Symbol-based multigrid methods for isogeometric analysis

Hendrik Speleers

Department of Mathematics,
University of Rome Tor Vergata,
Via della Ricerca Scientifica - 00133 Roma,
Italy
speleers@mat.uniroma2.it
joint work with M. Donatelli, C. Garoni, C. Manni and S. Serra-Capizzano

We consider the stiffness matrices coming from the Galerkin B-spline Isogeometric Analysis approximation of classical elliptic problems. By exploiting specific spectral properties compactly described by a symbol, we design efficient multigrid methods for the fast solution of the related linear systems. Despite the theoretical optimality, the convergence rate of the two-grid methods with classical stationary smoothers worsens exponentially when the spline degree increases. With the aid of the symbol we provide a theoretical interpretation of this exponential worsening. Moreover, by a proper factorization of the symbol we provide a preconditioned conjugate gradient "smoother", in the spirit of the multi-iterative strategy, that allows us to obtain a good convergence rate independent both of the matrix size and of the spline degree. A numerical experimentation confirms the effectiveness of our proposal and the numerical optimality with a uniformly high convergence rate, also for the V-cycle multigrid method and large spline degrees.