Chapter IV:
Notions of equilibrium

Introduction

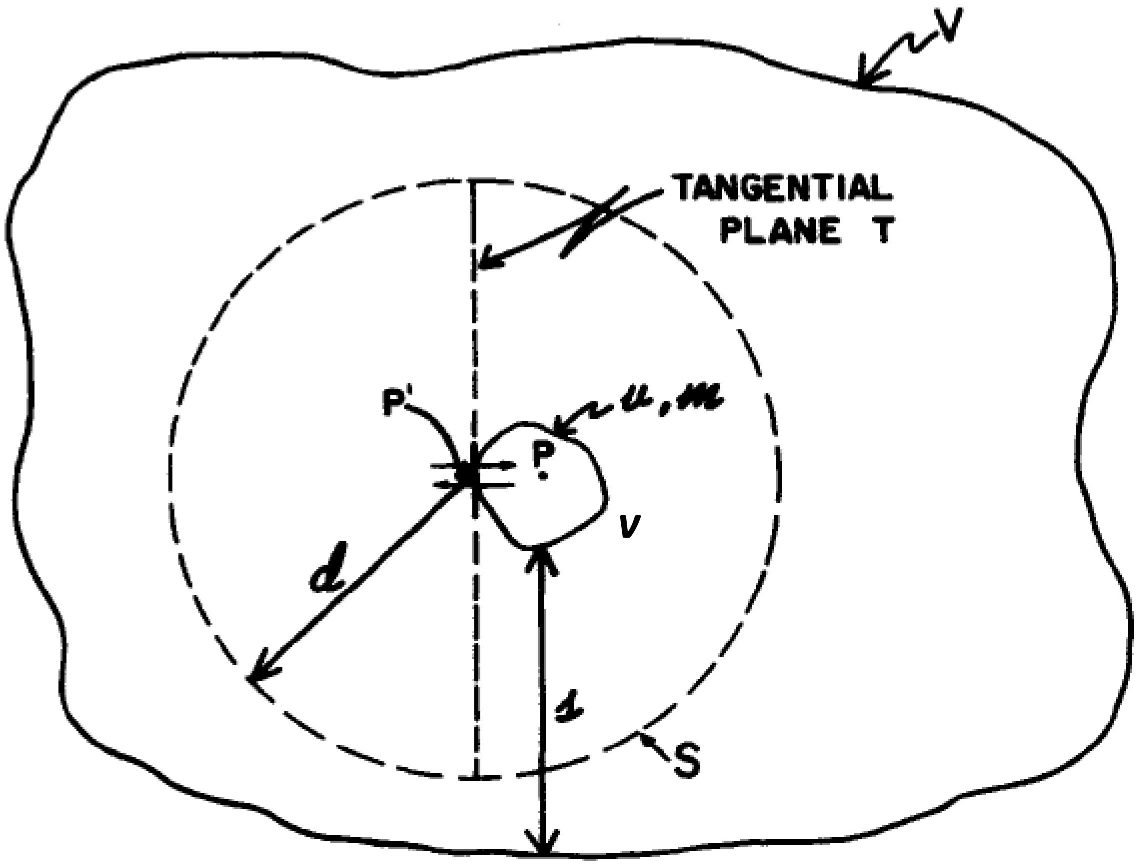
- Equilibrium relations: allow to relate some basic quantities

- Radiation Equilibrium (RE): makes the absorbed dose D equal to the net rest mass converted to energy per unit masse at the point of interest

- Charged-Particle Equilibrium (CPE): makes the absorbed dose D equal to the collision Kerma K_c

- Simplification of the dose calculation if CPE or RE exists

Radiation Equilibrium (RE): geometry



- A volume *V* contains a distributed radioactive source
- A smaller volume *v* exists about a point of interest *P*

Conditions on *V*

- *V* is large enough so that the maximum distance of penetration *d* of any emitted rays — excluding neutrinos — and of scattered and secondary radiations is smaller than *s*, the minimum separation of the boundaries of *v* and *V*
- For charged particles → a maximal range exists → the condition must be exactly realized
- For uncharged particles → attenuation ≈ exponential → the condition cannot be exactly realized → we can require *V* to be large enough to achieve any desired reduction in numbers of rays penetrating from its boundaries to reach *v*

Conditions on the distributed radiative source

1. The atomic composition of the medium is homogeneous

2. The medium is homogeneous → its density is thus homogeneous

3. The radioactive source is uniformly distributed

4. There are no electric or magnetic fields present to perturb the charged particles paths, except the fields associated with the randomly oriented individual atoms

Radiation Equilibrium

The radiation equilibrium in a volume *v* is achieved if, in the nonstochastic limit, for each type and energy of ray entering in the volume *v,* another identical ray leaves

Demonstration of RE

- We consider a point *P'* on the surface of volume *v,* a plane *T* that is tangent to *v* at point *P'* and a sphere *S* of radius d with center *P'* (completely included in *V* because $d < s$)
- The distribution of the source inside the sphere is perfectly symmetrical with respect to plane *T* → for each type and energy of ray crossing the plane in the immediate vicinity of P' in one direction there is an identical ray crossing it in the opposite direction
- Equivalent for all point at the surface of *v* → equality of radiant energies entering and leaving for each type of particles, charged (c) or uncharged (u)

Absorbed dose with RE (1)

$$
\begin{aligned}
(R_{in})_u &= (R_{out})_u \\
(R_{in})_c &= (R_{out})_c
\end{aligned}
$$

$$\bar{\epsilon} = R_{in} - R_{out} + \sum \bar{Q}$$

$$\Rightarrow \quad \bar{\epsilon} = \sum \bar{Q}$$

$$\Rightarrow \quad D = \frac{1}{\rho} \lim_{V \to 0} \overline{\left(\frac{\epsilon}{v}\right)} = \frac{1}{\rho} \lim_{V \to 0} \frac{\sum \overline{Q}}{v} = \frac{d \sum \overline{Q}}{\rho dv}$$

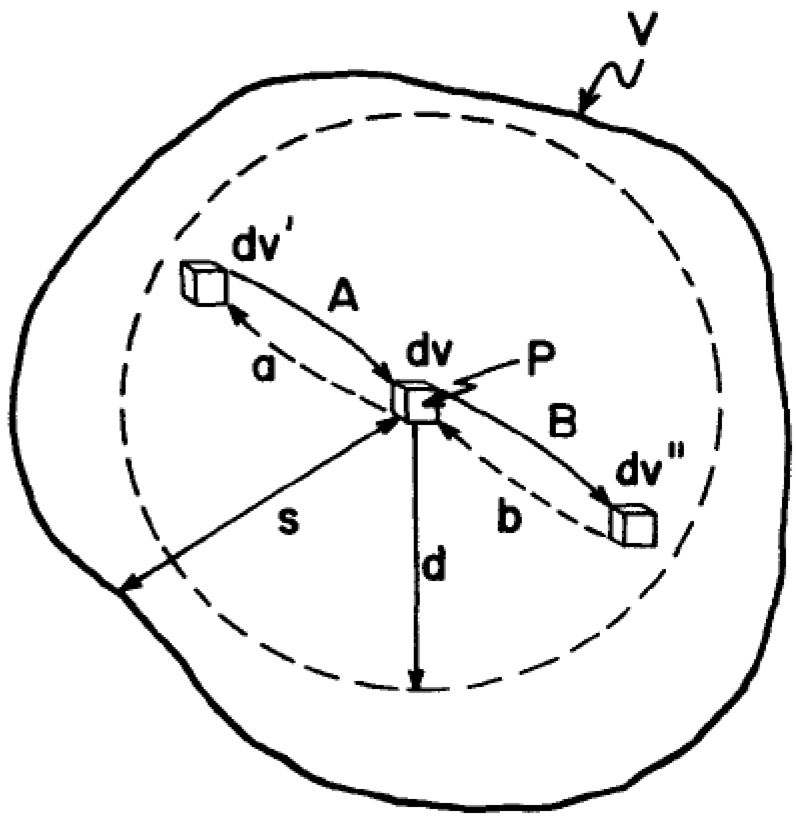
Absorbed dose with RE (2)

If radiation equilibrium exists at a point in a medium, the absorbed dose is equal to the expectation value of the energy released by the radioactive material per unit mass at that point, ignoring neutrinos

Comments on condition 4.

