Efficient solvers for coupled problems in geophysics

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Many problems in geophysics comprise interactions of processes, and are typically formulated as a system of coupled PDEs. In most cases this systems are transient and often also non-linear. Hence, developing efficient solvers is often a delicate task and must include combining suitable schemes for (i) time integration, (ii) linearization, and (iii) geometric and algebraic multigrid solvers. In this presentation we take an application oriented approach and focus on the problem classes of density-driven-flow and poroelasticity problems. For these two examples, we first comment on similarities and differences, and then provide details on their respective solution strategy:

For density-driven-flow problems, the fluid flow is density dependent and coupled non-linearly to a second quantity such as a substance concentration or heat, which is transported as well. For this problem class we investigate different non-linear solvers and decoupling strategies. In benchmark computations, an iterative coupling (single step nonlinear Gauss-Seidel) outperforms a partial Newton method (single step non-linear Jacobi). This effect is less pronounced for the iterative coupling, and mitigates after the first simulation phase when the velocity profile has stabilized. In the latter case corrections are computed independently, thus, this scheme shows to be inappropriate for this highly nonlinear problem class. The fully coupled Newton method, requiring a monolithic linear solver, however, proofs being superior to both approaches.

The poroelasticity problems considered in the second part of the talk are linear and are known to be decoupled efficiently by iterative coupling strategies. Since the mechanics sub-system can be considered as time-independent constraint, this the problem perfectly suited for (algebraic) multigrid solvers. We provide scalability results and also comment on the monolithic (fully coupled) solution.