}$$

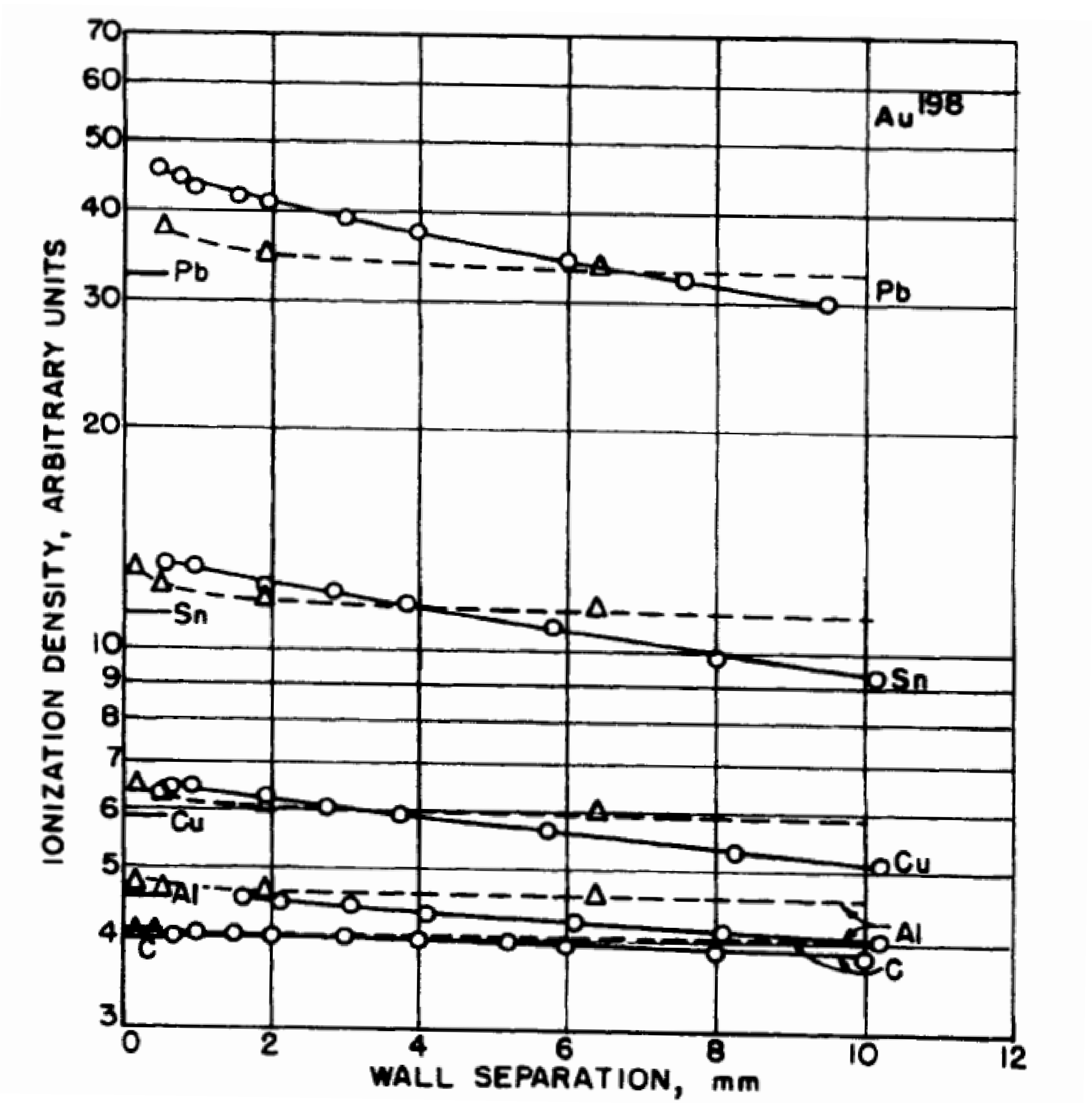
- In this case → Q_2/Q_1 is independent on the choice of gas g