- If presence of a magnetic and/or electric field homogeneous through *V* → the symmetry argument is no more valid → flow of **charged** particles in *P'* is no longer isotropic
- However isotropicity is not a requirement to obtain RE → any source anisotropicity that is **homogeneously** present everywhere in *V* will have no perturbing effect of RE in *v*
- For RE → not necessary that the entering and leaving flows are equal for all points of *v* → RE is reached when the entering and leaving flows *v* are equal for *v* (even if all particles enter by one side and leave by the other side)

Demonstration (1)



- We consider *dv*, an elemental volume in *P* and two elemental volumes, *dv'* and *dv''*, that are symmetrically positioned with respect to *dv*
- Distance between *dv* and *V*: $s > d$

Demonstration (2)

- The first 3 conditions are satisfied but not the 4th → presence of a homogeneous magnetic and/or electric field and the source it-self need not emit radiation isotropically (but the anisotropy is homogeneous through V)
- We suppose a movement G ⇒ D → *A = B* and *a = b* → *a+B = A+b* → flow from *dv* to *dv'+dv''* = flow from *dv'+dv''* to *dv* → the volumes *dv'* and *dv''* can be moved to all possible symmetrical locations inside V (location outside the sphere of radius *d* → nothing comes from *dv*, nothing arrives to *dv*) and the flows can be integrated → each flow in can be replaced by a flow out → RE in *P*

Charged-particle equilibrium (CPE)

Charged particle equilibrium (CPE) exists for the volume *v* if each charged particle of a given type and energy leaving *v* is replaced by an identical particle of the same energy entering

$$(R_{in})_c = (R_{out})_c$$

Clearly if **RE exists → CPE exists**
Si RE absent → CPE can exist

Distributed radioactive Sources

1. Distributed source of charged particles (+ negligible radiative losses): trivial case → only charged particles are emitted and radiative losses are negligible → with $s > d$ and the previous 4 conditions → CPE and RE exist for the volume v

2. Distributed source of uncharged particles: CPE implies that RE is obtained

3. Distributed source of charged and uncharged particles: charged particles and more-penetrating neutral particles are emitted → 3 possible cases

Distributed source of charged and uncharged particles (1)

a. *d* is the maximum range of the charged particles and *V* is just large enough to have *s* > *d* → uncharged particles escape from *V* without any interaction (→ without production of charged secondaries) → only the primary charged particles have to be considered in the symmetry argument → CPE exists but not RE

$$
\begin{aligned}
(R_{in})_u &< (R_{out})_u \\
(R_{in})_c &= (R_{out})_c
\end{aligned}
$$

The uncharged particles escape from *v* and are not replaced because there is no source outside of *V* →

$$\bar{\epsilon} = (R_{in})_u - (R_{out})_u + \sum \bar{Q}$$

$$D = \frac{d\left[(R_{in})_u - (R_{out})_u + \sum \overline{Q}\right]}{\rho dv}$$

Distributed source of charged and uncharged particles (2)

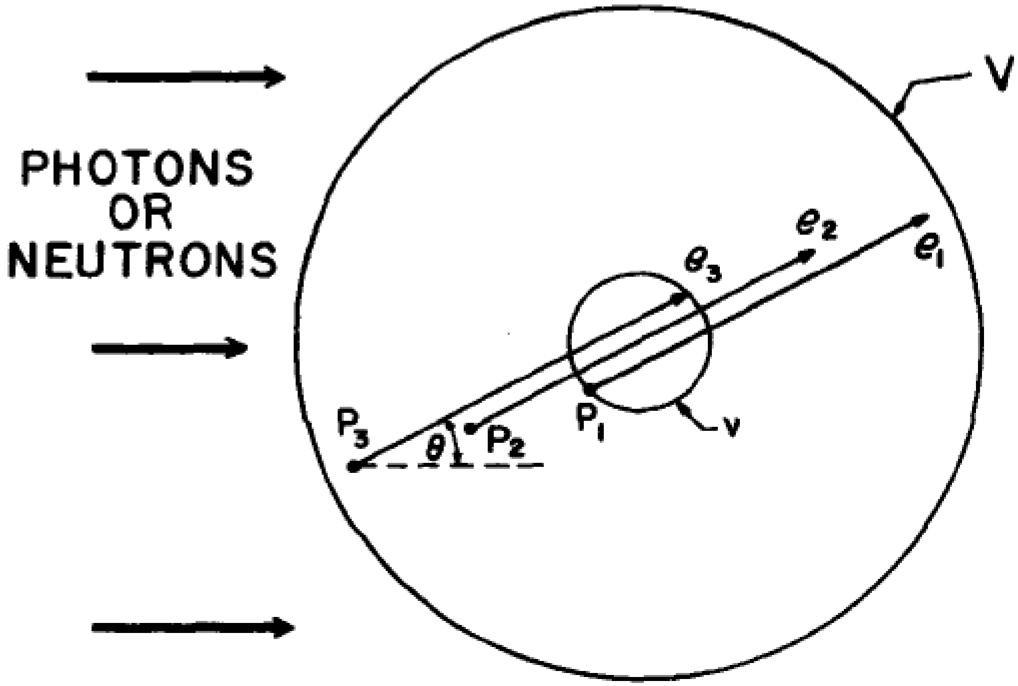
b. If V ↗ so that to be larger than the « effective » range of the uncharged particles and their secondaries → $(R_{in})_u$ ↗ up to have:

$$(R_{in})_u \quad = \quad (R_{out})_u$$

→ RE is restored

c. In intermediate cases (*V* large enough for CPE but not for RE): difficulties → some fraction of the *E* of the uncharged particles will be absorbed in *v* but it is relatively difficult to determine what that fraction is (see below)

External source of uncharged particles



In *V*: volume *v* such as the distance between the 2 boundaries is larger than the maximum range of all **charged particles** present

Necessary conditions for CPE

1. The atomic composition of the medium is homogeneous

2. The medium is homogeneous, its density is thus homogeneous

3. There exists a uniform field of indirectly ionizing radiation and the rays are very penetrating

4. There is no inhomogeneous electric/magnetic field

Demonstrations

- Demonstration 1: flow of uncharged particles is uniform + homogeneous medium → number of charged particles produced per unit volume is uniform everywhere in *V* however this production is not isotropic (anisotropic distributions of production of the charged particles by the uncharged particles) but this anisotropy is homogeneous → the charged particles slow down in a homogeneous medium → CPE for *v* (previous demonstration)
- Demonstration 2: charged particles follow a straight line and are all emitted with an angle θ → particles e_1, e_2 and e_3 → all 3 deposit the *E* that would be deposited by e_1 if all its *E* would be absorbed in *v* → this kind of combinations is always possible → CPE

Absorbed dose in CPE (1)

with CPE → $\bar{\epsilon} = (R_{in})_u - (R_{out})_u + \sum \bar{Q}$

avec → $\bar{\epsilon}_{tr}^{n} = (R_{in})_u - (R_{out})_u^{nonr} - (R_{out})_u^{rad} + \sum \bar{Q}$

⇒ $\bar{\epsilon}_{tr}^{n} = \bar{\epsilon} + (R_{out})_u - (R_{out})_u^{nonr} - (R_{out})_u^{rad}$

