Condition number estimates for higher order NURBS discretizations

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The discretization matrix A gets denser by increasing the polynomial degree p. Therefore, the cost for solving large problems becomes prohibitively expensive. The most practical way to solve them is to resort to a iterative method. Since the convergence rate of such methods ,e.g. multigrid methods, is strongly affected by the condition number of the system matrix A, it is important to assess this quantity as a function of the mesh size h for the h-refinement, or as a function of the degree p for the p-refinement.

In this talk, we will derive bounds for the minimum and maximum eigenvalues and the spectral condition number of matrices for higher order NURBS discretizations of elliptic partial differential equations in an open, bounded, simply connected Lipschitz domain $\Omega \subset \mathbb{R}^d$, $d \in \{2, 3\}$. We consider refinements based on mesh size h and polynomial degree p with maximum regularity of spline basis functions. For the h-refinement, the condition number of the stiffness matrix is bounded above by a constant times h^{-2} and the condition number of the mass matrix is uniformly bounded. For the p-refinement, the condition number grows exponentially and is bounded above by $p^{2d+2}4^{pd}$ and $p^{2d}4^{pd}$ for the stiffness and mass matrices, respectively. Rigorous theoretical proofs of these estimates will be provided and supporting numerical results.