H2-matrix preconditioners

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Generalised regularity results imply that the solution operators of strongly elliptic PDEs are rank-structured, i.e., that the interaction between well-separated subdomains can be approximated by low-rank operators. This holds true even for differential operators with discontinuous and anisotropic coefficients. The $\mathcal{H}^2$-matrix representation takes advantage of this property in order to find efficient preconditioners or solve eigenvalue problems.

In this talk, we consider new algorithms for constructing $\mathcal{H}^2$-matrix approximations of products, inverses and factorisations of the stiffness matrices corresponding to FEM and BEM problems. The algorithms rely on recursion and two fundamental algebraic operations: the simultaneous multiplication of an $\mathcal{H}^2$-matrix by k vectors and the update of an $\mathcal{H}^2$-matrix by a matrix of rank k. Both operations can be performed in linear complexity and give rise to higher-level algorithms of complexity $\mathcal{O}(nk^2 \log n)$.

In particular, we can construct efficient preconditioners for FEM and BEM problems in $\mathcal{O}(n \log n)$ operations requiring $\mathcal{O}(n)$ units of storage, and we can perform a step of the "slicing the spectrum" method for approximating arbitrary eigenvalues of PDEs in $\mathcal{O}(n \log n)$ operations.