Remind for the 3 last terms →

$(R_{out})_u$: Radiant *E* leaving *v* for uncharged particles

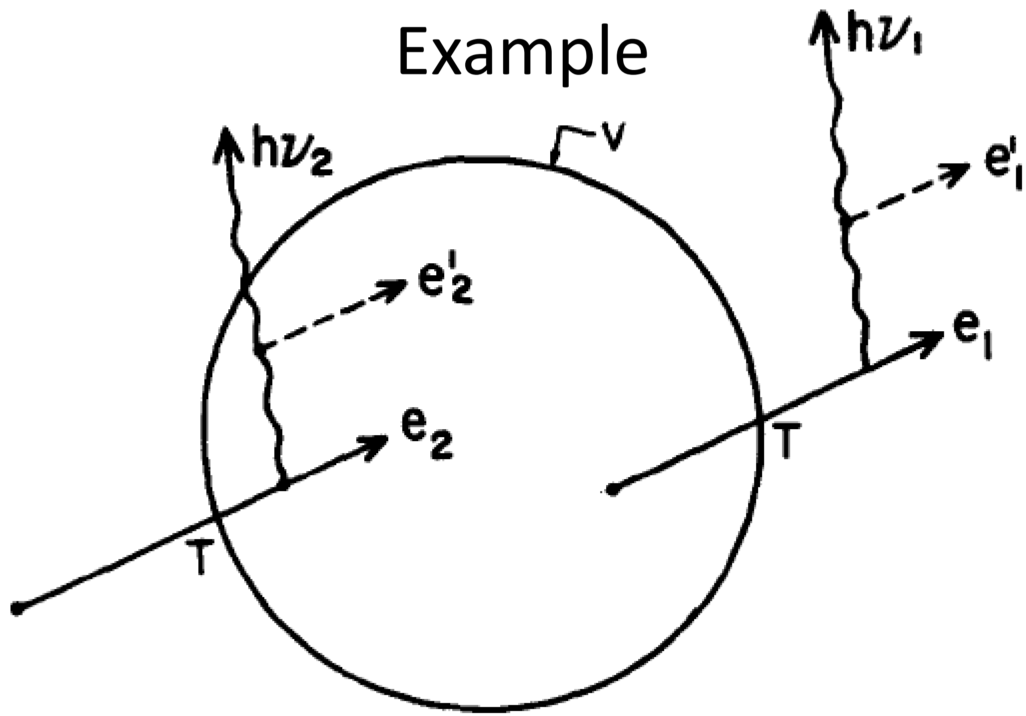
$(R_{out})_u^{nonr}$: Radiant *E* leaving for uncharged particles not including the energy originating from the radiative losses undergone by the charged particles set in motion in *v*

$(R_{out})_u^{rad}$: radiant *E* equal to the sum of all radiative losses in *V*, including *v*, undergone by the particles set in motion in *v*

Absorbed dose in CPE (2)

- For small *v* → $(R_{out})_u = (R_{out})_u^{nonr} + (R_{out})_u^{rad}$
- Demonstration: With CPE → for each charged particle entering *v* there is a corresponding particle with same *E* and same direction leaving *v*
- Due to hypothesis (homogeneousness) → for each event that happens in *v* it corresponds an identical event that happens ouside *v*
- Uncharged particles are very penetrating → for small *v* → the uncharged particles resulting from radiative losses in *v* escape from *v* without undergoing any interaction
- The radiative losses in *v* for an entering e^- contribute to $(R_{out})_u$ and the identical radiative losses outside *v* due e^- set in motion in *v* contribute to $(R_{out})_u^{rad}$ (for $(R_{out})_u^{nonr} = 0$)
- The homogeneousness implies that both contributions are equal on the condition that the uncharged particles produced in *v* escape from *v*

Example



<u>Case 1</u> (solid line): $h\nu_2$ escape → $(R_{out})_u = h\nu_2$ but $(R_{out})_u^{rad} = h\nu_1$ and by definition: $h\nu_1 = h\nu_2$ (with $(R_{out})_u^{nonr} = 0$)

<u>Case 2</u> (dashed line): $h\nu_2$ is absorbed and produces e'_2 → $(R_{out})_u = 0$ but $(R_{out})_u^{rad} = h\nu_1$ as before (with always $(R_{out})_u^{nonr} = 0$) → equation not satisfied → *v* must be small enough to allow that radiative losses escape

Absorbed dose in CPE (3)

$$\bar{\epsilon} = \bar{\epsilon}^{n}_{tr}$$

With *v* → *dv* (→ **small by** définition), we can write →

$$\frac{d\bar{\epsilon}}{dm} = \frac{d\bar{\epsilon}^{n}_{tr}}{dm}$$

$$D \stackrel{\text{CPE}}{=} K_c$$

Absorbed dose in CPE (4)

With charged-particle equilibrium, the absorbed dose in a medium irradiated by a uniform flow of uncharged particles is equal to the collision Kerma

Dose ↔ Energy fluence/fluence

Photons ⇒

$$K_c = \Psi \left(\frac{\mu_{en}}{\rho}\right)_{E,Z}$$

$$\Rightarrow \quad D \stackrel{\text{CPE}}{=} \Psi \left(\frac{\mu_{en}}{\rho}\right)_{E,Z}$$

Neutrons ⇒

$$K = K_c$$

$$K = \Phi (F_n)_{E,Z}$$

$$K = \Phi \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} E$$

$$\Rightarrow \quad D \stackrel{\text{CPE}}{=} \Phi \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} E$$

Application to 2 ≠ media

- We consider a given energy fluence (photons) or fluence (neutrons) in 2 media (A and B) having two ≠ average mass energy-absorption coefficients (photons) or ≠ mean Kerma factors (neutrons) →
- Photons:

$$\frac{D_A}{D_B} \stackrel{\text{CPE}}{=} \frac{(K_c)_A}{(K_c)_B} = \frac{\overline{(\mu_{en}/\rho)}_A}{\overline{(\mu_{en}/\rho)}_B}$$

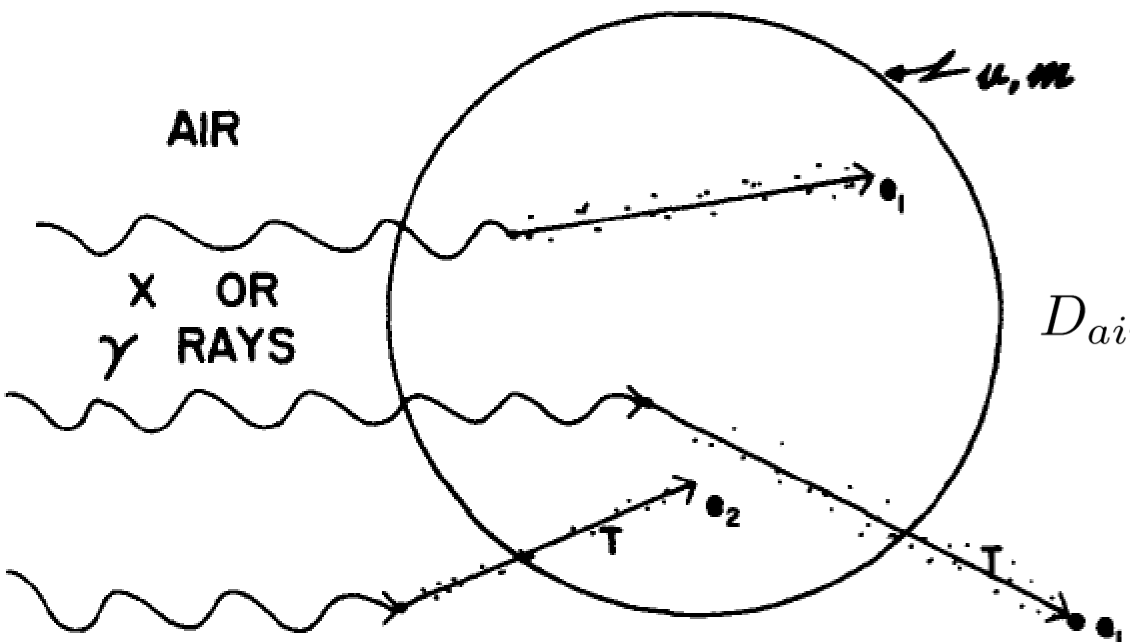
- Neutrons:

$$\frac{D_A}{D_B} \stackrel{\text{CPE}}{=} \frac{K_A}{K_B} = \frac{\overline{(F_n)}_A}{\overline{(F_n)}_B}$$

