HYMLS: A Multilevel ILU approach for coupled fluid and transport equations

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Many flow problems deal with transport of matter and/or heat. This constitutes a challenging multiphysics problem if the transported entity also influences the flow. From a computing efficiency view point, it is best to treat the associated equations in a coupled manner [5]. If one employs a domain decomposition approach, all the unknowns related to one domain should be in the memory of the node which treats that part. Moreover, communication should be avoided as much as possible during the construction of the right-hand side, the construction of the Jacobian matrix and the solution process. Along this line we developed a finite volume package FVM and a solver HYMLS, both based on elements of the EPETRA-package (available within Trilinos (see http://trilinos.sandia.gov/)).

HYMLS is a linear system solver for steady state incompressible Navier-Stokes equations coupled to transport equations in 2 and 3D [1,2,3]. Recently, we constructed a multilevel variant of it, which makes it possible to solve 3D problems of over 10 million unknowns quickly on a parallel computer. The behavior of the method is very much like that of multigrid methods. The solver is very robust. For the problem described in [4], it allowed a quick increase in the Reynolds number to get into the interesting region around Re=2000. Here we will show the performance of the method on the Rayleigh-Bénard convection in a cube, with six no-slip walls [6].

To study the stability of the solutions we determine the eigenvalues using the ANASAZI-package, which contains a generalized version of the Arnoldi method. Also here we employ HYMLS to solve the linear systems that result from a Cayley transform of the generalized eigenvalue problem. In the talk we will give a more detailed explanation of the used algorithms and their performance.

References

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