BootCMatch: an alpha-AMG solver based on Compatible Weighted Matching

Pasqua D'Ambra

Institute for High-Performance Computing and Networking,
National Research Council of Italy,
Via Pietro Castellino, 111,
I-80131 Napoli, Italy
pasqua.dambra@cnr.it
joint work with Panayot S. Vassilevski

In [2] we introduced a new adaptive Algebraic Multigrid (αAMG) method to solve symmetric positive definite (s.p.d.) systems of linear equations without exploiting any a-priori knowledge or assumptions on characteristics of the algebraic smoothness (or near null components of the system). Our method relies on a bootstrap strategy aimed to compute a sequence of AMG hierarchies composed in a multiplicative way. The goal is to obtain a composite solver with a desired convergence rate. Starting from a general (random) given vector, at each step of the bootstrap procedure, a new algebraically smooth vector related to the current composite solver is computed. Each successive hierarchy is built by using pairwise aggregation of unknowns driven by a weighted matching algorithm with weights depending on the most recently computed algebraically smooth vector utilizing a notion of the compatible relaxation [1]. Matching algorithms in a matrix graph were successfully exploited in reordering schemes designed to enhance matrix diagonal dominance in sparse direct methods [3]; we apply linear-time complexity approximate weighted matching in a graph [4] to form aggregates of unknowns and build the coarse-vector space by simple piecewise constant interpolation of the current algebraically smooth vector. This coarsening process, which we referred to as *compatible weighted matching*, is completely automatic and algebraic, and it replaces the commonly used characterization of strength of connections in both the coarse space selection and in the interpolation scheme.

In the present work, we describe an extension of the method, including aggressive coarsening obtained by combining multiple sweeps of the pairwise aggregation procedure, as well as utilizing more accurate interpolation operators obtained by weighted-Jacobi smoothing of the piecewise constant interpolation operators. This leads to smoothed aggregation type adaptive AMG (or SA-αAMG) method, which exhibits improved convergence properties and reduced building setup cost.

We present some main features of the *BootCMatch: αAMG based on Bootstrap Compatible Matching*, a C-language implementation of the method, and discuss performance results on a suite of symmetric positive-definite linear systems arising from discretization of elliptic PDEs as well as on problems from the University of Florida Sparse Matrix Collection.

References

[1] A. Brandt, *General Highly Accurate Algebraic Coarsening*, Electronic Transactions on Numerical Analysis, Vol. 10, pp. 1-20, 2000.

[2] P. D'Ambra, P. S. Vassilevski, *Adaptive AMG with Coarsening based on Compatible Weighted Matching* , to appear in Computing and Visualization in Science.

[3] I. S. Duff, J. Koster, *On Algorithms for Permuting Large Entries to the Diagonal of a Sparse Matrix*, SIAM J. on Matrix Analysis and Applications, Vol. 22, pp. 973-996, 2001.

[4] R. Preis, *Linear Time 1/2-Approximation Algorithm for Maximum Weighted Matching in General Graphs*, in STACS'99, LNCS, Springer-Verlag, Vol. 1563, pp. 259-269, 1999.