- $D_A \neq D_B$ because of atomic compositions ≠ or energy spectra ≠

Application of CPE to exposure

- Definition of exposure → measure of ionizations produced **everywhere** by all secondaries e⁻ liberated in a given volume
- With CPE (in a **small** ionization chamber) → measure of the ionization collected in a given air volume



$$D_{air} \overset{\text{CPE}}{=} (K_c)_{air} = X \left(\frac{W}{e} \right)_{air}$$

Causes of CPE failure for uncharged particles

1. Inhomogeneity of atomic composition within V

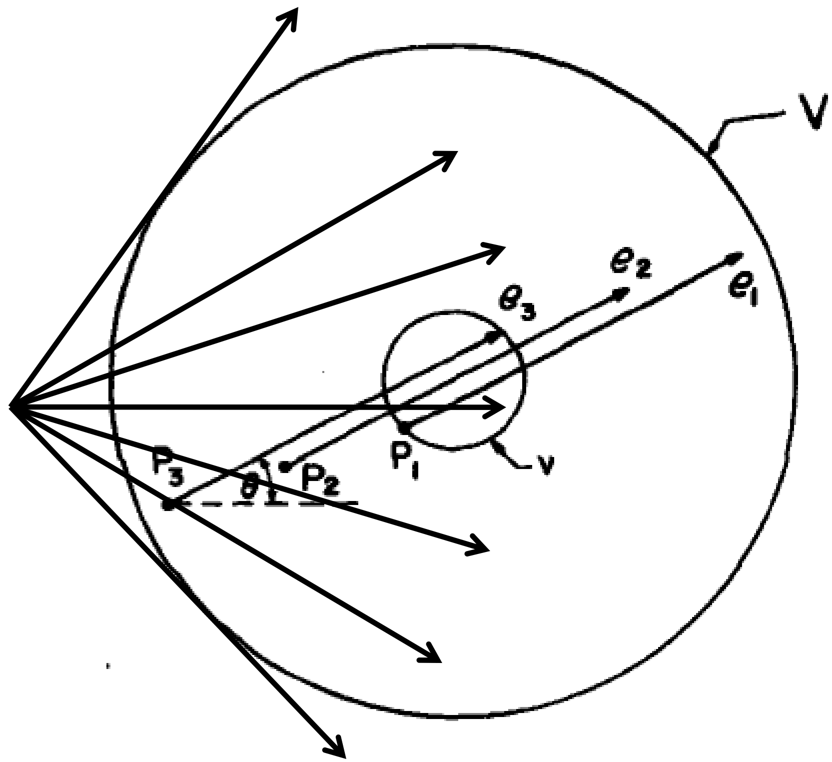
2. Inhomogeneity of density in V

3. Non-uniformity of the field of uncharged particles in V

4. Presence of a non-homogeneous electric/magnetic field in V

Causes of CPE failure: practically (1)

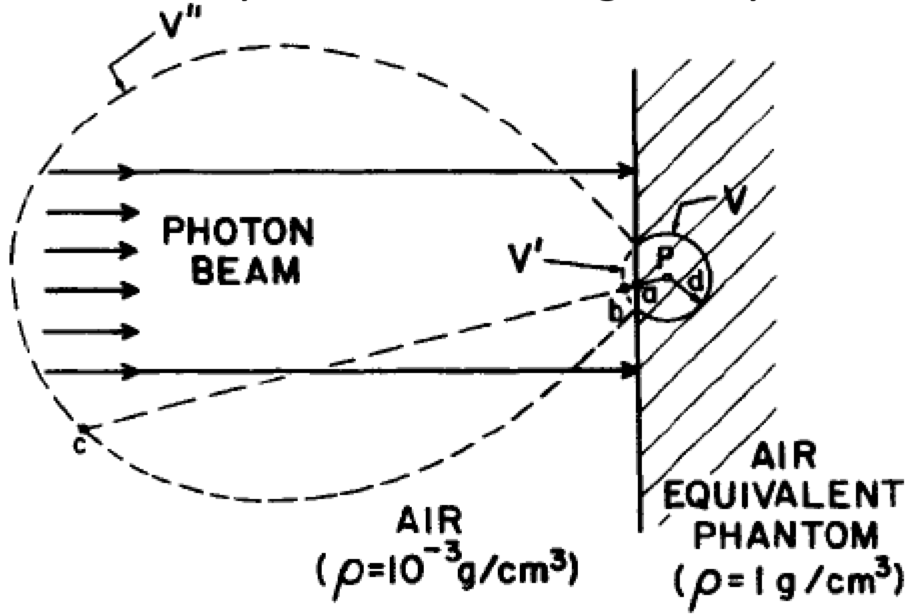
- Proximity to a source



- If the source is too close → non-uniform energy fluence → number of e_3 > number of e_1 → CPE fails

Causes of CPE failure: practically (2)

- Proximity to a boundary or an inhomogeneity



- Phantom has the atomic composition of air → only density changes (factor 1000)
- To replace *V'* missing in the solid → *V''* 1000 x larger (density 1000 x smaller) → e^- starting from *b* if solid has to start from *c* in air → beam not wide enough to irradiate *c* although it irradiates *b* → *V''* not irradiated homogeneously → failure of CPE at the boundary of the surface

Causes of CPE failure: practically (3)

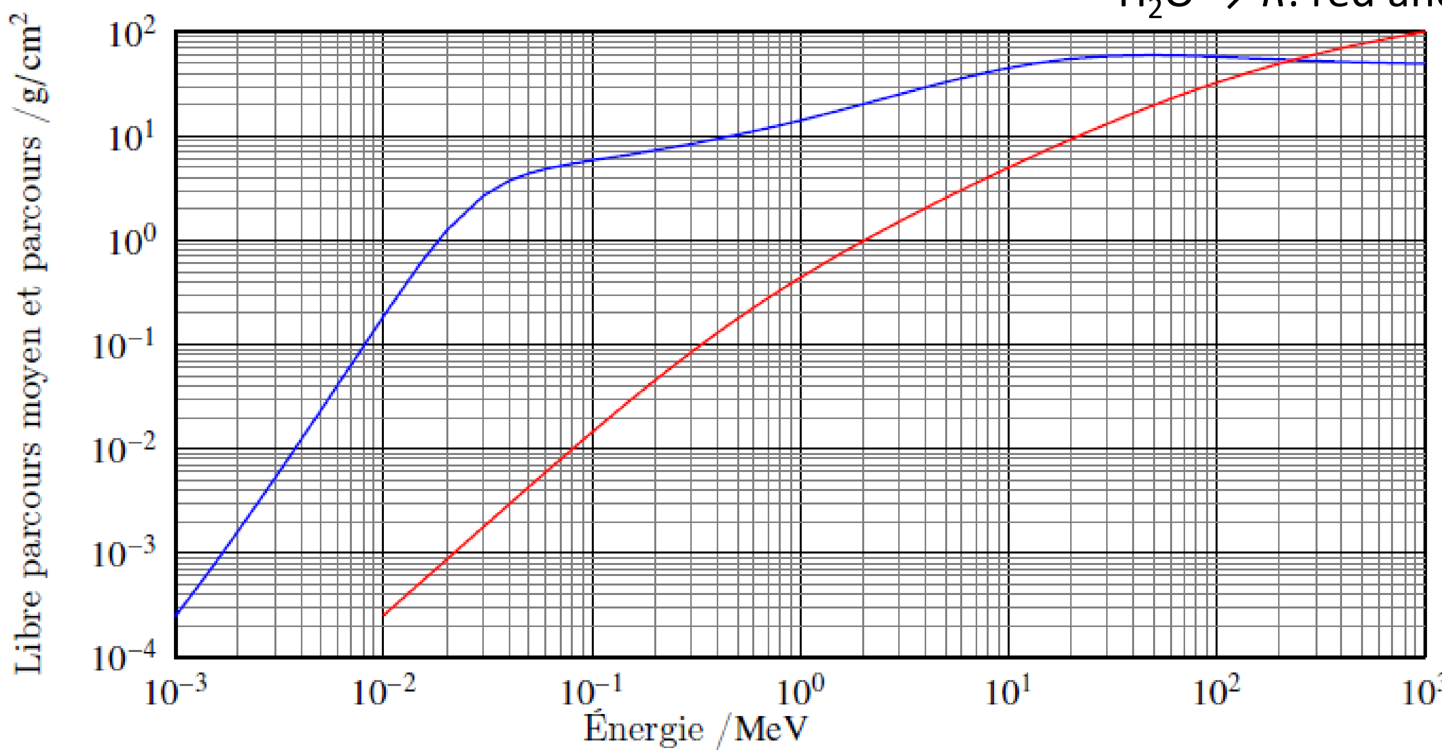
- Moreover → even if *V''* were uniformly irradiated → e^- scattered close to *c* will miss the phantom whereas the same e^- scattered close to *b* with the same angle will reach the volume

