A multigrid preconditioner for the Hellan-Herrmann-Johnson mixed method for biharmonic problems

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In this talk we consider the biharmonic Dirichlet problem

$$\Delta^2 y = f \quad \text{in } \Omega, \qquad u = \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega$$

on a polygonal two-dimensional domain Ω with boundary $\partial\Omega$. The Hellan-Herrmann-Johnson mixed method uses the Hessian $\mathbf{u} = \Delta^2 y$ as auxiliary variable. The well-posedness of the associated continuous mixed variational problem for $(\mathbf{u}, y)$ is shown in a nonstandard Sobolev space. Motivated by this analysis of the continuous problem a similar result is derived for the discretized problem. The resulting preconditioner is of optimal efficiency and is solely based on standard multigrid methods for second-order elliptic problems.

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