# A Review on AMG: From Academia to Industry


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The development of algebraic multigrid (AMG) started over 30 years ago, driven by the attempt to automate and generalize geometric multigrid for the efficient solution of elliptic partial differential equations. The original AMG approach was effectively restricted to particular classes of problems, an important one being the class of linear algebraic systems with matrices close to rowsum zero M-matrices. In such cases, the original AMG is very mature and can handle large linear systems much more efficiently than any one-level method. While geometric multigrid solvers, when available, are generally still faster than their algebraic counterpart, the strengths of AMG-based solvers are its robustness and ease of use, its applicability in complex geometric situations with unstructured grids, and its capability to even solve certain (non-PDE) problems which are beyond the reach of geometric multigrid.

In spite of its potential, it took until around 1995 before there was a remarkable increase of interest in AMG, essentially caused by two facts: First, on the scientific side, the increasing geometrical complexity of applications, discretized on large and unstructured grids, technically limited the immediate use of geometric multigrid. Here AMG-based techniques appeared to be a promising alternative to tackle such problems. Second, in industrial simulation, models have been rapidly growing in geometric complexity, heterogeneity and size, causing the computational time required to solve linear systems of equations to become the major bottleneck. The classical one-level solvers used in industrial software packages seriously limited the practicability of numerical simulation. The potential of AMG-based solvers – their numerical efficiency, robustness and scalability – together with their ease-of-use as "plug-in" solvers have caused a growing industrial interest in such solvers.

Fostered by this situation, R&D on AMG-based and related methods has become a significant part of the general R&D on multigrid. Various extensions of the original AMG approach have been introduced, aiming at increasing its range of applicability. Several other possibilities to generalize AMG have been investigated; research on various new and related approaches has started and is still ongoing today. Most important from a practical point of view, substantial progress has been achieved regarding the efficient treatment of coupled systems of PDEs.

In this presentation we will give a review on the history of AMG in general and its impact on various branches of industrial simulation in particular. In contrast to geometric multigrid methods, AMG methods have found their way into many industrial and commercial simulation programs. We will summarize on our experiences and difficulties to bridge the gap between academia and industry.