Local Fourier Analysis of Pattern Structured Operators

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Multigrid methods [?] are used to compute the solution u of the system of equations

$$Lu = f\ ,$$

where L is typically a discretization of a partial different equations (PDE) and f a corresponding, given right hand side. Local Fourier Analysis (LFA) [?, ?, ?] is well known to provide quantitative estimates for the speed of convergence of multigrid methods, by analyzing the involved operators in the frequency domain.

For the initial formulation of LFA [?] it was crucial to assume that all involved operators have constant coefficients. For many PDE operators the coefficients vary continuously in space. Thus if the grid is fine enough the discrete operator L will only vary slightly between neighboring grid points and hence can be well approximated by an operator with *locally* constant coefficients. Thus constant coefficient are often reasonable assumption.

However, when analyzing more complex problems or even the multigrid method as a whole this assumption is too restrictive. Interpolation and restriction operators typically act differently on variables that have a coarse grid representative and those who do not have one. Another example are patter relaxation schemes like the Red-Black Gauß-Seidel method where red points of the grid are treated differently from the black ones.

It is possible to analyze these cases [?, ?] when allowing for interaction of certain frequencies (see also [?, ?]). Even more, it turns out that when we allow for more frequencies to interact we can analyze operators given by increasingly complex patterns. In our talk we will illustrate a general framework for analyzing pattern structured operators, i.e., operators whose action is invariant under certain shifts of the input function. Furthermore, we discuss different applications.

References

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