Second B-G corollary for neutrons

$$\frac{Q_2}{Q_1} = \frac{V_2}{V_1} \frac{\overline{F}_{n\,w_2}}{\overline{F}_{n\,w_1}} \frac{{}_m\overline{S}_g^{w_1}}{{}_m\overline{S}_g^{w_2}} \frac{(\overline{W}/e)_1}{(\overline{W}/e)_2}$$

The ratio of *(W/e)* has to be kept in equation if w_1 and w_2 are sufficiently ≠ to produce spectra of heavy charged particles characterized by ≠ *(W/e)* values inside a given gas

Criticism of B-G relation



The B-G theory does not take into account the δ e$^-$ → Modification of the fluence and large modification of the energy of the e$^-$ incident on the cavity

Example: Ionization densities for an ionization chamber (air) with walls made in variable materials as a function of the distance between the walls for γ with energy of 412 keV

Cavity theory of Spencer (Spencer-Attix)

- Consider the effects of δ e^-
- Consider the effects of size of the cavities

- Conditions:
  - The 2 B-G conditions are respected
  - CPE
  - No Bremsstrahlung

Threshold energy

- The cavity is characterized by the Δ parameter (depending on the size of the cavity) defined as the mean kinetic *E* of the e^- with a projected range just large enough to cross the cavity
- The spectric fluence ${}^{\delta}\Phi_E$ (including δ e^-) is divided into 2 components:
  1. The « fast » group: e^- with $E \geq \Delta$ that can transport *E* and that cross the cavity
  2. The « slow » group: e^- with $E < \Delta$ that are assumed to have zero range → they drop their *E* on the spot where $E < \Delta$ → they are not able to enter the cavity nor to transport *E*

Calculation of the dose according S-A theory (1)

- The absorbed dose at a point in w with CPE →

$$D_w \overset{\mathrm{CPE}}{=} \int_{\Delta}^{E_{max}} {}^{\delta}\Phi_E \left(\frac{dE_\Delta}{\rho dx}\right)_{elec,w} dE$$

- $dE_\Delta/\rho dx$: restricted stopping power for e^- including only losses with $E < \Delta$ → only e^- of weak E contribute to the dose → the other ones transport their E elsewhere
- The integral starts at Δ because the e^- with $E < \Delta$ have no range

Calculation of the dose according S-A theory (2)

- A similar expression is obtained for *D* inside the cavity →

$$D_g \overset{\text{CPE}}{=} \int_{\Delta}^{E_{max}} {}^{\delta}\Phi_E \left(\frac{dE_\Delta}{\rho dx}\right)_{elec,g} dE$$

$$\frac{D_g}{D_w} \overset{\text{CPE}}{=} \frac{\int_{\Delta}^{E_{max}} {}^{\delta}\Phi_E \left(\frac{dE_\Delta}{\rho dx}\right)_{elec,g} dE}{\int_{\Delta}^{E_{max}} {}^{\delta}\Phi_E \left(\frac{dE_\Delta}{\rho dx}\right)_{elec,w} dE}$$

