# Multigrid Method for Systems of Nonlinear Equations arising from Poroelasticity Problem


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Shale gas is natural gas which is formed by being trapped within shale layer formations. Shale layers have typically very low permeability which dramatically reduces the mobility of this so-called unconventional gas. Hydraulic fracturing has been regarded as one of key methods of extracting these gas resources. It is a process in which the energy from the injection of a highly pressurized fluid creates fractures within the rock.

The coupled seepage and stress process in saturated geological media can be interpreted by means of Biot's theory of consolidation. It describes the time-dependent interaction between the deformation of porous material and the fluid flow pressure inside of it. Our 2D problem can be formulated as a system of partial differential equations for the unknowns displacements $u$,$v$ and pore pressure of the fluid $p$. The governing equations are given by:

$$\begin{cases} -(\lambda+2\mu)u_{xx} - \mu u_{yy} - (\lambda+\mu)v_{xy} + \alpha p_x = f_1, \\ -(\lambda+\mu)u_{xy} - \mu v_{xx} - (\lambda+2\mu)v_{yy} + \alpha p_y = f_2, \\ \frac{1}{Q}p_t + (u_x+v_y)_t - k(\sigma,p)(p_{xx}+p_{yy}) = f_3 \end{cases}$$

Here $\lambda$ and $\mu$ are Lame coefficients. $k$ is the coefficient of permeability which depends on the stress and fluid pressure, resulting in a nonlinear set of equation.

Numerical approximation is necessary to solve this problem. As it is a nonlinear system of equations, we would like to use a nonlinear multigrid method, such as FAS and Newton-multigrid, as an iterative solution method for the discretized partial differential equations. It is the challenge to determine suitable multigrid components. We would like to discrete equations on collocated grids. However, such discretization may be unstable, because some oscillations may appear in the first time steps of numerical solution. After this phase, the solution becomes smoother and these oscillations tend to disappear. We need to take some special care in order to construct a stable discretization for the whole process. This can be achieved by adding an artificial elliptic pressure term to the seepage equation. The artificial term is $\varepsilon\frac{\partial \triangle p}{\partial t}$, with $\varepsilon = \frac{h^2}{4(\lambda+2\mu)}$. When the grid size $h \to 0$, the artificial pressure term tends to 0. Since this term is proportional to $h^2$, second order accuracy can also be maintained. With respect to the smoother, we choose the so-called box relaxation which solves the discrete equations locally cell by cell. In practice, this means that five unknowns centered around a pressure point are relaxed simultaneously. So in one smoothing iteration all displacement unknowns are updated twice, whereas pressure unknowns are updated once. Numerical experiments will show the good convergence of multigrid and also the simulation of crack formation.