Finite element multigrid framework for mimetic finite difference discretizations

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We are interested in the efficient multigrid solution of the algebraic systems of equations resulting from the mimetic finite difference (MFD) schemes for elliptic partial differential equations. Such discretizations work on general unstructured and irregular grids not necessarily aligned with coordinate axes. Moreover, the mimetic finite differences result in discrete grid operators which satisfy the compatibility conditions (exact sequence properties) connecting grad, div and curl operators on the continuous level.

We show how such MFD schemes can be derived using standard finite element spaces in H(curl). In this way, using the finite element framework, we are able to analyze the convergence of the MFD discretizations and design multigrid methods for the solution of the resulting linear systems. We propose, and, via the local Fourier analysis (LFA) framework we also analyze geometric multigrid algorithms for such problems. Finally, we present several numerical tests which demonstrate the efficiency of the proposed multigrid methods and the sharpness of the LFA estimates of the convergence rate.