A Hybrid Multigrid Algorithm for Elliptic Problems using Adaptive Higher-Order Cut Cells

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We propose a hybrid geometric-algebraic multigrid approach for solving elliptic equations on domains with complex geometries. The discretization uses a novel higher-order ($p = 2, 4, 6$) *finite volume, cell average* cut cell representation to discretize the variable coefficient elliptic operator on a Cartesian mesh. The benefit of this is a regular stencil in most of the domain, thereby avoiding difficulties associated with algebraic coarsening of a global mesh. However, the challenge introduced with cut cells is two-fold: the resulting operators are typically not symmetric, and small cut cells can generate very large negative eigenvalues in the finite volume formulation. In addition, traditional geometric coarsening introduces very different representations of the cut cell operator on coarser meshes, and can produce topological constraints on the coarsest mesh, where we can apply an algebraic solver more effectively. We find that the hybrid approach realizes many of the benefits of geometric coarsening while retaining the robustness of algebraic multigrid. We demonstrate this approach for a number of examples of complex geometries in an adaptive mesh hierarchy implemented in the *Chombo* parallel framework. Our results show that we obtain both higher-order accuracy and near-optimal multigrid convergence rates, without being constrained by traditional aggregation methods or adjoint operators for restriction and prolongation.