Theoretical Advances in non-Galerkin Algebraic Multigrid

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Algebraic multigrid (AMG) is a popular and effective solver for systems of linear equations that arise from discretized partial differential equations. While AMG has been effectively implemented on large scale parallel machines, challenges remain, especially when moving to exascale. In particular, stencil sizes (the number of nonzeros in a row) tend to increase further down in the coarse grid hierarchy and this growth leads to more communication. Thus, as problem size increases and the number of levels in the hierarchy grows, the overall efficiency of the parallel AMG method decreases, sometimes dramatically. This growth in stencil size is due to the standard Galerkin coarse grid operator, P^TAP, where P is the prolongation (i.e., interpolation) operator. For example, the coarse grid stencil size for a simple 3D 7-point finite differences approximation to diffusion can increase into the thousands on present day machines, causing an associated increase in communication costs. Previous work by the authors has successfully truncated coarse grid stencils in an algebraic fashion. First, the sparsity pattern of the non-Galerkin coarse grid is determined by employing a heuristic minimal "safe" pattern together with strength-of-connection ideas. Second, the nonzero entries are determined by collapsing the stencils in the Galerkin operator. The purpose of this talk is to provide some theoretical foundation for the method. In particular if the original two-grid Galerkin method is optimal, then the two-grid non-Galerkin method is shown to also be optimal. The impact of the theory on the algorithm, together with supporting serial and parallel results will also be given.