Subdivision surfaces refinement for generating multigrid hierarchies with application in neuroscientific numerical simulations

Martin Stepniewski

Goethe-Center for Scientific Computing,
Goethe University Frankfurt am Main,
Kettenhofweg 139,
60325 Frankfurt am Main, Germany
martin.stepniewski@gcsc.uni-frankfurt.de
joint work with Gillian Queisser

An important goal in neuroscience is understanding how networks of neurons are processing information. Chemical synapses define interfaces where signals are exchanged between neurons and other (neural) cells and thus play an essential role in this context. To pursue this goal we are developing detailed mathematical models of synaptic processes. Amongst others the models comprise the description of three-dimensional reaction-diffusion dynamics with non-linear (inner) boundary conditions. This leads to systems of coupled partial differential equations, which have to be discretized in space and time. Multigrid methods are a highly efficient way of finally solving the resulting and in practice vast systems of linear equations.

However, realistic neurobiological applications of numerical simulation occur on arbitrarily complex domains including neuron networks, single neurons, cell structures like axons, dendrites, dendritic spines or cell organelles facing the challenge of unstructured computational grids with severe anisotropies, invaginations, nestings and branches. To meet this challenge accurate and robust refinement techniques are essential.

We present a modified multigrid method with a new refinement strategy based on grid hierarchies which are generated by using Loop's smooth subdivision surface refinement [1] of the boundary and ordninary linear refinement of the inner grid in combination with a Laplacian smoothing [2]. Starting with a triangulated surface geometry, which approximates the boundary of the corresponding neurobiological domain, Loop's refinement scheme defines a smooth subdivision surface as limit of successive refinements and vertex repositionings by distinct position masks. The vertices of the initial surface geometry are first projected onto their position on the subdivision surface and then a constrained Delaunay tetrahedrization [3] is generated as coarse grid. The multigrid hierarchy is now created by linear refinement operations projecting the boundary vertices of each refinement level onto their final position on the subdivision surface resulting in a particularly smooth approximation of the computational domain. To prevent degenerate volume elements to emerge especially in the vincinity of the boundary an optimization-based Laplacian smoothing is used to reposition inner vertices of each refinement level.

References

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