- Failure of CPE due to change of density at the boundary and to geometric factors

Causes of CPE failure: practically (4)

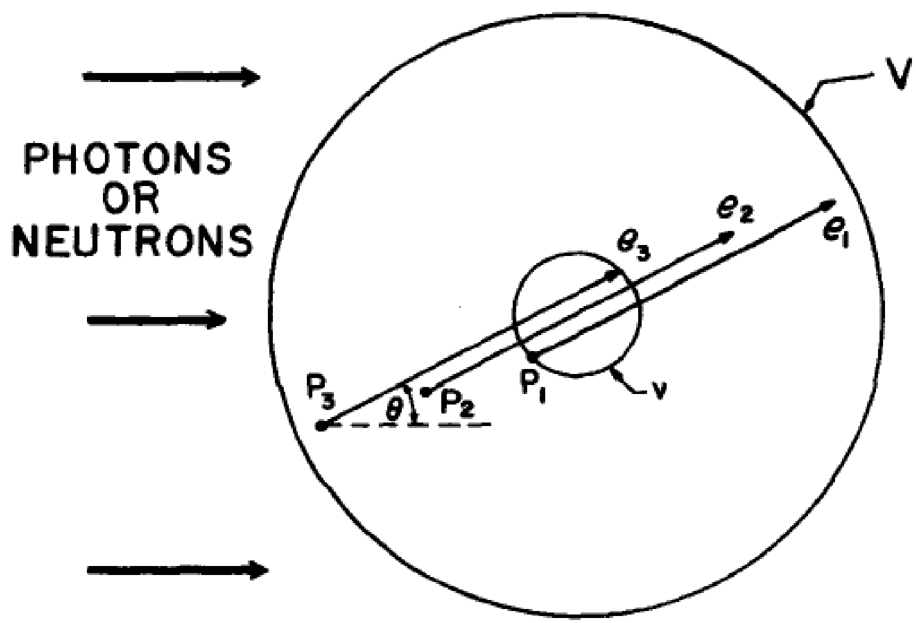
- High energy → failure of CPE
- The range R of an e^- increases more rapidly as a function of E than the mean free path λ of a photon → For $E \approx 200$ MeV → $R = \lambda$

H_2O → R: red and λ : blue



Causes of CPE failure: practically (5)

- $s > d$ is always necessary → but along d: attenuation of the beam of uncharged particles



- Number of charged particles generated at P_3 > at P_1 → fails of CPE (all the more so since E ↗)

Causes of CPE failure: practically (6)

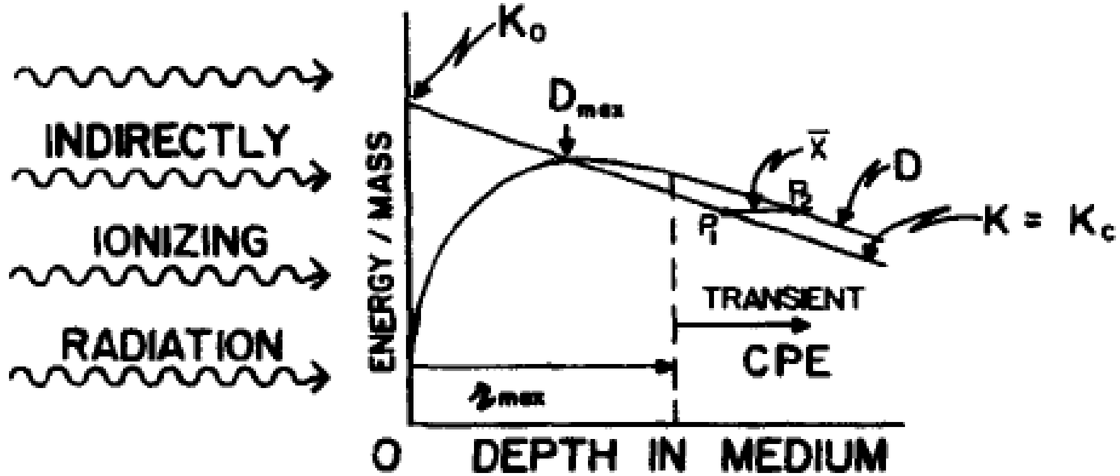
- we consider thus a measure of exposure can be made only for γ or RX with *E* < *3* MeV

- For *E* > *3* MeV → the definition of exposure is always valid but its measure cannot be made (because it depends on CPE)

- If it is possible to attain « another » known relation between D_{air} and $(K_c)_{air}$ (possibly more complicate than a simple =)→ exposure can still be measured → situation called Transient Charged-Particle Equilibrium or TCPE

Transient Charged-Particle Equilibrium (1)

- TCPE exists if, at all points within a region, there is a relation such as $D = \beta K_c$ with $\beta > 1$

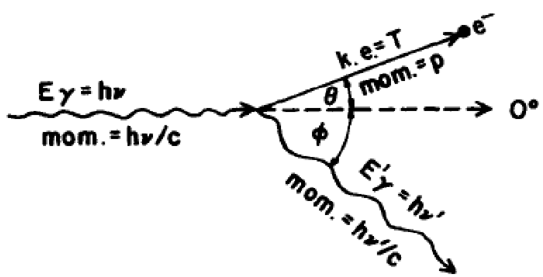


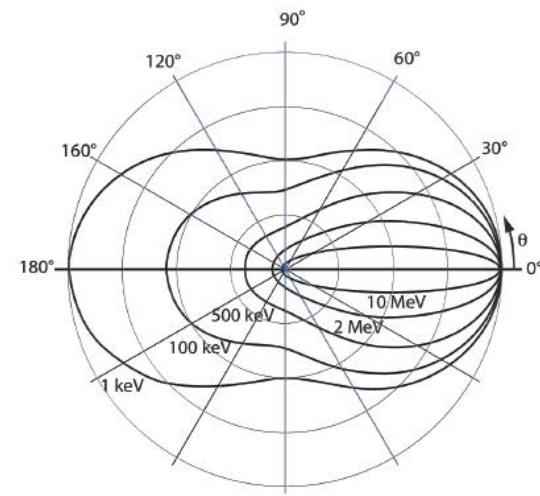
- We consider a broad beam geometry: incident beam of uncharged particles ⊥ to the material and we have $K = K_c$
- Kerma at surface: K_0 → exponential attenuation (K curve)

