A new multigrid strategy for Stokes problems

Yvan Notay

Métrologie Nucléaire,
Université Libre de Bruxelles,
50 av. F.D. Roosevelt,
1050 Bruxelles, Belgium
ynotay@ulb.ac.be

Standard discretizations of Stokes problems lead to linear systems of equations in saddle point form:

$$\begin{pmatrix} A & B^T \\ B & -C \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \end{pmatrix} ,$$

where the matrix block C is either zero in case of a stable discretization, or a small stabilization term. Due to this possible zero block, the direct application of algebraic multigrid methods is notoriously difficult.

In this talk, we propose a new approach to overcome this difficulty. It consist in first transforming the above system by pre- and post-multiplication with simple (and algebraic) sparse block triangular matrices; doing thus a form of pre-conditioning in the literal sense, designed to make sure that the transformed matrix is "well adapted" to multigrid.

More precisely, after transformation, all the diagonal blocks are symmetric positive definite, and resemble or correspond to a discrete Laplace operator. The idea is then to associate to each block a prolongation that works well for it, and to combine these to obtain a global prolongation. Observe that this can be achieved with virtually any algebraic or even geometric multigrid method.

Finally, nothing more is needed: for damped Jacobi-smoothing, uniform two-grid convergence can be guaranteed for the global system under the sole assumption that the two-grid schemes for the different diagonal blocks are themselves uniformly convergent – a requirement easy to meet given that these blocks are discrete Laplace-like matrices.

The approach will be illustrated by a few examples, showing further that time-dependent problems and variable viscosity can be handled in a natural way, without requiring a particular treatment.