Structure Preserving Algebraic Multigrid

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We develop an algebraic multigrid method for solving linear systems of equation with non-Hermitian matrices that possess a simple symmetrizing operator, e.g., Saddle-point problems, Hamiltonian matrices. In particular we develop a method for the Wilson discretization of the 2-dimensional Dirac equation. The proposed approach uses a bootstrap setup algorithm based on a multigrid eigensolver. It computes test vectors which define the least squares interpolation operators by working mainly on coarse grids, leading to an efficient and integrated self learning process for defining algebraic multigrid interpolation.

The algorithm is motivated by the γ_5-symmetry of the Dirac equation, which carries over to the Wilson discretization. This discrete γ_5-symmetry is used to reduce a general Petrov Galerkin bootstrap setup algorithm to a Galerkin method for the Hermitian and indefinite formulation of the Wilson matrix. Kaczmarz relaxation is used as the multigrid smoothing scheme in both the setup and solve phases of the resulting Galerkin algorithm. Extensive numerical results are presented to motivate the design and demonstrate the effectiveness of the proposed approach.