Transient Charged-Particle Equilibrium (2)

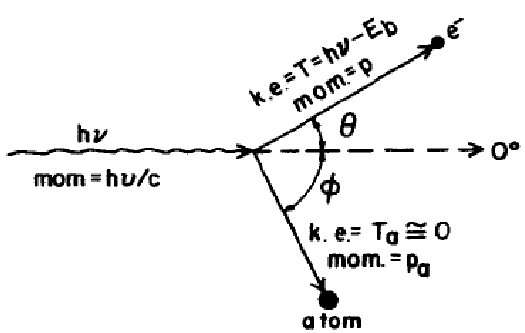
- The dose *D* ↗ first with increasing depth (with $\beta < 1$) → population of charged particles moving to the right ↗ for a number of uncharged particles interactions ↗ → contribution to the dose ↗

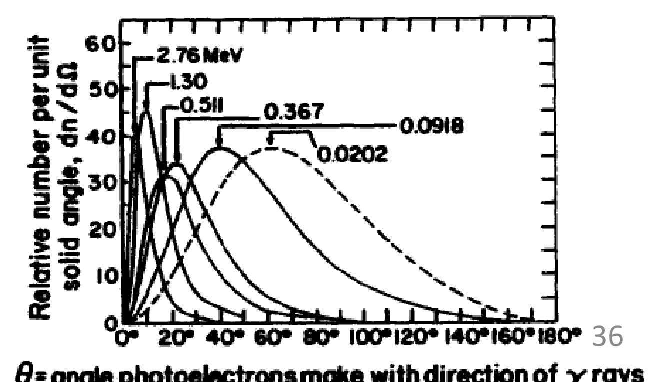
<u>Compton</u>





<u>Photoelectric</u>





Transient Charged-Particle Equilibrium (3)

- The dose reaches a maximum D_{max} corresponding to an equilibrium → the increase of charged particles is counterbalanced by the attenuation of the beam of uncharged particles in the material
- D_{max} occurs approximately for D crossing K (except if the incident beam is « contaminated » by charged particles → D_{max} is shifted to the surface and $D \neq K$ at D_{max})
- r_{max} corresponds to the maximum distance the secondary charged particles starting at the surface can penetrate in the direction of the incident rays
- After r_{max} → D ↘ and becomes ∥ to K (even if the slopes can change together with depth) → TCPE
- $D > K$ due to backscattering

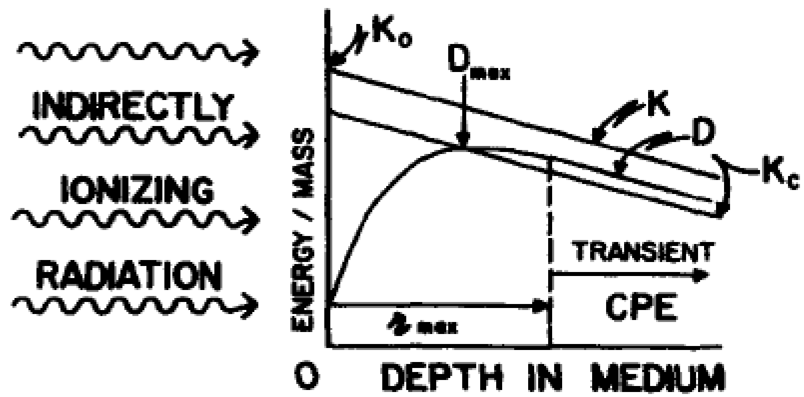
Transient Charged-Particle Equilibrium (4)

$$D \overset{\text{TCPE}}{=} K_c e^{\mu' \overline{x}}$$
$$\overset{\text{TCPE}}{=} K_c (1 + \mu' \overline{x})$$

- Radiative interactions are neglected
- μ‘ is the slope of *K* (K_c) and *D*
- $\overline{x}$ is the mean distance the secondary charged particles carry their kinetic energy in the direction of the primary rays (distance between P_1 and P_2 for which *K* and *D* are =)
- $\overline{x}$ and μ‘ are not generally known

Transient Charged-Particle Equilibrium (5)

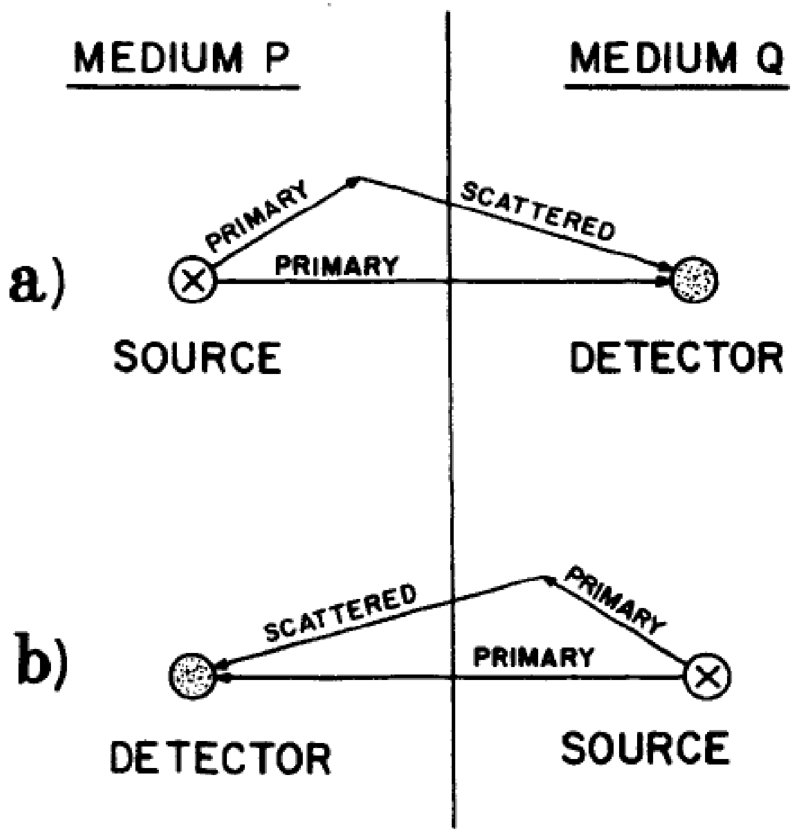
- If K_r cannot be neglected →



- K is larger than K_c and D by the amount: $K_r = [(\mu_{tr}-\mu_{en})/\mu_{tr}]K$ if the photons produced during the radiative losses escape from the medium

Reciprocity theorem: infinite homogeneous medium

- Simplest case: *Reversing the positions of a point detector and a point source within an infinite homogeneous medium does not change the amount of radiation detected*



Reciprocity theorem: infinite non-homogeneous medium

- With P ≠ Q (properties of scattering and/or attenuation are ≠) → the transmission of primary rays is equal