Ratio ${}^{\delta}\Phi_E/\Phi_E$

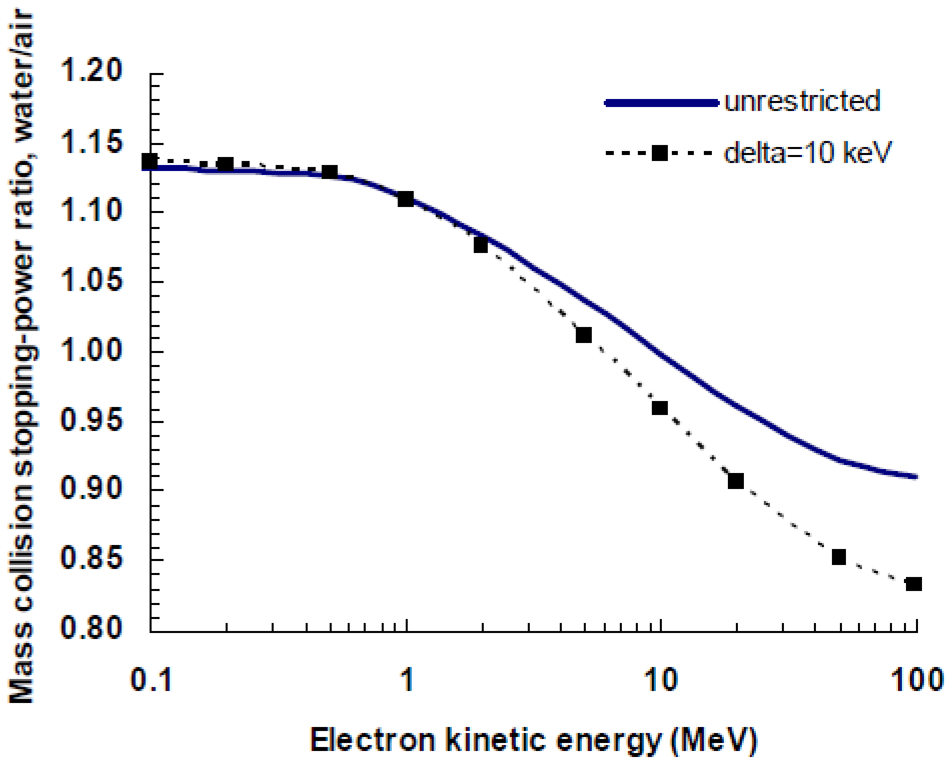
E/E_0	C	Al	Cu	Sn	Pb
1.00	1.00	1.00	1.00	1.00	1.00
0.50	1.00	1.00	1.00	1.00	1.00
0.25	1.05	1.05	1.06	1.06	1.07
0.125	1.21	1.23	1.25	1.27	1.29
0.062	1.60	1.66	1.73	1.79	1.85
0.031	2.4	2.6	2.8	2.9	3.1
0.016	4.4	4.7	5.2	5.5	6.0
0.008	8.5	9.4	10.5	11.3	12.3
0.004	17	19	22	24	—

