MGOPT Methods for Optimization Problems Arising in Non-Newtonian Fluids Simulation

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This work is concerned with the application of multigrid methods to the numerical solution of the following class of optimization problems: find $y \in W_0^{1,\nu}(\Omega)$ such that

$$\min_{y \in W_0^{1,\nu}} J(y) := \frac{1}{\nu} \int_\Omega |\nabla y|^\nu \, dx + g \int_\Omega |\nabla y| \, dx - \int_\Omega f y \, dx,$$

where $g > 0$ and $f \in W^{-1,\nu'}(\Omega)$ is a given function. This kind of problems arise in the modelization of non-Newtonian fluids e.g. the Herschel-Bulkley model . We propose a Huber regularization of the non-differentiable term in the functional J. Well posedness of the regularized problems is proved, and convergence of the regularized solutions to the solution of the original problem is verified. Further, we discuss the finite element discretization of the problem. Our main interest is to develop a multigrid algorithm to solve these kind of problems at big scale. Therefore, we propose a multigrid for optimization method (MGOPT) for the numerical solution of the discretized problem. This method is based on the well known full approximation storage (FAS) scheme. As an important feature of this work, we propose to use a preconditioned descent method combined with an innovative linesearch method as the smoothing process. Finally, several numerical experiments are carried out to show the efficiency of the approach. Particularly, we discuss the application of our algorithm to simulate the flow of non-Newtonian fluids.

References

[1] J.C. DE LOS REYES AND S. GONZÁLEZ ANDRADE, *Numerical simulation of two-dimensional Bingham fluid flow by semismooth Newton methods*, Journal of Computational and Applied Mathematics, 235 (2010) 11-32.

[2] R.R. HUILGOL AND Z. YOU, *Applications of the augmented Lagrangian method to steady pipe flows of Bingham, Casson and Herschel-Bulkley fluids*, Journal of Non-Newtonian Fluid Mechanics, 128 (2005) 126-143.

[3] S.G. Nash,*A multigrid approach to discretized optimization problems*, Optim. Methods Soft., 14 (2000) 99-116.

[4] M. Vallejos and A. Borzì,Multigrid optimization methods for linear and bilinear elliptic control problems, Computing, 82 (2008) 31-52.