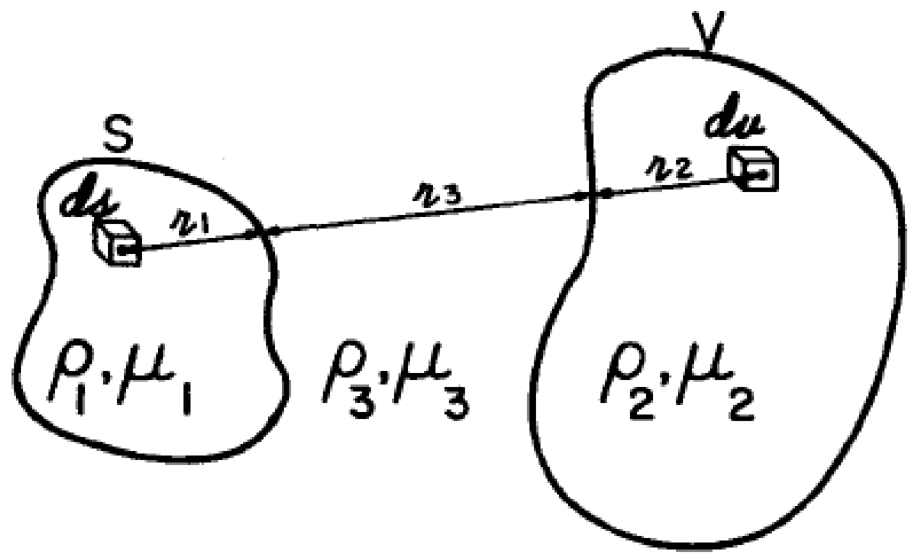
- With P ≠ Q → creation and/or transmission of scattered rays *may* differ

- If primary rays are dominating or if the propagation of secondaries is not « too » ≠ in 2 media → the theorem is still valid

Reciprocity theorem: extended source and detector

- The integral dose in a volume *V* due to a y-ray source uniformly distributed throughout source volume *S* is equal to the integral dose that would occur in *S* if the same activity density per unit mass were distributed throughout *V*
- Mayneord theorem→ « dose » stated in roentgens→ implicitly *S* and *V* are in air
- Theorem valid if μ_{en}/ρ is equal for media *S* and *V*
- Theorem also valid for neutrons

Demonstration of the theorem of Mayneord (1)



- Each region is homogeneous
- S contains a uniformly distributed γ-ray source with specific activity *A’* (Bq/kg) → the elementary volume *ds* (m^3) contains an activity $dA = A'\rho_1 ds$ (Bq)
- Each atomic decay emits one γ-ray with the single energy *E* (MeV), the theorem is still valid for multienergy sources
- Narrow beam attenuation is considered → μ_i

Demonstration of the theorem of Mayneord (2)

- Consider a volume element *dv* in *V* at a distance $r = r_1+r_2+r_3$ away from *ds*
- Without attenuation → the fluence Φ (photons) at *dv* for an irradiation time Δt (s) →

$$\Phi = \frac{dA\Delta t}{4\pi r^2}$$

- The energy fluence (MeV/m^2) →

$$\Psi = \Phi E$$

- The collision Kerma at *dv* (J/kg)

$$K_c = \Psi \left(\frac{\mu_{en}}{\rho_2} \right)_{E,V}$$

Demonstration of the theorem of Mayneord (3)

- With CPE → $K_c = D$ → the dose D (MeV/kg) at dv due to the source at ds (without attenuation) →

$$D = \frac{A'\rho_1 \Delta t E(\mu_{en}/\rho_2)_{E,V} ds}{4\pi r^2}$$

- If attenuation of only primaries is considered → the total dose D_{tot} at dv due to the source at S →

$$D_{tot} = \frac{A'\rho_1 \Delta t E(\mu_{en}/\rho_2)_{E,V}}{4\pi} \int_S \frac{e^{-(r_1\mu_1 + r_2\mu_2 + r_3\mu_3)}}{r^2} ds$$

- The integral dose (MeV): $D(V,S) = \int D_{tot}\rho_2 dv$ →

$$D(V,S) = \frac{A'\rho_1 \rho_2 \Delta t E(\mu_{en}/\rho_2)_{E,V}}{4\pi} \int_V \int_S \frac{e^{-(r_1\mu_1 + r_2\mu_2 + r_3\mu_3)}}{r^2} dv ds$$

Demonstration of the theorem of Mayneord (4)

- The integral dose *D(S,V)* at *S* for a source with same specific activity *A'* at *V* →

$$D(S,V) = \frac{A'\rho_1\rho_2\Delta t E(\mu_{en}/\rho_1)_{E,S}}{4\pi}\int_S\int_V \frac{e^{-(r_1\mu_1+r_2\mu_2+r_3\mu_3)}}{r^2}dsdv$$

- *D(S,V) = D(V,S)* if $(\mu_{en}/\rho_1)_{E,S} = (\mu_{en}/\rho_2)_{E,V}$
- Theorem of Mayneord is demonstrated for primary rays only, for 2 volumes with equal μ_{en}/ρ and with CPE
- If homogeneous medium → reciprocity guaranteed for scattered (symmetry arguments)

Corollaries of the theorem of Mayneord

- If *S* and *V* contain identical, uniformly distributed total activities, they will each deliver to the other the same average absorbed dose.
- If all the activity in *S* is concentrated at an internal point *P*, then the dose at *P* due to the distributed source in *V* equals the average dose in *V* resulting from an equal source at *P*
- The dose at any internal point *P* in *S* due to a uniformly distributed source throughout *S* itself is equal to the average absorbed dose in *S* resulting from the same total source concentrated at *P*
- → useful in calculations of internal dose due to distributed sources in the body (although, strictly speaking, only valid for primaries or for an infinite homogeneous medium)

Reciprocity theorem for charged particles

- For charged particles in an infinite homogeneous medium → substitution of the exponential attenuation by an empirical derived function (method of Loevinger) → reciprocity theorem valid for primaries and scattered

- Reciprocity theorem only depends on symmetry arguments

Application: absorbed dose in a radioactive medium

- Examples of CPE and RE conditions for radioactive media
- Importance of the size of the medium
- More examples during exercices

Size of the medium (1)

We consider a radioactive medium of spherical shape and with radius *r* emitting both charged particles and γ (as it is often the case)→ the size of the radioactive object is essential

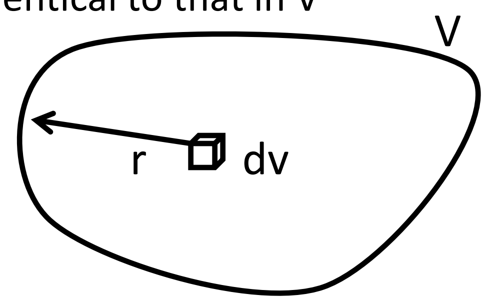
1. $d < r \ll 1/\mu$ → CPE: all γ escape (and are not backscattered) and all charged particles created at any point *P* that is at least a distance *d* from the boundary of *V* give all their *E* → *D* at *P* equal to the *E* per unit mass of medium that is given to charged particles in radioactive decay (radiative losses are neglected: true for small Z)
2. $r \gg 1/\mu$ → RE at any internal point *P* that is far enough from the boundary → *D* at *P* equal to the *E* per unit mass of medium that is given to charged particles plus γ in radioactive decay

Size of the medium (2)

3. $r \sim 1/\mu$ → more complicate: only a part of the γ gives its *E* to the medium → definition of the absorbed fraction - *AF*:

$$AF = \frac{\gamma\text{-ray radiant energy absorbed in target volume}}{\gamma\text{-ray radiant energy emitted by the source}}$$

Infinite homogeneous medium identical to that in V



$$AF_{dv,V} = \frac{\overline{\epsilon}_{dv,V}}{\overline{R}_{dv}}$$

Reciprocity theorem → $\overline{\epsilon}_{dv,V} = \overline{\epsilon}_{V,dv}$

(only approached if vacuum outside V)

$$AF_{dv,V} = \frac{\overline{\epsilon}_{V,dv}}{\overline{R}_{dv}}$$

→ *AF* = *D* at *P* / *D* at *P* with RE

Absorbed fraction

- Thus → if *x%* of the emitted γ-rays at *dv* escape from *V* → decrease of *x%* in *D* at *P* by comparison to the dose with RE → $AF_{dv,V} = 1\text{-}x\%$
- With $\overline{\mu}'$ the mean effective attenuation coefficient for an energy fluence of γ along the distance *r* inside the medium → the fraction of γ escaping following the *r* direction is $\exp\left(-\overline{\mu}' r\right)$