Mean values for E_0 = 1.31, 0.65 and 0.33 MeV

Comparison between D_g/D_w for S-A and B-G

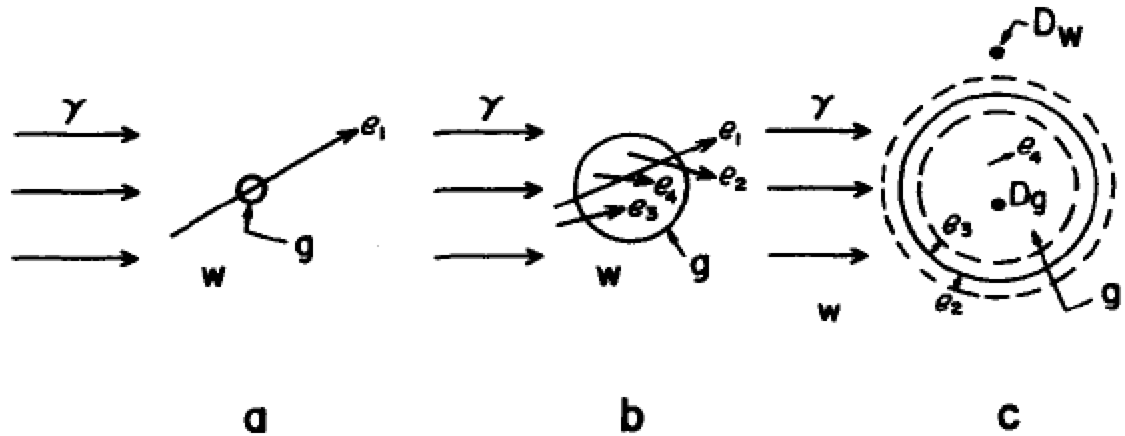
			D_g/D_w						
			Spencer						
Wall Medium	E_0 (keV)	Δ (keV) = Range[b] (mm) =	2.5 0.015	5.1 0.051	10.2 0.19	20.4 0.64	40.9 2.2	81.8 7.2	Bragg-Gray
C	1308		1.001	1.002	1.003	1.004	1.004	1.005	1.005
	654		0.990	0.991	0.992	0.992	0.993	0.994	0.994
	327		0.985	0.986	0.987	0.988	0.988	0.989	0.989
Al	1308		1.162	1.151	1.141	1.134	1.128	1.123	1.117
	654		1.169	1.155	1.145	1.137	1.131	1.126	1.125
	327		1.175	1.161	1.151	1.143	1.136	1.130	1.134
Cu	1308		1.456	1.412	1.381	1.359	1.340	1.327	1.312
	654		1.468	1.421	1.388	1.363	1.345	1.329	1.327
	327		1.485	1.436	1.400	1.375	1.354	1.337	1.353
Sn	1308		1.786	1.694	1.634	1.592	1.559	1.535	1.508
	654		1.822	1.723	1.659	1.613	1.580	1.551	1.547
	327		1.861	1.756	1.687	1.640	1.602	1.571	1.595
Pb	1308		—	2.054	1.940	1.865	1.811	1.770	1.730
	654		—	2.104	1.985	1.904	1.848	1.801	1.796
	327		—	2.161	2.030	1.946	1.881	1.832	1.876

Ratio of stopping powers: restricted/non-restricted



Cavity theory of Burlin (general cavity theory)

