Block H-LU preconditioners for higher-order FEM

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The finite element discretization of partial differential equations requires the selection of suitable finite element spaces. While higher order finite elements lead to solutions of higher accuracy, their associated discrete linear systems of equations are often more difficult to solve than those of lower order elements.

Here, we present efficient preconditioners for these types of linear systems of equations. More specifically, we will use hierarchical ($\mathcal{H}$-) matrices to build block $\mathcal{H}$-LU preconditioners. $\mathcal{H}$-matrices provide a powerful technique to compute and store approximations to dense matrices in a data-sparse format. The basic idea is the approximation of matrix data in hierarchically structured subblocks by low rank representations. The preconditioners will be of a "hybrid blackbox" nature: The setup of the preconditioner will occur in a "blackbox" fashion, i.e., only the stiffness matrix is needed as input. However, the "hybrid" part implies that certain knowledge of the origin of the system is available and will possibly be exploited. Such knowledge could include a certain sparsity structure (e.g. produced through particular types of finite elements) or even a certain block structure (e.g. in mixed finite elements). We conclude with numerical results.