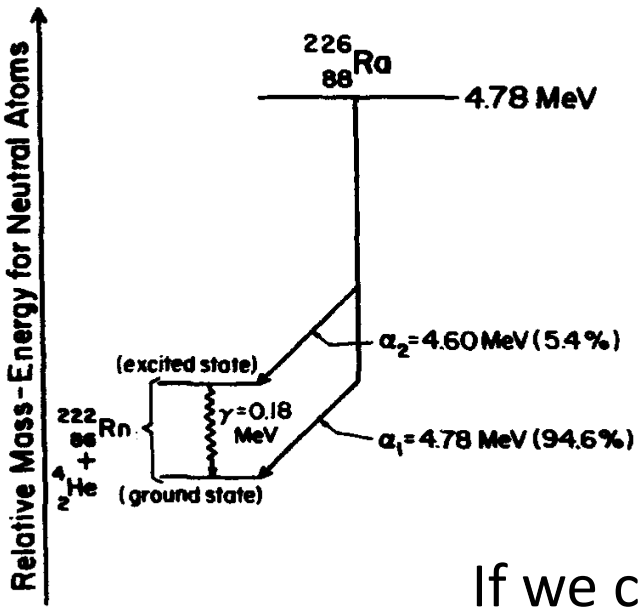
$$AF_{dv,V} = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}\left(1 - e^{-\overline{\mu}' r}\right)\sin\theta d\theta d\phi$$

- If we consider the straight-ahead approximation → $\mu_{en} \simeq \overline{\mu}'$ and

$$\overline{r} = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} r \sin\theta d\theta d\phi \quad \Longrightarrow \quad AF_{dv,V} \simeq 1 - e^{-\mu_{en}\overline{r}}$$

$\alpha + \gamma$ disintegration

$$^{226}_{88}\mathrm{Ra} \rightarrow \, ^{222}_{86}\mathrm{Rn} + \, ^{4}_{2}\mathrm{He} + 4.78\,\mathrm{MeV}$$



E_{α} = 0.946 x 4.78 + 0.054 x 4.60 = 4.77 MeV

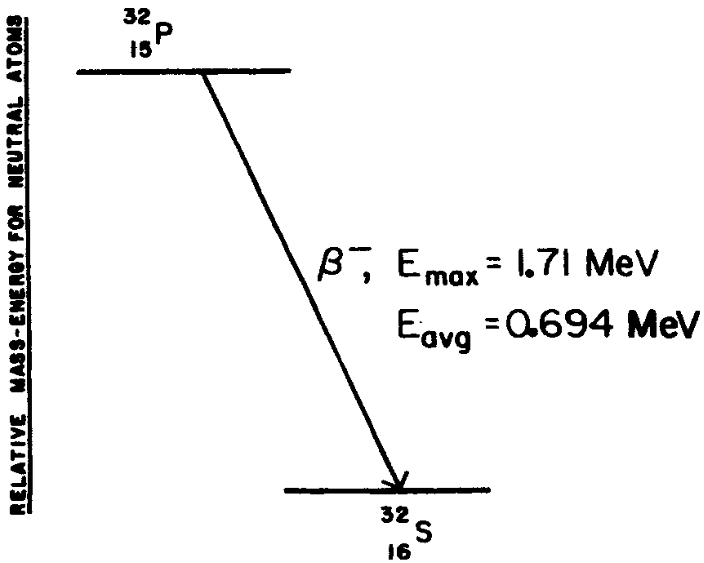
If we consider *n* disintegrations /g

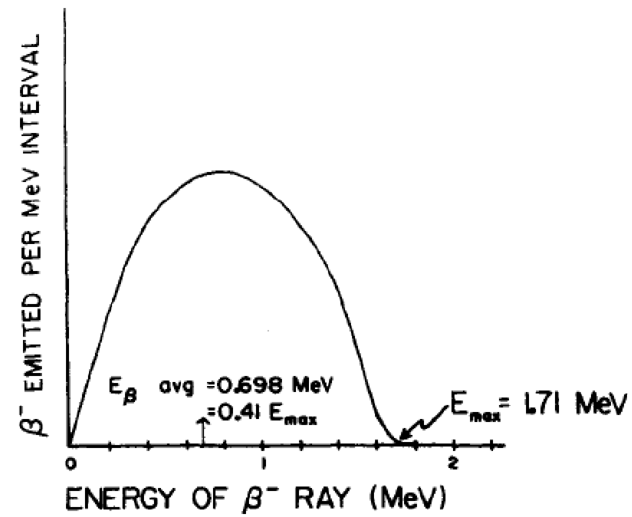
CPE: 4.77 *n* MeV/g

RE: 4.78 *n* MeV/g

β disintegration

$$^{32}_{15}\mathrm{P} + \mathrm{e}^{-} \rightarrow ^{32}_{16}\mathrm{S} + \beta^{+} + ^{0}_{0}\nu + 1.71\,\mathrm{MeV}$$





$E_{\beta} = E_{avg} = 0.694$ MeV ≈ 1/3 E_{max}

If we consider *n* disintegrations /g → CPE: 0.694 *n* MeV/g

Application: absorbed dose in a thin foil for charged particles

- For a beam of charged particles with energy E_{in} and fluence Φ incident $\perp$ on a material with atomic number Z, with density ρ, thin enough (thickness *l*) to have:
  1. $S_{elec} \approx$ constant and depends on E_{in}
  2. Every particle passes straight through the foil → scattering is negligible
  3. The kinetic *E* carried out of the foil by the δ e^- is negligible (the foil is thick compared to the range of the δ e^- or the film is « sandwiched » between two foils of the same Z → CPE)
- Backscattering of particles is negligible
  - For ions → just
  - For e^- → same number of backscattered in the front or at the back of the foil → equilibrium

Dose in a thin foil charged particles

- For ions (heavy) → 3 assumptions are reasonable
- For e^- (light) → assumption 2. is the weakest → requires sometimes corrections
- The loss *E* ΔE is given by (MeV/m^2) →

$$\Delta E = \Phi \left(\frac{dE}{\rho dx} \right)_{elec} \rho l$$

  with $(dE/\rho dx)_{elec}$, the mass electronic stopping power of the medium evaluated at E_{in}

- With assumption 3. → the *E* lost in the foil is the imparted *E* →

$$D = \frac{\Delta E}{\rho l} = \Phi \left(\frac{dE}{\rho dx} \right)_{elec}$$

Comments on this result

- *D* independent on *l* → to tilt the foil does not alter the dose
- If assumption 3. is no more satisfied (too thin isolated foil) → δ e^- escape and carry out their *E* → the equation for *D* becomes →

$$D = \Phi \left(\frac{dE}{\rho dx} \right)_{\Delta}$$

- With Δ chosen to be the *E* of those escaping δ e^-
- Very difficult to have a foil *really* isolated

Dose in a thick target for heavy ions

- Use of the database for the CSDA range → Knowing E_{in} and the thickness *l* of the medium → we have E_{exit} →

$$\Delta T = E_{in} - E_{exit}$$

- The imparted energy (MeV/m²) →

$$\Delta E = \Phi \Delta T$$

- The dose is →

$$D = \frac{\Delta E}{\rho l} = \frac{\Phi \Delta T}{\rho l}$$

Dose in a thick target for electrons

- We assume the target > maximal projected range of the e^-
- Complication due to Bremsstrahlung → definition of the radiation field $Y(E_{in})$ for an e^- with kinetic energy E_{in} : fraction of E_{in} lost during radiative collisions along the travel of the e^- →

$$\Delta E = \Phi E_{in}[1 - Y(E_{in})]$$

- The dose is →

$$D = \frac{\Delta E}{\rho l} = \frac{\Phi E_{in}[1 - Y(E_{in})]}{\rho l}$$