- The S-A theory is not suited for very large cavities

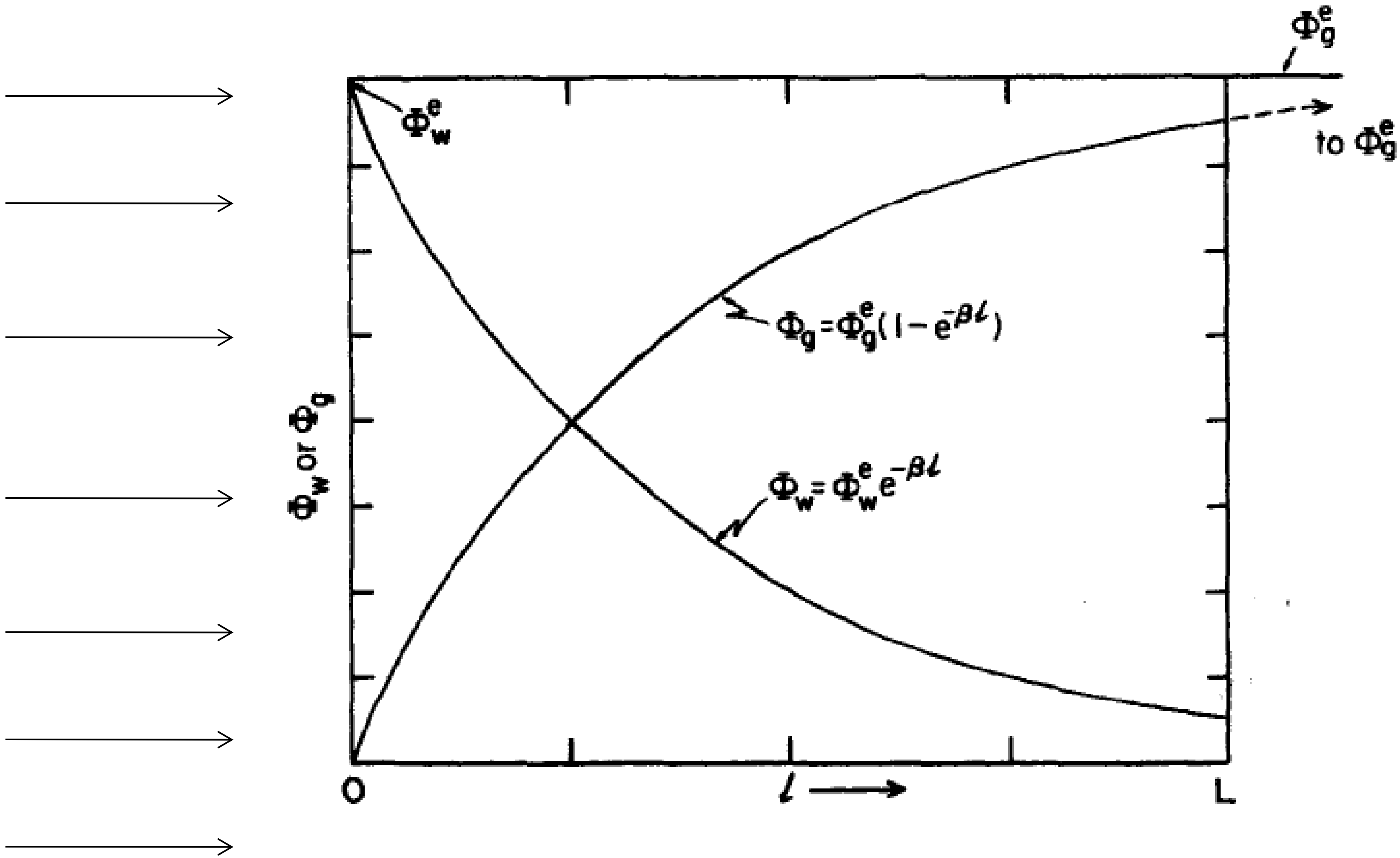


- Size effect:
  a. Small cavity: *D* delivered by « crossers » (e_1)
  b. Intermediate-sized cavity: *D* delivered by crossers (e_1), « starters » (e_2) , « stoppers » (e_3) and « insiders » (e_4) → dose non-uniform
  c. Large cavity: *D* delivered by insiders (e_4) (γ rays)

Burlin conditions

1. Media w (walls) and g are homogeneous
2. A homogeneous γ-ray field exists everywhere throughout w and g (no attenuation)
3. CPE exists at all points in w and g that are farther than the maximum electron range from the cavity boundary
4. The spectra of secondary electrons generated in w and g are the same
5. The fluence e^- entering from the wall is attenuated exponentially as it passes through g (The *E* distribution stays identical)
6. The fluence of e^- that originate in the cavity builds up to its equilibrium value exponentially as a function of distance into the cavity
7. The coefficients od attenuation in 5. and et of increase in 6. are identical: β

Condition 7. for w = g



Burlin cavity relation

$$\frac{\overline{D_g}}{D_w} = d\left({}_m\overline{S}_w^g\right) + (1-d)\left(\frac{\overline{\mu_{en}}}{\rho}\right)_w^g$$

- *d*: parameter of the cavity: 1 for small cavities and 0 for large cavities
- $\overline{D}_g$: mean dose in the medium g of the cavity
- $D_w = (K_c)_w$ (with CPE)
- ${}_m\overline{S}_w^g$: ratio of the electronic stopping powers in g and w (obtained from B-G or S-A)
- $(\overline{\mu_{en/\rho}})_w^g$:ratio of the mass energy-absorption coefficients in g and w

