Multigrid Method for Solving Elliptic Monge-Ampere Equation Arising from Image Registration

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The Monge-Ampère equation is a nonlinear second order partial differential equation, which arises in differential geometry and other applications. In image registration, the problem is to transform one image to align with another image. One approach is based on the Monge-Kantorovich mass transfer problem. The goal is to find the optimal mapping M which minimizes the Kantorovich-Wasserstein distance. The optimal mapping can be written as $M = \nabla\psi$, where ψ satisfies the following Monge-Ampère equation

$$det(D^2\psi(x)) = \frac{I_1(x)}{I_2(\nabla\psi)},$$

where I_1 and I_2 are the given images. Here $det(D^2\psi(x))$ denotes the determinant of the Hessian of ψ. In this talk, we will present a multigrid method for solving the Monge-Ampère equation. Our approach is to reformulate the Monge-Ampère equation as a Hamilton-Jacobi-Bellman (HJB) equation. We will develop a monotone discretization scheme such that it will converge to a viscosity solution. We will then present a relaxation scheme which is a very slowly convergent method as a standalone solver but it is very effective for reducing high frequency errors. We will adopt it as a smoother for multigrid and demonstrate its smoothing properties. Finally, numerical results will be presented to illustrate the effectiveness of the method.