A robust structured incomplete Cholesky preconditioner

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We consider a new algebraic factorization preconditioner for the iterative solution of large sparse symmetric positive definite linear systems. The preconditioner is based on a sparse variant of Cholesky factorization in which the off-diagonal part of the block rows of the factor is approximated by low-rank matrices. We use low-rank approximations that satisfy a specific orthogonality condition: the approximation is orthogonal to the corresponding approximation error. The resulting factorization is then shown breakdown-free, and further, the corresponding condition number is bounded as a function of the accuracy of individual approximations.

On the practical side, the preconditioner exploits, in an algebraic manner, the low rank structure available in PDE applications. This is achieved through the reordering of unknowns which is based on the sparsity pattern of the system matrix, and which preserves the sparsity patter of the resulting factor. A preliminary implementation of the method is presented and compared with similar Cholesky and incomplete Cholesky factorizations based on dropping of individual entries.