Distance *d* for w = g

- According the definition of Burlin and with *L*, the mean chord of the cavity (*L = 4V/S* for a convex cavity):

$$
\begin{aligned}
d &\equiv \frac{\overline{\Phi}_w}{\overline{\Phi}_w^e} = \frac{\int_0^L \Phi_w^e e^{-\beta l} dl}{\int_0^L \Phi_w^e dl} = \frac{1 - e^{-\beta L}}{\beta L} \\
1 - d &\equiv \frac{\overline{\Phi}_g}{\overline{\Phi}_g^e} = \frac{\int_0^L \Phi_w^e (1 - e^{-\beta l}) dl}{\int_0^L \Phi_w^e dl} = \frac{\beta L + e^{-\beta L} - 1}{\beta L}
\end{aligned}
$$

Distance *d* for w ≠ g

- For w ≠ g → β no more identical for increasing and attenuation → not considered by Burlin but →

$$\frac{\overline{\Phi}_g}{\overline{\Phi}_g^e} \equiv d' \quad \neq \quad (1-d)$$

$$d' + d \quad \neq \quad 1$$

- Burlin choice of β →

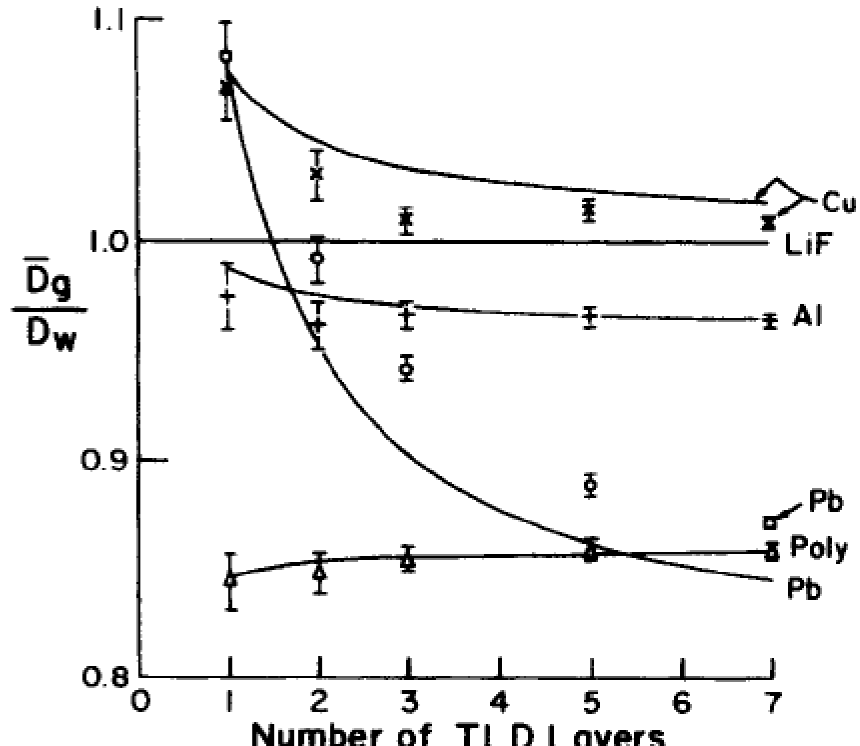
$$e^{-\beta t_{max}} = 0.01$$

with t_{max} the maximal depth of e^- penetration

- Janssens proposes 0.04 (better agreement with experiment)

Application of the Burlin relation

- Dose measured in stacks of LiF thermoluminescence dosimeters sandwiched between equilibrium-thickness walls of various media and irradiated by ^{60}Co-γ



Burlin relation for incident e^-

$$\frac{\overline{D_g}}{D_w} = d\left({}_m\overline{S}^g_w\right)$$

- Small cavity → $d = 1$ → B-G relation
- Large cavity → $d = 0$ and $D_g \cong 0$ → E deposited in a superficial layer of the cavity → zero effect on the mean dose

Other cavity theories

- Other theories exist → more and more complex → sometimes difficult to apply
- Monte Carlo methods make the extreme complexity of cavity theories useless →
  - Simpler to apply
  - Faster
  - Only methods applicable for complex geometries
- The simple cavity theories continue to be useful for simples cases or for a first estimation of complicate cases

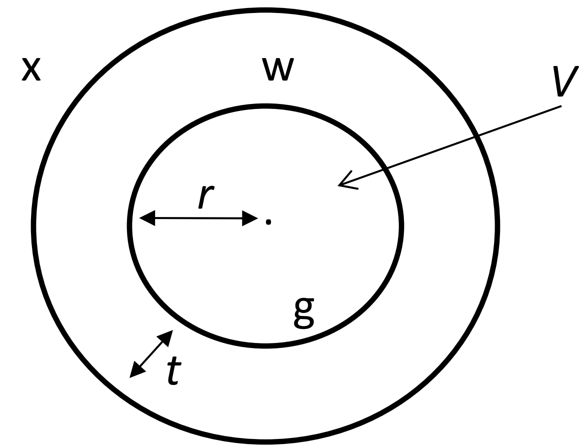
Fano theorem

In an infinite medium of given atomic composition exposed to a uniform field of indirectly ionizing radiation, the field of secondary radiation is also uniform and *independent of the* density of the medium, as well as of density variations from point to point.

Consequences of the Fano theorem

- The fluence of charged particles in all points where CPE exists is independent of density variations within the volume of origin of the particles
- The first B-G condition (small cavity to do not perturb the charged-particle field) may be ignored and replaced by the condition that the walls and the cavity have concordant atomic compositions (water and polystyrene for instance)
- Attention: the polarization effect is neglected → a charged particle crossing a medium polarizes it → density effect → the theorem is valid only if the density effect is ≈ in both media
- Fano theorem demonstrated with transport equation

Simple dosimeter in terms of cavity theory



- Volume *V* (cavity) filled with a medium g (gas, liquid, solid) and surrounded by a wall of medium w
- The wall is at the same time a source of charged secondary particles which are responsible of the dose inside V, a protection against the charged particles originating from the outside x, a filter, a protection against external damages,…

For photons and neutrons (1)

- Photons:

$$D \overset{\text{CPE}}{=} K_c = \Psi\left(\frac{\mu_{en}}{\rho}\right)$$

$$D \overset{\text{TCPE}}{=} K_c(1+\mu'\overline{x}) = K_c\beta = \Psi\left(\frac{\mu_{en}}{\rho}\right)\beta$$

- Neutrons:

$$D \overset{\text{CPE}}{=} K = \Phi F_n$$

$$D \overset{\text{TCPE}}{=} K_c(1+\mu'\overline{x}) = K\beta = \Phi F_n\beta$$

For photons and neutrons (2)

- If t is large enough to exclude charged particles originating from outside and at least as large as the maximum range of the charged particles created inside w
- If r is small enough to satisfy the 1st B-G condition
- If w is irradiated in an uniform way

→ CPE exists inside the cavity
→ The reading of the dosimeter gives D_g
→ B-G, Spencer or Burlin (attention to d) allows to obtain D_w
→ If a medium x replaces g (in the same conditions) →

$$D_x \overset{\mathrm{CPE}}{=} D_w \frac{\overline{(\mu_{en}/\rho)_x}}{\overline{(\mu_{en}/\rho)_w}} \quad \text{for photons}$$

$$D_x \overset{\mathrm{CPE}}{=} D_w \frac{(\overline{F}_n)_x}{(\overline{F}_n)_w} \quad \text{for neutrons}$$

For charged particles

- If *r* is small enough to satisfy the 1st B-G condition
- If *t* is small enough to do not perturb the field

→ Practical rule: the wall and the cavity cannot exceed $\sim$ 1% of the range of the incident charged particles

- If CPE exists for the cavity → occurs if the δ e$^-$ created in the walls counterbalance those escaping from the cavity → obligatory agreement (in atomic number and density) between the materials of the walls and of the cavity

$$\Rightarrow \quad D = \Phi \left(\frac{dE}{\rho dx} \right)_{elec}$$

- For incident e$^-$ → problem of scattering!